MATHEMATICS

IIT FOUNDATION

CLASS 12

For JEE Main & JEE Advanced

PYQs & Practice with Solutions

(12 Chapters — 2,080 Questions with Solutions, including 611 PYQs)

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ISBN: 978-81-180191-5-2

Published by Om Muruga Publication





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IIT Foundation Mathematics Class 12 — for JEE Main & JEE Advanced

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Level tags on every question: [Easy] direct application (board level and easier JEE Main) · [Medium] standard JEE Main level · [Hard] tougher JEE Main / JEE Advanced level.

Previous Year Questions are from JEE Main 2019–2026, JEE Advanced, AIEEE (2002–2012) and IIT-JEE (1979–2012); each carries its exam, year and shift.

CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

CBSE XII: Unit I – Inverse Trigonometric Functions | TN XII: Unit 4 – Inverse Trigonometric Functions | JEE Main 2027: Unit 3 (Trigonometry)

A. HIGHLIGHTS

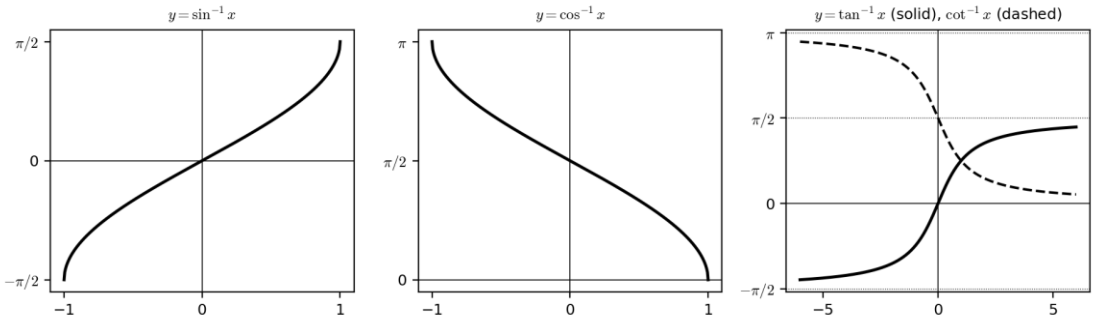
1. Why Principal Values?

The trigonometric functions are periodic and hence many-one, so they have no inverse on their whole domain. Restricting each to an interval on which it is one-one and takes all its values (its **principal value branch**) makes it a bijection, and the inverse is defined there. For example, $\sin:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to[-1,1]$ is bijective, and its inverse is $\sin^{-1}$ (also written $\arcsin$).

> $\sin^{-1}x$ is NOT $(\sin x)^{-1} = 1/\sin x$. It is the angle, in the principal range, whose sine is x.

2. Domains and Principal Ranges

Function	Domain	Principal range
$\sin^{-1}x$	[−1, 1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1}x$	[−1, 1]	$[0,\pi]$
$\tan^{-1}x$	$\mathbb{R}$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$\cot^{-1}x$	$\mathbb{R}$	$(0,\pi)$
$\sec^{-1}x$	$\mathbb{R}-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$\text{cosec}^{-1}x$	$\mathbb{R}-(-1,1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$



Graphs of the principal value branches

The graph of $y=f^{-1}(x)$ is the mirror image of the graph of the restricted $y=f(x)$ in the line $y=x$.

Illustration. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$.

Solution: $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ (in $[0,\pi]$) and $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$. Sum = $\frac{\pi}{2}$.

3. Composition with the Original Function

$\sin(\sin^{-1}x) = x$ for $x \in [-1,1]$, and similarly for the others on their domains. But $\sin^{-1}(\sin x) = x$ **only** for $x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$; outside, bring the angle into the principal range first.

Expression	**Value for x in**
$\sin^{-1}(\sin x)$	x in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$; $\pi - x$ in $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$; $x - 2\pi$ in $\left[\frac{3\pi}{2},\frac{5\pi}{2}\right]$
$\cos^{-1}(\cos x)$	x in $[0,\pi]$; $2\pi - x$ in $[\pi, 2\pi]$
$\tan^{-1}(\tan x)$	$x - k\pi$, choosing k so the value lies in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Illustration. Find $\sin^{-1}(\sin 5)$ and $\cos^{-1}(\cos 5)$.

Solution: 5 lies in $\left[\frac{3\pi}{2},\frac{5\pi}{2}\right]$, so $\sin^{-1}(\sin 5) = 5 - 2\pi$; 5 lies in $[\pi, 2\pi]$, so $\cos^{-1}(\cos 5) = 2\pi - 5$.

JEE EXTENSION – PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS: CBSE XII now examines only definitions, principal values and graphs; TN XII and JEE Main also use the properties below.

4. Properties (JEE Extension)

Property	**Result**
Negative arguments	$\sin^{-1}(-x) = -\sin^{-1}x$, $\tan^{-1}(-x) = -\tan^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $\cot^{-1}(-x) = \pi - \cot^{-1}x$
Reciprocal arguments	$\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$, $\sec^{-1}x = \cos^{-1}\frac{1}{x}$; $\cot^{-1}x = \tan^{-1}\frac{1}{x}$ $(x > 0)$
Complementary pairs	$\sin^{-1}x + \cos^{-1}x = \tan^{-1}x + \cot^{-1}x = \sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$
Sum of $\tan^{-1}$	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$ $(xy < 1)$; add π if $x, y > 0$ and $xy > 1$
Difference of $\tan^{-1}$	$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ $(xy > -1)$
Double angle	$2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$ (for suitable values of x)
Conversions	$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ $(0 \le x < 1)$

Telescoping sums: write each term as a difference: $\tan^{-1}\frac{b-a}{1+ab} = \tan^{-1}b - \tan^{-1}a$. For example $\tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}(n+1) - \tan^{-1}n$.

Illustration. Evaluate $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$.

Formula used: sum formula ($xy = \frac{1}{6} < 1$).

Solution: $\tan^{-1}\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \tan^{-1}1 = \frac{\pi}{4}$.

SYLLABUS NOTE: Derivatives and integrals involving inverse trigonometric functions are treated in Chapters 5 and 7.

B. FORMULA OVERVIEW

No.	Concept	Formula / Result
1	Principal ranges	$\sin^{-1}$: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$; $\cos^{-1}$: $[0,\pi]$; $\tan^{-1}$: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$; $\cot^{-1}$: $(0,\pi)$
2	Inverse then function	$\sin(\sin^{-1}x) = x$, $x \in [-1,1]$
3	Function then inverse	$\sin^{-1}(\sin x) = x$ only in the principal range
4	Negative argument	$\sin^{-1}(-x) = -\sin^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}x$
5	Complementary	$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
6	Complementary	$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$
7	Sum	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$, $xy < 1$
8	Difference	$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$
9	Double angle	$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$, $\lvert x\rvert < 1$
10	Telescoping	$\tan^{-1}\frac{1}{1+n+n^2}$ equals $\tan^{-1}(n+1) - \tan^{-1}n$

Sections C, D and E – Solved Questions, Previous Year Questions and Practice Questions – follow. [N] = Numerical-value question.

C. SOLVED IIT-LEVEL QUESTIONS

Level tags: **[Easy]** board level · **[Medium]** JEE Main level · **[Hard]** tougher JEE Main / JEE Advanced level.

Principal Values

Q1. [Medium] The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is:

(A) $\frac{4\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $-\frac{\pi}{6}$

Formula used: Principal range of $\sin^{-1}$: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Solution: $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ and $-\frac{\pi}{3}$ is in the range.

Ans: (C) $-\frac{\pi}{3}$

Q2. [Easy] The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is:

(A) $-\frac{\pi}{4}$ (B) $\frac{5\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Formula used: $\cos^{-1}(-x) = \pi - \cos^{-1}x$.

Solution: $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Ans: (D) $\frac{3\pi}{4}$

Q3. [Easy] The principal value of $\tan^{-1}(-1)$ is:

(A) $-\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{7\pi}{4}$

Formula used: Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Solution: $\tan\left(-\frac{\pi}{4}\right) = -1$.

Ans: (A) $-\frac{\pi}{4}$

Q4. [Easy] The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is:

(A) $-\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{6}$ (D) $\frac{7\pi}{6}$

Formula used: $\cot^{-1}(-x) = \pi - \cot^{-1}x$; range $(0, \pi)$.

Solution: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Ans: (B) $\frac{5\pi}{6}$

Q5. [Medium] The value of $\sec^{-1}2 + \text{cosec}^{-1}(-2)$ is:

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) 0

Formula used: $\sec^{-1}x = \cos^{-1}\frac{1}{x}$, $\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$.

Solution: $\cos^{-1}\frac{1}{2} + \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$.

Ans: (C) $\frac{\pi}{6}$

Q6. [Easy] The value of $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ is:

(A) 0 (B) $-\frac{\pi}{2}$ (C) π (D) $\frac{\pi}{2}$

Formula used: Principal values.

Solution: $\frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$.

Ans: (B) $-\frac{\pi}{2}$

Q7. [Medium] The range of $\cot^{-1}x$ (principal branch) is:

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $[0, \pi]$

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $(0, \pi)$

Formula used: Definition.

Solution: cot is one-one on $(0, \pi)$ and takes all real values there.

Ans: (D) $(0, \pi)$

Q8. [Hard] Considering the principal values of the inverse trigonometric functions, the value of the expression $\tan\left(2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) - 2\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)\right)$ is equal to: **[JEE Main 2026, 28 Jan Shift 2]**

(A) $\frac{16}{63}$ (B) $-\frac{16}{63}$ (C) $-\frac{33}{56}$ (D) $\frac{33}{56}$

Formula used: Convert both to $\tan^{-1}$: $\sin^{-1}\frac{2}{\sqrt{13}} = \tan^{-1}\frac{2}{3}$, $\cos^{-1}\frac{3}{\sqrt{10}} = \tan^{-1}\frac{1}{3}$.

Solution: $\tan(2A) = \frac{4/3}{1-4/9} = \frac{12}{5}$; $\tan(2B) = \frac{2/3}{1-1/9} = \frac{3}{4}$.

$\tan(2A - 2B) = \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{9}{5}} = \frac{33/20}{14/5} = \frac{33}{56}$.

Ans: (D) $\frac{33}{56}$

Q9. [Medium] If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to: **[JEE Main 2019, 09 Jan Shift 1]**

(A) 7π (B) 0 (C) π (D) 10

Formula used: Bring 10 into the principal range.

Solution: $\sin 10 = -\sin(10 - 3\pi)$ with $10 - 3\pi \approx 0.58$, so $x = 3\pi - 10$; $\cos 10 = \cos(4\pi - 10)$ with $4\pi - 10 \in [0, \pi]$, so $y = 4\pi - 10$.

$y - x = \pi$.

Ans: (C) π

Domain and Range

Q10. [Easy] The domain of $\sin^{-1}(2x - 1)$ is:

(A) [−1, 1] (B) [0, 1] (C) [−1, 0] (D) [0, 2]

Formula used: $-1 \le 2x - 1 \le 1$.

Solution: $0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$.

Ans: (B) [0, 1]

Q11. [Easy] The domain of $\cos^{-1}(x^2 - 1)$ is:

(A) [−1, 1] (B) $\left[-\sqrt{2}, \sqrt{2}\right]$ (C) [0, √2] (D) [−2, 2]

Formula used: $-1 \le x^2 - 1 \le 1$.

Solution: $0 \le x^2 \le 2$.

Ans: (B) $\left[-\sqrt{2}, \sqrt{2}\right]$

Q12. [Easy] The domain of $\sin^{-1}x + \cos^{-1}(x - 1)$ is:

(A) [−1, 1] (B) [0, 2] (C) [0, 1] (D) [−1, 2]

Formula used: Intersect both domains.

Solution: $x \in [-1,1]$ and $x - 1 \in [-1,1] \Rightarrow x \in [0,2]$; common: [0, 1].

Ans: (C) [0, 1]

Q13. [Hard] The range of $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is:

(A) $[0, \pi]$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

(C) $\left(0, \frac{\pi}{2}\right)$ (D) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Formula used: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ on [−1, 1].

Solution: $f(x) = \frac{\pi}{2} + \tan^{-1}x$, $x \in [-1,1]$; $\tan^{-1}x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

Ans: (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

Q14. [Hard] The maximum value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ is:

(A) $\frac{\pi^2}{8}$ (B) $\frac{5\pi^2}{4}$ (C) $\frac{\pi^2}{2}$ (D) π^2

Formula used: Put $t = \sin^{-1}x$, $\cos^{-1}x = \frac{\pi}{2} - t$.

Solution: $t^2 + \left(\frac{\pi}{2} - t\right)^2$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is greatest at $t = -\frac{\pi}{2}$: $\frac{\pi^2}{4} + \pi^2 = \frac{5\pi^2}{4}$.

Ans: (B) $\frac{5\pi^2}{4}$

Q15. [Hard] Let $[\cdot]$ denote the greatest integer function. If the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+2[x]}{3}\right)$ is $[\alpha, \beta]$, then $12(\alpha + \beta)$ is equal to: **[JEE Main 2026, 04 Apr Shift 1]**

(A) 6 (B) 8 (C) 9 (D) 4

Formula used: Need $-3 \le 4x + 2[x] \le 3$.

Solution: For $[x] = 0$ $(0 \le x < 1)$: $4x \le 3 \Rightarrow x \le \frac{3}{4}$. For $[x] = -1$ $(-1 \le x < 0)$: $4x - 2 \ge -3 \Rightarrow x \ge -\frac{1}{4}$. Other integer parts fail.

Domain $\left[-\frac{1}{4}, \frac{3}{4}\right]$: $12(\alpha + \beta) = 6$.

Ans: (A) 6

Q16. [Hard] If the domain of the function $f(x) = \sin^{-1}\left(\frac{1}{x^2-2x-2}\right)$ is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$, then $\alpha + \beta + \gamma + \delta$ is equal to: **[JEE Main 2026, 24 Jan Shift 2]**

(A) 2 (B) 3 (C) 4 (D) 5

Formula used: Need $|x^2 - 2x - 2| \ge 1$.

Solution: $x^2 - 2x - 2 \ge 1 \Rightarrow x \le -1$ or $x \ge 3$; $x^2 - 2x - 2 \le -1 \Rightarrow 1 - \sqrt{2} \le x \le 1 + \sqrt{2}$.

$\alpha = -1$, $\beta + \gamma = 2$, $\delta = 3$: sum 4.

Ans: (C) 4

Q17. [Hard] If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is (α, β], then $3\alpha + 10\beta$ is equal to: **[JEE Main 2024, 04 Apr Shift 1]**

(A) 95 (B) 97 (C) 98 (D) 100

Formula used: Intersect both conditions.

Solution: $\left|\frac{3x-22}{2x-19}\right| \le 1 \Rightarrow 3 \le x \le \frac{41}{5}$.

$\frac{(3x-5)(x-1)}{(x-5)(x+2)} > 0$ excludes [3,5]: domain (5, 41/5].

$3\alpha + 10\beta = 15 + 82 = 97$.

Ans: (B) 97

Q18. [Hard] If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is (α, β], then the value of $5\beta - 4\alpha$ is equal to: **[JEE Main 2024, 30 Jan Shift 2]**

(A) 12 (B) 10 (C) 11 (D) 9

Formula used: Intersect both conditions.

Solution: $\left|\frac{2x-1}{x+2}\right| \le 1 \Rightarrow -\frac{1}{3} \le x \le 3$; $\frac{2x+3}{(4x-3)(x+1)} > 0$ on this range needs $x > \frac{3}{4}$.

Domain (3/4, 3]: $15 - 3 = 12$.

Ans: (A) 12

Q19. [Hard] Let m and M respectively be the minimum and the maximum values of $f(x) = \sin^{-1}2x + \sin 2x + \cos^{-1}2x + \cos 2x$, $x \in \left[0, \frac{\pi}{8}\right]$. Then $m + M$ is equal to: **[JEE Main 2022, 30 Jun Shift 1]**

(A) $1 + \sqrt{2} + \pi$ (B) $\left(1 + \sqrt{2}\right)\pi$ (C) $\pi + \sqrt{2}$ (D) $1 + \pi$

Formula used: $\sin^{-1}2x + \cos^{-1}2x = \frac{\pi}{2}$.

Solution: $f = \frac{\pi}{2} + \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$; on $\left[0, \frac{\pi}{8}\right]$: min $\frac{\pi}{2} + 1$ (at 0), max $\frac{\pi}{2} + \sqrt{2}$ (at $\frac{\pi}{8}$).

$m + M = \pi + 1 + \sqrt{2}$.

Ans: (A) $1 + \sqrt{2} + \pi$

Q20. [Hard] The domain of the function $f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ is: **[JEE Main 2021, 31 Aug Shift 2]**

(A) $\left[0, \frac{1}{2}\right]$ (B) $\left[0, \frac{1}{4}\right]$

(C) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$ (D) $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$

Formula used: Intersect both conditions.

Solution: $\left|\frac{x-1}{x+1}\right|\le 1\Rightarrow x\ge 0$.

$\frac{3x^2+x-1}{(x-1)^2}\le 1\Rightarrow 2x^2+3x-2\le 0\Rightarrow x\le\frac{1}{2}$; at least $-1\Rightarrow 4x^2-x\ge 0\Rightarrow x\le 0$ or $x\ge\frac{1}{4}$.

Domain $\{0\}\cup\left[\frac{1}{4},\frac{1}{2}\right]$.

Ans: (C) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$

Q21. [Hard] The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is: **[IIT-JEE 1999]**

(A) zero (B) one (C) two (D) infinite

Formula used: Domain: $x(x+1)\ge 0$ and $x^2+x+1\le 1$.

Solution: Both together force $x^2+x=0$: $x=0$ or -1.

At both points the equation reads $0+\frac{\pi}{2}=\frac{\pi}{2}$ ✓: two solutions.

Ans: (C) two

Composition with Trigonometric Functions

Q22. [Easy] $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ equals:

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{6}$ (D) $-\frac{5\pi}{6}$

Formula used: $\cos^{-1}(\cos x)=2\pi-x$ for $x\in[\pi,2\pi]$.

Solution: $2\pi-\frac{7\pi}{6}=\frac{5\pi}{6}$.

Ans: (B) $\frac{5\pi}{6}$

Q23. [Medium] $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ equals:

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Formula used: $\pi-x$ for $x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$.

Solution: $\pi-\frac{2\pi}{3}=\frac{\pi}{3}$.

Ans: (A) $\frac{\pi}{3}$

Q24. [Medium] $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ equals:

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D) $-\frac{3\pi}{4}$

Formula used: Subtract π to reach $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Solution: $\frac{3\pi}{4}-\pi=-\frac{\pi}{4}$.

Ans: (C) $-\frac{\pi}{4}$

Q25. [Medium] $\sin^{-1}(\sin 10)$ equals:

(A) 10 (B) $10-3\pi$ (C) $3\pi-10$ (D) $10-2\pi$

Formula used: 10 lies in $\left[\frac{5\pi}{2},\frac{7\pi}{2}\right]$, where $\sin^{-1}(\sin x)=3\pi-x$.

Solution: $\frac{5\pi}{2}\approx 7.85<10<11.0\approx\frac{7\pi}{2}$; value $3\pi-10\approx-0.575$, which is in the principal range.

Ans: (C) $3\pi-10$

Q26. [Easy] $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ equals:

(A) $\frac{13\pi}{6}$ (B) $\frac{11\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $\frac{\pi}{6}$

Formula used: Reduce by 2π.

Solution: $\cos\frac{13\pi}{6}=\cos\frac{\pi}{6}$ and $\frac{\pi}{6}\in[0,\pi]$.

Ans: (D) $\frac{\pi}{6}$

Q27. [Medium] $\cos\left(\sin^{-1}\frac{3}{5}\right)$ equals:

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) $-\frac{4}{5}$

Formula used: Right triangle; angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ has $\cos\ge 0$.

Solution: $\sqrt{1-\frac{9}{25}}=\frac{4}{5}$.

Ans: (A) $\frac{4}{5}$

Q28. [Easy] $\tan\left(\cos^{-1}\frac{3}{5}\right)$ equals:

(A) $\frac{3}{4}$ (B) $\frac{4}{5}$ (C) $\frac{5}{3}$ (D) $\frac{4}{3}$

Formula used: Right triangle with adjacent 3, hypotenuse 5.

Solution: Opposite = 4: $\tan=\frac{4}{3}$.

Ans: (D) $\frac{4}{3}$

Q29. [Medium] $\sin(\cot^{-1}x)$, $x\in\mathbb{R}$, equals:

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}$

(C) $\frac{1}{x}$ (D) $\frac{1}{\sqrt{1+x^2}}$

Formula used: $\cot^{-1}x \in (0,\pi)$, so its sine is positive.

Solution: If $\cot\theta = x$, then $\text{cosec}^2\theta = 1 + x^2$ and $\sin\theta = \frac{1}{\sqrt{1+x^2}}$.

Ans: (D) $\frac{1}{\sqrt{1+x^2}}$

Q30. [Easy] $\sin(\cos^{-1}x)$, $x \in [-1,1]$, equals:

(A) $-\sqrt{1-x^2}$ (B) x (C) $\sqrt{1-x^2}$ (D) $\frac{1}{x}$

Formula used: $\cos^{-1}x \in [0,\pi]$, where $\sin \geq 0$.

Solution: $\sin\theta = \sqrt{1-\cos^2\theta}$.

Ans: (C) $\sqrt{1-x^2}$

Q31. [Easy] $\cos\left(2\sin^{-1}\frac{1}{3}\right)$ equals:

(A) $\frac{7}{9}$ (B) $\frac{2}{9}$ (C) $\frac{8}{9}$ (D) $\frac{1}{9}$

Formula used: $\cos 2\theta = 1 - 2\sin^2\theta$.

Solution: $1 - \frac{2}{9} = \frac{7}{9}$.

Ans: (A) $\frac{7}{9}$

Q32. [Medium] $\tan\left(2\tan^{-1}\frac{1}{5}\right)$ equals:

(A) $\frac{2}{5}$ (B) $\frac{5}{12}$ (C) $\frac{12}{5}$ (D) $\frac{10}{23}$

Formula used: $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$.

Solution: $\frac{2/5}{1-1/25} = \frac{2/5}{24/25} = \frac{5}{12}$.

Ans: (B) $\frac{5}{12}$

Q33. [Medium] $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ equals:

(A) $\frac{3-\sqrt{5}}{2}$ (B) $\frac{3+\sqrt{5}}{2}$

(C) $\frac{\sqrt{5}}{3}$ (D) $\frac{2}{3}$

Formula used: $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$.

Solution: $\cos\theta = \frac{\sqrt{5}}{3}$, $\sin\theta = \frac{2}{3}$; $\frac{1-\sqrt{5}/3}{2/3} = \frac{3-\sqrt{5}}{2}$.

Ans: (A) $\frac{3-\sqrt{5}}{2}$

Q34. [Easy] $\cos\left(\tan^{-1}\frac{3}{4}\right)$ equals:

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$

Formula used: Right triangle with sides 3, 4, 5.

Solution: Adjacent 4, hypotenuse 5.

Ans: (A) $\frac{4}{5}$

Q35. [Medium] $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$ is equal to: **[JEE Main 2025, 28 Jan Shift 1]**

(A) 1 (B) 0 (C) $\frac{32}{65}$ (D) $\frac{33}{65}$

Formula used: $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{56}{65}$.

Solution: $\sin^{-1}\frac{56}{65} + \sin^{-1}\frac{33}{65} = \frac{\pi}{2}$ since $56^2 + 33^2 = 65^2$.

$\cos\frac{\pi}{2} = 0$.

Ans: (B) 0

Q36. [Hard] A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is: **[JEE Main 2021, 24 Feb Shift 2]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $2\sqrt{2}-1$ (C) $\sqrt{7}-1$ (D) $\frac{1}{\sqrt{7}}$

Formula used: Let $\theta = \sin^{-1}\frac{\sqrt{63}}{8}$: $\cos\theta = \frac{1}{8}$.

Solution: $\cos\frac{\theta}{2} = \sqrt{\frac{9}{16}} = \frac{3}{4}$.

$\tan\frac{\theta}{4} = \sqrt{\frac{1-3/4}{1+3/4}} = \frac{1}{\sqrt{7}}$.

Ans: (D) $\frac{1}{\sqrt{7}}$

Q37. [Hard] If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \cdots$, then $\tan(S)$ is equal to: **[JEE Main 2020, 05 Sep Shift 1]**

(A) $\frac{5}{6}$ (B) $\frac{5}{11}$ (C) $-\frac{6}{5}$ (D) $\frac{10}{11}$

Formula used: $\tan^{-1}\frac{1}{1+n(n+1)} = \tan^{-1}(n+1) - \tan^{-1}n$.

Solution: $S = \tan^{-1}11 - \tan^{-1}1$.

$\tan S = \frac{11-1}{1+11} = \frac{5}{6}$.

Ans: (A) $\frac{5}{6}$

Q38. [Hard] The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1 + \sum_{k=1}^{n}2k\right)\right)$ is: **[JEE Advanced 2013]**

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

Formula used: Telescoping as in the previous type.

Solution: Sum = $\tan^{-1}24 - \tan^{-1}1$; $\tan = \frac{23}{25}$; $\cot = \frac{25}{23}$.

Ans: (B) $\frac{25}{23}$

Q39. [Hard] The value of $\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$ is: **[IIT-JEE 1983]**

(A) $\frac{6}{17}$ (B) $\frac{17}{6}$ (C) $\frac{16}{7}$ (D) none of these

Formula used: $\cos^{-1}\frac{4}{5} = \tan^{-1}\frac{3}{4}$.

Solution: $\tan = \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}} = \frac{17}{6}$.

Ans: (B) $\frac{17}{6}$

Properties and Equations (JEE Extension)

Q40. [Medium] If $\sin^{-1}x = \frac{\pi}{5}$, then $\cos^{-1}x$ equals:

(A) $\frac{\pi}{5}$ (B) $\frac{4\pi}{5}$ (C) $\frac{\pi}{10}$ (D) $\frac{3\pi}{10}$

Formula used: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$.

Solution: $\frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$.

Ans: (D) $\frac{3\pi}{10}$

Q41. [Easy] $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$ equals:

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Formula used: Sum formula ($xy < 1$).

Solution: $\frac{5/6}{1-1/6} = 1$.

Ans: (C) $\frac{\pi}{4}$

Q42. [Medium] $\tan^{-1}2 + \tan^{-1}3$ equals:

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{2}$

Formula used: Sum formula with $xy > 1$, $x, y > 0$: add π.

Solution: $\pi + \tan^{-1}\frac{5}{1-6} = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Ans: (C) $\frac{3\pi}{4}$

Q43. [Medium] $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$ equals:

(A) $\tan^{-1}\frac{1}{3}$ (B) $\frac{\pi}{4}$

(C) $\tan^{-1}\frac{1}{2}$ (D) $\tan^{-1}\frac{17}{36}$

Formula used: Sum formula.

Solution: $\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{18}} = \frac{17/36}{17/18} = \frac{1}{2}$.

Ans: (C) $\tan^{-1}\frac{1}{2}$

Q44. [Medium] $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$ equals:

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Formula used: $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$.

Solution: $2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{3}{4}$; $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{25/28}{25/28} = \frac{\pi}{4}$.

Ans: (A) $\frac{\pi}{4}$

Q45. [Medium] $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}$ equals:

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π (D) $\frac{\pi}{3}$

Formula used: Convert to $\tan^{-1}$.

Solution: $\sin^{-1}\frac{4}{5} = \tan^{-1}\frac{4}{3}$ and $2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{3}{4}$; $\tan^{-1}x + \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$ $(x > 0)$.

Ans: (A) $\frac{\pi}{2}$

Q46. [Hard] $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$ equals:

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

Formula used: Double the double angle.

Solution: $2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{5}{12}$, $4\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{120}{119}$. $\tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239} = \tan^{-1}\frac{120\cdot239-119}{119\cdot239+120} = \tan^{-1}1$.

Ans: (D) $\frac{\pi}{4}$

Q47. [Medium] $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$ equals:

(A) $\frac{17}{6}$ (B) $\frac{6}{17}$ (C) $\frac{7}{16}$ (D) $\frac{16}{7}$

Formula used: $\tan(A + B)$.

Solution: $\tan A = \frac{3}{4}$, $\tan B = \frac{2}{3}$: $\frac{3/4+2/3}{1-1/2} = \frac{17/12}{1/2} = \frac{17}{6}$.

Ans: (A) $\frac{17}{6}$

Q48. [Medium] $\sum_{n=1}^{10}\tan^{-1}\frac{1}{1+n+n^2}$ equals:

(A) $\tan^{-1}10$ (B) $\frac{\pi}{4}$

(C) $\tan^{-1}\frac{10}{11}$ (D) $\tan^{-1}\frac{5}{6}$

Formula used: Telescoping: $\tan^{-1}(n+1) - \tan^{-1}n$.

Solution: Sum = $\tan^{-1}11 - \tan^{-1}1 = \tan^{-1}\frac{10}{12} = \tan^{-1}\frac{5}{6}$.

Ans: (D) $\tan^{-1}\frac{5}{6}$

Q49. [Easy] If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ $(xy < 1)$, then:

(A) $x + y = 1$ (B) $x + y + xy = 1$ (C) $xy = 1$ (D) $x - y + xy = 1$

Formula used: Sum formula.

Solution: $\frac{x+y}{1-xy} = 1$.

Ans: (B) $x + y + xy = 1$

Q50. [Medium] The solution of $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$ is:

(A) $x = -1$ (B) $x = \frac{1}{6}$ or -1

(C) $x = 1$ (D) $x = \frac{1}{6}$

Formula used: Sum formula, then check.

Solution: $\frac{5x}{1-6x^2} = 1 \Rightarrow 6x^2 + 5x - 1 = 0 \Rightarrow x = \frac{1}{6}$ or -1.

$x = -1$ makes the left side negative: rejected.

Ans: (D) $x = \frac{1}{6}$

Q51. [Easy] The solution of $\sin^{-1}x = \cos^{-1}x$ is:

(A) $x = 0$ (B) $x = 1$

(C) $x = \frac{1}{\sqrt{2}}$ (D) $x = \pm\frac{1}{\sqrt{2}}$

Formula used: Sum is $\frac{\pi}{2}$.

Solution: Each equals $\frac{\pi}{4}$.

Ans: (C) $x = \frac{1}{\sqrt{2}}$

Q52. [N] [Easy] If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then $x + y + z$ is ____.

Formula used: Each term is at most $\frac{\pi}{2}$.

Solution: All three must equal $\frac{\pi}{2}$: $x = y = z = 1$.

Ans: 3

Q53. [N] [Easy] If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then $xy + yz + zx$ is ____.

Formula used: Each term is at most π.

Solution: $x = y = z = -1$: $1 + 1 + 1 = 3$.

Ans: 3

Q54. [Medium] For $x < 0$, $\tan^{-1}x + \cot^{-1}x$ (principal values) equals:

(A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) 0

Formula used: The complementary relation holds for all real x with $\cot^{-1}$ in $(0, \pi)$.

Solution: E.g. $x = -1$: $-\frac{\pi}{4} + \frac{3\pi}{4} = \frac{\pi}{2}$.

Ans: (A) $\frac{\pi}{2}$

Q55. [Hard] If $\sin\left(\tan^{-1}\left(x\sqrt{2}\right)\right) = \cot\left(\sin^{-1}\sqrt{1-x^2}\right)$, $x \in (0,1)$, then the value of x is: **[JEE Main 2026, 06 Apr Shift 2]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{8}$

Formula used: $\sin(\tan^{-1}t) = \frac{t}{\sqrt{1+t^2}}$; $\cot(\sin^{-1}u) = \frac{\sqrt{1-u^2}}{u}$.

Solution: $\frac{x\sqrt{2}}{\sqrt{1+2x^2}} = \frac{x}{\sqrt{1-x^2}} \Rightarrow 2(1-x^2) = 1 + 2x^2 \Rightarrow x^2 = \frac{1}{4}$.

$x = \frac{1}{2}$.

Ans: (A) $\frac{1}{2}$

Q56. [Hard] Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is: **[JEE Main 2015]**

(A) $\frac{3x-x^3}{1+3x^2}$ (B) $\frac{3x+x^3}{1+3x^2}$

(C) $\frac{3x-x^3}{1-3x^2}$ (D) $\frac{3x+x^3}{1-3x^2}$

Formula used: $\tan^{-1}x + 2\tan^{-1}x = 3\tan^{-1}x$ for $|x| < \frac{1}{\sqrt{3}}$.

Solution: $y = \tan(3\tan^{-1}x) = \frac{3x-x^3}{1-3x^2}$.

Ans: (C) $\frac{3x-x^3}{1-3x^2}$

Q57. [Medium] If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is: **[AIEEE 2007]**

(A) 4 (B) 5 (C) 1 (D) 3

Formula used: $\text{cosec}^{-1}\frac{5}{4} = \sin^{-1}\frac{4}{5}$.

Solution: $\sin^{-1}\frac{x}{5} = \cos^{-1}\frac{4}{5} = \sin^{-1}\frac{3}{5}$: $x = 3$.

Ans: (D) 3

Q58. [Hard] The value of x for which $\sin\left(\cot^{-1}(1+x)\right) = \cos(\tan^{-1}x)$ is: **[IIT-JEE 2004]**

(A) $\frac{1}{2}$ (B) 1 (C) 0 (D) $-\frac{1}{2}$

Formula used: $\sin(\cot^{-1}t) = \frac{1}{\sqrt{1+t^2}}$, $\cos(\tan^{-1}x) = \frac{1}{\sqrt{1+x^2}}$.

Solution: $(1+x)^2 = x^2 \Rightarrow x = -\frac{1}{2}$.

Ans: (D) $-\frac{1}{2}$

D. PREVIOUS YEAR QUESTIONS (PYQs)

Principal Values and Compositions

1. [Easy] Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$, is ____. **[JEE Main 2024, 09 Apr Shift 2]**

Formula used: $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$.

Solution: $2\sin^{-1}x + \frac{3\pi}{2} - 3\sin^{-1}x = \frac{2\pi}{5} \Rightarrow \sin^{-1}x = \frac{11\pi}{10}$.

This exceeds $\frac{\pi}{2}$: no solution.

Ans: 0

2. [Easy] If $a = \sin^{-1}(\sin 5)$ and $b = \cos^{-1}(\cos 5)$, then $a^2 + b^2$ is equal to: **[JEE Main 2024, 31 Jan Shift 2]**

(A) $4\pi^2 + 25$ (B) $8\pi^2 - 40\pi + 50$ (C) $4\pi^2 - 20\pi + 50$ (D) 25

Formula used: Bring 5 into the principal ranges.

Solution: $5 \in \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$: $a = 5 - 2\pi$. $5 \in [\pi, 2\pi]$: $b = 2\pi - 5$.

$a^2 + b^2 = 2(2\pi - 5)^2 = 8\pi^2 - 40\pi + 50$.

Ans: (B) $8\pi^2 - 40\pi + 50$

3. [Easy] $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to: **[JEE Main 2022, 27 Jun Shift 1]**

(A) $\frac{11\pi}{12}$ (B) $\frac{17\pi}{12}$ (C) $\frac{31\pi}{12}$ (D) $-\frac{3\pi}{4}$

Formula used: Principal values.

Solution: $\frac{\pi}{3} + \frac{5\pi}{6} - \frac{\pi}{4} = \frac{4\pi+10\pi-3\pi}{12} = \frac{11\pi}{12}$.

Ans: (A) $\frac{11\pi}{12}$

4. [Easy] $\cos^{-1}\left(\cos(-5)\right) + \sin^{-1}(\sin 6) - \tan^{-1}(\tan 12)$ is equal to: (The inverse trigonometric functions take the principal values) **[JEE Main 2021, 01 Sep Shift 2]**

(A) $3\pi + 1$ (B) $3\pi - 11$ (C) $4\pi - 11$ (D) $4\pi - 9$

Formula used: Reduce each angle into the principal range.

Solution: $\cos(-5) = \cos 5$: $\cos^{-1} = 2\pi - 5$. $\sin^{-1}(\sin 6) = 6 - 2\pi$. $\tan^{-1}(\tan 12) = 12 - 4\pi$.

Total = $2\pi - 5 + 6 - 2\pi - 12 + 4\pi = 4\pi - 11$.

Ans: (C) $4\pi - 11$

5. [Medium] The value of $\tan^{-1}\left[\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right]$ is equal to: **[JEE Main 2022, 25 Jun Shift 2]**

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$ (C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Formula used: $\cos\frac{15\pi}{4} = \frac{1}{\sqrt{2}}$; $\tan\frac{\pi}{8} = \sqrt{2} - 1$.

Solution: Argument = $\frac{\frac{1}{\sqrt{2}}-1}{\frac{1}{\sqrt{2}}} = 1 - \sqrt{2} = -\tan\frac{\pi}{8}$.

Value $-\frac{\pi}{8}$.

Ans: (B) $-\frac{\pi}{8}$

6. [Medium] If the inverse trigonometric functions take principal values, then $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\frac{4}{3}\right) + \frac{2}{5}\sin\left(\tan^{-1}\frac{4}{3}\right)\right)$ is equal to: **[JEE Main 2022, 26 Jun Shift 2]**

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Formula used: $\tan^{-1}\frac{4}{3}$: $\cos = \frac{3}{5}$, $\sin = \frac{4}{5}$.

Solution: $\frac{3}{10}\cdot\frac{3}{5} + \frac{2}{5}\cdot\frac{4}{5} = \frac{9+16}{50} = \frac{1}{2}$; $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$.

Ans: (C) $\frac{\pi}{3}$

7. [Hard] If $\sin^{-1}\frac{\alpha}{17} + \cos^{-1}\frac{4}{5} - \tan^{-1}\frac{77}{36} = 0$, $0 < \alpha < 13$, then $\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha)$ is equal to: **[JEE Main 2023, 31 Jan Shift 1]**

(A) π (B) 16 (C) 0 (D) $16-5\pi$

Formula used: Convert to $\tan^{-1}$.

Solution: $\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}=\tan^{-1}\frac{77\cdot4-36\cdot3}{36\cdot4+77\cdot3}=\tan^{-1}\frac{200}{375}=\tan^{-1}\frac{8}{15}$, so $\frac{\alpha}{17}=\frac{8}{17}$, $\alpha=8$.

$8\in\left[\frac{5\pi}{2},\frac{7\pi}{2}\right]$, so $\sin^{-1}(\sin8)=3\pi-8$; $8\in[2\pi,3\pi]$, so $\cos^{-1}(\cos8)=8-2\pi$. Sum = π.

Ans: (A) π

8. [Medium] $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to: **[JEE Main 2022, 26 Jul Shift 1]**

(A) 1 (B) 2 (C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Formula used: $\sec^{-1}\frac{\sqrt{5}}{2}=\tan^{-1}\frac{1}{2}$; combine $\tan^{-1}$ terms.

Solution: $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\tan^{-1}\frac{13/40}{39/40}=\tan^{-1}\frac{1}{3}$; doubled: $\tan^{-1}\frac{3}{4}$.

$\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{2}=\tan^{-1}\frac{5/4}{5/8}=\tan^{-1}2$. Value 2.

Ans: (B) 2

9. [Medium] $50\tan\left(3\tan^{-1}\frac{1}{2}+2\cos^{-1}\frac{1}{\sqrt{5}}\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$ is equal to ____. **[JEE Main 2022, 29 Jun Shift 1]**

Formula used: $\cos^{-1}\frac{1}{\sqrt{5}}=\tan^{-1}2=\frac{\pi}{2}-\tan^{-1}\frac{1}{2}$; $\tan\frac{\theta}{2}=\frac{\sec\theta-1}{\tan\theta}$.

Solution: First angle = $3\theta+\pi-2\theta=\pi+\theta$ with $\theta=\tan^{-1}\frac{1}{2}$: $50\times\frac{1}{2}=25$.

Second: $\tan\phi=2\sqrt{2}$, $\sec\phi=3$: $\tan\frac{\phi}{2}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$; $4\sqrt{2}\times\frac{1}{\sqrt{2}}=4$. Total 29.

Ans: 29

10. [Easy] The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is: **[JEE Main 2022, 27 Jun Shift 2]**

(A) $\frac{25}{26}$ (B) $\frac{50}{51}$ (C) $\frac{26}{25}$ (D) $\frac{52}{51}$

Formula used: Telescoping.

Solution: Sum = $\tan^{-1}51-\tan^{-1}1=\tan^{-1}\frac{50}{52}$; $\cot=\frac{52}{50}=\frac{26}{25}$.

Ans: (C) $\frac{26}{25}$

11. [Easy] If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p$, then the value of $\tan p$ is: **[JEE Main 2021, 26 Aug Shift 2]**

(A) 100 (B) $\frac{51}{50}$ (C) $\frac{50}{51}$ (D) $\frac{101}{102}$

Formula used: $\frac{1}{2r^2}=\frac{2}{4r^2}=\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}$.

Solution: Each term = $\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$; sum = $\tan^{-1}101-\tan^{-1}1=\tan^{-1}\frac{100}{102}$.

$\tan p=\frac{50}{51}$.

Ans: (C) $\frac{50}{51}$

12. [Easy] If $\cot^{-1}(\alpha)=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\cdots$ upto 100 terms, then α is: **[JEE Main 2021, 17 Mar Shift 1]**

(A) 1.01 (B) 1.00 (C) 1.02 (D) 1.03

Formula used: $\cot^{-1}2k^2=\tan^{-1}\frac{1}{2k^2}=\tan^{-1}(2k+1)-\tan^{-1}(2k-1)$.

Solution: Sum = $\tan^{-1}201-\tan^{-1}1=\tan^{-1}\frac{200}{202}$; $\alpha=\cot(\text{sum})=\frac{202}{200}=1.01$.

Ans: (A) 1.01

13. [Hard] Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$, then $\lim_{k\to\infty}S_k$ is equal to: **[JEE Main 2021, 16 Mar Shift 1]**

(A) $\tan^{-1}\frac{3}{2}$ (B) $\frac{\pi}{2}$ (C) $\cot^{-1}\frac{3}{2}$ (D) $\tan^{-1}3$

Formula used: Divide by 3^{2r}: term = $\tan^{-1}\frac{t}{1+\cdots}$ with $t=\left(\frac{2}{3}\right)^r$.

Solution: With $t=\left(\frac{2}{3}\right)^r$, dividing by 9^r: term = $\tan^{-1}\frac{t}{3+2t^2}=\tan^{-1}\frac{t-\frac{2t}{3}}{1+t\cdot\frac{2t}{3}}=\tan^{-1}t-\tan^{-1}\frac{2t}{3}$.

So the rth term is $\tan^{-1}\left(\frac{2}{3}\right)^r-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}$; the sum telescopes to $\tan^{-1}\frac{2}{3}=\cot^{-1}\frac{3}{2}$.

Ans: (C) $\cot^{-1}\frac{3}{2}$

14. [Hard] The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)+\cdots$ is: **[JEE Main 2025, 04 Apr Shift 2]**

(A) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$ (B) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

(C) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$

Formula used: $\cot^{-1}\frac{4n^2+3}{4}=\tan^{-1}\frac{1}{1+\left(n-\frac{1}{2}\right)\left(n+\frac{1}{2}\right)}$.

Solution: Each term = $\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$; the series telescopes to $\frac{\pi}{2}-\tan^{-1}\frac{1}{2}$.

Ans: (B) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$

15. [Hard] Considering only the principal values of the inverse trigonometric functions, the value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is: **[JEE Advanced 2024]**

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$ (C) $\frac{-5}{24}$ (D) $\frac{5}{24}$

Formula used: $\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$; $\cos^{-1}\frac{2}{\sqrt{5}}=\tan^{-1}\frac{1}{2}$.

Solution: $\tan\left(2\tan^{-1}\frac{1}{2}\right)=\frac{4}{3}$.

$\tan(A-B)=\frac{\frac{3}{4}-\frac{4}{3}}{1+1}=-\frac{7}{24}$.

Ans: (B) $\frac{-7}{24}$

16. [Medium] $\text{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to: **[JEE Main 2021, 25 Feb Shift 2]**

(A) $\frac{56}{33}$ (B) $\frac{65}{56}$ (C) $\frac{65}{33}$ (D) $\frac{75}{56}$

Formula used: $2\cot^{-1}5=2\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{5}{12}$.

Solution: $A=\tan^{-1}\frac{5}{12}$, $B=\cos^{-1}\frac{4}{5}$: $\sin(A+B)=\frac{5}{13}\cdot\frac{4}{5}+\frac{12}{13}\cdot\frac{3}{5}=\frac{56}{65}$.

cosec = $\frac{65}{56}$.

Ans: (B) $\frac{65}{56}$

17. [Hard] If $\alpha=\cos^{-1}\left(\frac{3}{5}\right)$, $\beta=\tan^{-1}\left(\frac{1}{3}\right)$, where $0<\alpha,\beta<\frac{\pi}{2}$, then $\alpha-\beta$ is equal to: **[JEE Main 2019, 08 Apr Shift 1]**

(A) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (B) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(C) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (D) $\tan^{-1}\left(\frac{9}{14}\right)$

Formula used: $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

Solution: $\sin\alpha=\frac{4}{5}$, $\cos\alpha=\frac{3}{5}$; $\sin\beta=\frac{1}{\sqrt{10}}$, $\cos\beta=\frac{3}{\sqrt{10}}$.

$\sin(\alpha-\beta)=\frac{12-3}{5\sqrt{10}}=\frac{9}{5\sqrt{10}}$.

Ans: (B) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

18. [Easy] The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is: **[JEE Main 2019, 10 Jan Shift 2]**

(A) $\frac{23}{22}$ (B) $\frac{22}{23}$ (C) $\frac{19}{21}$ (D) $\frac{21}{19}$

Formula used: $1+\sum 2p=1+n(n+1)$; $\cot^{-1}\left(1+n(n+1)\right)=\tan^{-1}(n+1)-\tan^{-1}n$.

Solution: Sum = $\tan^{-1}20-\tan^{-1}1$; $\tan=\frac{19}{21}$, so $\cot=\frac{21}{19}$.

Ans: (D) $\frac{21}{19}$

19. [Easy] The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is: **[AIEEE 2008]**

(A) $\frac{5}{17}$ (B) $\frac{6}{17}$ (C) $\frac{3}{17}$ (D) $\frac{4}{17}$

Formula used: $\text{cosec}^{-1}\frac{5}{3}=\tan^{-1}\frac{3}{4}$.

Solution: $\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\frac{17/12}{1/2}=\frac{17}{6}$; $\cot=\frac{6}{17}$.

Ans: (B) $\frac{6}{17}$

20. [Medium] Let $[\cdot]$ denote the greatest integer function. If the domain of the function $f(x)=\sin^{-1}\left(\frac{x+[x]}{3}\right)$ is [α, β), then $\alpha^2+\beta^2$ is: **[JEE Main 2026, 06 Apr Shift 1]**

(A) 2 (B) 5 (C) 10 (D) 13

Formula used: Need $-3\le x+[x]\le 3$.

Solution: $[x]=1$: $x\le 2$ (all of [1, 2)); $[x]=-1$: $x\ge -2$, so all of [−1, 0); $[x]=-2$ gives $x+[x]<-3$; $[x]=2$ gives $x+[x]\ge 4$.

Domain [−1, 2): $\alpha^2+\beta^2=1+4=5$.

Ans: (B) 5

21. [Medium] If the domain of the function $f(x)=\cos^{-1}\left(\frac{2x-5}{11-3x}\right)+\sin^{-1}(2x^2-3x+1)$ is the interval $[\alpha,\beta]$, then $\alpha+2\beta$ is equal to: **[JEE Main 2026, 21 Jan Shift 1]**

(A) 1 (B) 3 (C) 5 (D) 2

Formula used: Intersect the two conditions.

Solution: $|2x^2 - 3x + 1| \le 1 \Rightarrow 0 \le x \le \frac{3}{2}$.

$\left|\frac{2x-5}{11-3x}\right| \le 1$ holds on $\left[0, \frac{3}{2}\right]$ (it gives $x \le \frac{16}{5}$ and $x \le 6$). Domain $\left[0, \frac{3}{2}\right]$: $\alpha + 2\beta = 3$.

Ans: (B) 3

22. [Easy] If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $\mathbb{R} - (\alpha, \beta)$, then $12\alpha\beta$ is equal to:
[JEE Main 2024, 09 Apr Shift 1]

(A) 24 (B) 32 (C) 40 (D) 36

Formula used: $|x - 1| \le |2x + 3|$.

Solution: $(x-1)^2 \le (2x+3)^2 \Rightarrow 3x^2 + 14x + 8 \ge 0 \Rightarrow x \le -4$ or $x \ge -\frac{2}{3}$.

$(\alpha, \beta) = \left(-4, -\frac{2}{3}\right)$: $12\alpha\beta = 32$.

Ans: (B) 32

23. [Medium] If the domain of the function $f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ is (α, β], then $36|\alpha + \beta|$ is equal to:
[JEE Main 2023, 15 Apr Shift 1]

(A) 45 (B) 63 (C) 72 (D) 54

Formula used: Intersect the three conditions.

Solution: $-1 \le 4x + 3 \le 1 \Rightarrow -1 \le x \le -\frac{1}{2}$; $-3 \le 10x + 6 \le 3 \Rightarrow -\frac{9}{10} \le x \le -\frac{3}{10}$; $(4x + 3)(x + 2) > 0 \Rightarrow x > -\frac{3}{4}$.

Domain (−3/4, −1/2]: $36 \cdot \frac{5}{4} = 45$.

Ans: (A) 45

24. [Medium] If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval (α, β], then $\alpha + \beta$ is equal to: **[JEE Main 2021, 22 Jul Shift 2]**

(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) $\frac{3}{2}$

Formula used: Need $x^2 - x + 1 \le 1$ and $\sin^{-1}\frac{2x-1}{2} > 0$.

Solution: $x^2 - x \le 0 \Rightarrow 0 \le x \le 1$; $\frac{2x-1}{2} > 0 \Rightarrow x > \frac{1}{2}$.

Domain (1/2, 1]: $\alpha + \beta = \frac{3}{2}$.

Ans: (D) $\frac{3}{2}$

Equations and Properties

25. [Medium] For $n \in \mathbb{N}$, if $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$, then n is equal to ____.
[JEE Main 2024, 06 Apr Shift 1]

Formula used: $\cot^{-1}x = \tan^{-1}\frac{1}{x}$ $(x > 0)$; sum formula.

Solution: $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} = \tan^{-1}\frac{7}{11}$; adding $\tan^{-1}\frac{1}{5}$: $\tan^{-1}\frac{46}{48} = \tan^{-1}\frac{23}{24}$.

$\tan^{-1}\frac{1}{n} = \frac{\pi}{4} - \tan^{-1}\frac{23}{24} = \tan^{-1}\frac{1}{47}$. $n = 47$.

Ans: 47

26. [Hard] Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [−1, 1] such that $\cos^{-1}x - \sin^{-1}y = \alpha$, $-\frac{\pi}{2} \le \alpha \le \pi$. Then, the minimum value of $x^2 + y^2 + 2xy\sin\alpha$ is: **[JEE Main 2024, 04 Apr Shift 2]**

(A) 0 (B) −1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Formula used: Put $x = \cos A$, $y = \sin B$ with $A - B = \alpha$.

Solution: With $x = \cos A$ $(A \in [0, \pi])$, $y = \sin B$ $(B \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right])$ and $\alpha = A - B$, the expression equals $(x + y\sin\alpha)^2 + y^2\cos^2\alpha \ge 0$.

Equality holds, e.g., at $x = y = 0$ $(\alpha = \frac{\pi}{2})$.

Minimum value 0.

Ans: (A) 0

27. [Hard] For $\alpha, \beta, \gamma \ne 0$. If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equal to: **[JEE Main 2024, 31 Jan Shift 1]**

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (D) $\sqrt{3}$

Formula used: Angles of a triangle; sine rule and cosine rule.

Solution: Let $A = \sin^{-1}\alpha$ etc.; $A + B + C = \pi$, and α, β, γ are proportional to the sides.

$(\alpha+\beta)^2-\gamma^2=3\alpha\beta \Rightarrow \frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta}=\frac{1}{2}=\cos C$, so $C=\frac{\pi}{3}$ and $\gamma=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$.

Ans: (A) $\frac{\sqrt{3}}{2}$

28. [Medium] Let $x=\frac{m}{n}$ (m,n are co-prime natural numbers) be a solution of the equation $\cos(2\sin^{-1}x)=\frac{1}{9}$ and let α,β $(\alpha>\beta)$ be the roots of the equation $mx^2-nx-m+n=0$. Then the point (α,β) lies on the line: **[JEE Main 2024, 29 Jan Shift 2]**

(A) $3x+2y=2$ (B) $5x-8y=-9$ (C) $3x-2y=-2$ (D) $5x+8y=9$

Formula used: $\cos2\theta=1-2\sin^2\theta$.

Solution: $1-2x^2=\frac{1}{9}\Rightarrow x=\frac{2}{3}$; $m=2$, $n=3$.

$2x^2-3x+1=0$: $\alpha=1$, $\beta=\frac{1}{2}$; $5+4=9$ ✓.

Ans: (D) $5x+8y=9$

29. [Easy] Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x)+\tan^{-1}(2x)=\frac{\pi}{4}$ is: **[JEE Main 2024, 27 Jan Shift 2]**

(A) More than 2 (B) 1 (C) 2 (D) 0

Formula used: Sum formula; reject roots with $xy\geq1$.

Solution: $\frac{3x}{1-2x^2}=1\Rightarrow 2x^2+3x-1=0\Rightarrow x=\frac{-3\pm\sqrt{17}}{4}$.

One positive root: 1.

Ans: (B) 1

30. [Easy] For x in (−1, 1], the number of solutions of the equation $\sin^{-1}x=2\tan^{-1}x$ is equal to ____. **[JEE Main 2023, 13 Apr Shift 2]**

Formula used: $2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}$ for $|x|\leq1$.

Solution: $x=\frac{2x}{1+x^2}\Rightarrow x=0$ or $x^2=1$; in (−1, 1]: $x=0,1$.

Check $x=1$: $\frac{\pi}{2}=2\cdot\frac{\pi}{4}$ ✓. Two solutions.

Ans: 2

31. [Medium] If S is the set of all real x with $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$, then $\sum_{x\in S}\left(\sin\left((x^2+x+5)\frac{\pi}{2}\right)-\cos\left((x^2+x+5)\pi\right)\right)$ is equal to ____. **[JEE Main 2023, 13 Apr Shift 1]**

Formula used: $\sin^{-1}\frac{t}{\sqrt{t^2+1}}=\tan^{-1}t$.

Solution: Equation: $\tan^{-1}(x+1)-\tan^{-1}x=\frac{\pi}{4}\Rightarrow \frac{1}{1+x(x+1)}=1\Rightarrow x=0,-1$.

For both, $x^2+x+5=5$: $\sin\frac{5\pi}{2}-\cos5\pi=1+1=2$. Sum = 4.

Ans: 4

32. [Medium] Let S be the set of all solutions of the equation $\cos^{-1}(2x)-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\pi$, $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$. Then $\sum_{x\in S}2\sin^{-1}(x^2-1)$ is equal to: **[JEE Main 2023, 01 Feb Shift 1]**

(A) 0 (B) $-\frac{2\pi}{3}$

(C) $\pi-\sin^{-1}\frac{\sqrt{3}}{4}$ (D) $\pi-2\sin^{-1}\frac{\sqrt{3}}{4}$

Formula used: Ranges: $\cos^{-1}\in[0,\pi]$.

Solution: $\cos^{-1}(2x)\leq\pi$ and $2\cos^{-1}\sqrt{1-x^2}\geq0$, so equality needs $\cos^{-1}(2x)=\pi$ and $\cos^{-1}\sqrt{1-x^2}=0$, i.e. $x=-\frac{1}{2}$ and $x=0$ simultaneously — impossible.

$S=\phi$; the empty sum is 0.

Ans: (A) 0

33. [Hard] Let S be the set of all $x\in(0,1)$ such that $2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. If $n(S)$ denotes the number of elements in S then: **[JEE Main 2023, 01 Feb Shift 2]**

(A) $n(S)=2$ and only one element in S is less than $\frac{1}{2}$ (B) $n(S)=1$ and the element in S is more than $\frac{1}{2}$

(C) $n(S)=1$ and the element in S is less than $\frac{1}{2}$ (D) $n(S)=0$

Formula used: $\cos^{-1}\frac{1-x^2}{1+x^2}=2\tan^{-1}x$ $(x\geq0)$; $\tan^{-1}\frac{1-x}{1+x}=\frac{\pi}{4}-\tan^{-1}x$.

Solution: $\frac{\pi}{2} - 2\tan^{-1}x = 2\tan^{-1}x \Rightarrow \tan^{-1}x = \frac{\pi}{8} \Rightarrow x = \sqrt{2} - 1 \approx 0.414$.

One solution, less than $\frac{1}{2}$.

Ans: (C) $n(S) = 1$ and the element in S is less than $\frac{1}{2}$

34. [Hard] If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$, $-1 < x < 1$, $x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to ____. **[JEE Main 2023, 25 Jan Shift 1]**

Formula used: Treat $x > 0$ and $x < 0$ separately.

Solution: $x > 0$: both terms equal $2\tan^{-1}x$: $4\tan^{-1}x = \frac{\pi}{3} \Rightarrow x = \tan\frac{\pi}{12} = 2 - \sqrt{3}$.

$x < 0$: $\cot^{-1}$ of a negative number = $\pi + 2\tan^{-1}x$: $4\tan^{-1}x + \pi = \frac{\pi}{3} \Rightarrow x = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$.

Sum = $2 - \sqrt{3} - \frac{1}{\sqrt{3}} = 2 - \frac{4}{\sqrt{3}}$, so $\alpha = 2$.

Ans: 2

35. [Medium] $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}$ is equal to: **[JEE Main 2023, 24 Jan Shift 1]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Formula used: Simplify each argument.

Solution: $\frac{1+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)} = \frac{1}{\sqrt{3}}$: first term $\frac{\pi}{6}$.

$\frac{8+4\sqrt{3}}{6+3\sqrt{3}} = \frac{4}{3}$, $\sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$. Total $\frac{\pi}{3}$.

Ans: (C) $\frac{\pi}{3}$

36. [Medium] Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to: **[JEE Main 2022, 28 Jul Shift 1]**

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Formula used: Write everything in terms of $\theta = \sin^{-1}x$.

Solution: $\frac{\pi}{2} - 3\theta = \cos^{-1}2x$; taking cosines: $\sin 3\theta = 2\sin\theta \Rightarrow \sin\theta(1 - 4\sin^2\theta) = 0 \Rightarrow x = 0, \pm\frac{1}{2}$.

All three satisfy the original equation (e.g. $x = -\frac{1}{2}$: $\frac{2\pi}{3} + \frac{\pi}{3} = \pi = \cos^{-1}(-1)$). Sum = 0.

Ans: (A) 0

37. [Medium] Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then $\sum_{k=1}^{2021}\tan^{-1}\left(\frac{1}{1+a_k a_{k+1}}\right)$ is equal to: **[JEE Main 2023, 30 Jan Shift 2]**

(A) $\frac{\pi}{4} - \cot^{-1}(2022)$ (B) $\cot^{-1}(2022) - \frac{\pi}{4}$

(C) $\tan^{-1}(2022) - \frac{\pi}{4}$ (D) $\frac{\pi}{4} - \tan^{-1}(2022)$

Formula used: $\tan^{-1}\frac{a_{k+1}-a_k}{1+a_k a_{k+1}} = \tan^{-1}a_{k+1} - \tan^{-1}a_k$.

Solution: $a_k = k$, so the sum telescopes to $\tan^{-1}2022 - \frac{\pi}{4} = \frac{\pi}{4} - \cot^{-1}2022$ (both forms are equal; option (A) is listed).

Ans: (A) $\frac{\pi}{4} - \cot^{-1}(2022)$

38. [Hard] The set of all values of k for which $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = k\pi^3$, $x \in \mathbb{R}$, is the interval: **[JEE Main 2022, 24 Jun Shift 1]**

(A) [1/32, 7/8) (B) $\left(\frac{1}{24}, \frac{13}{16}\right)$

(C) $\left[\frac{1}{48}, \frac{13}{16}\right]$ (D) [1/32, 9/8)

Formula used: $a + b = \frac{\pi}{2}$: $a^3 + b^3 = \frac{\pi^3}{8} - \frac{3\pi}{2}ab$.

Solution: With $t = \tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$: $k\pi^3 = \frac{\pi^3}{8} - \frac{3\pi}{2}t\left(\frac{\pi}{2} - t\right)$.

Minimum at $t = \frac{\pi}{4}$: $k = \frac{1}{8} - \frac{3}{32} = \frac{1}{32}$; as $t \to -\frac{\pi}{2}$, $k \to \frac{1}{8} + \frac{3}{4} = \frac{7}{8}$ (not attained).

Ans: (A) [1/32, 7/8)

39. [Easy] The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is: **[JEE Main 2021, 20 Jul Shift 1]**

(A) 1 (B) 2 (C) 4 (D) 0

Formula used: Domain first.

Solution: Need $x(x+1) \geq 0$ and $x^2 + x + 1 \leq 1$, so $x(x+1) = 0$: $x = 0$ or -1.

Then the left side is $0 + \frac{\pi}{2} \neq \frac{\pi}{4}$: no real roots.

Ans: (D) 0

40. [Hard] The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is: **[JEE Main 2021, 17 Mar Shift 1]**

(A) $-\frac{32}{4}$ (B) $-\frac{31}{4}$ (C) $-\frac{30}{4}$ (D) $-\frac{33}{4}$

Formula used: For $x<1$, $\cot^{-1}\frac{1}{x-1}=\pi+\tan^{-1}(x-1)$; for $x>1$ it is $\tan^{-1}(x-1)$.

Solution: Solving $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$ (mod π): $\frac{2x}{2-x^2}=\frac{8}{31}\Rightarrow 4x^2+31x-8=0\Rightarrow x=\frac{1}{4}$ or -8.

Checking with principal values, only $x=-8$ satisfies the equation. Sum = $-8=-\frac{32}{4}$.

Ans: (A) $-\frac{32}{4}$

41. [Medium] Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to: **[JEE Main 2021, 16 Mar Shift 2]**

(A) 2 (B) 1 (C) 3 (D) 0

Formula used: Take sines: $\sin(A+B)=\sin A\cos B+\cos A\sin B$.

Solution: $\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$ gives $x=0$ or $x=\pm1$.

All three satisfy the equation (e.g. $x=1$: $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}=\frac{\pi}{2}$). Answer 3.

Ans: (C) 3

42. [Hard] Considering the principal values of the inverse trigonometric functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right)$, $-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$, is equal to: **[JEE Main 2025, 04 Apr Shift 1]**

(A) $-\frac{5\pi}{6}-\sin^{-1}x$ (B) $\frac{\pi}{4}+\sin^{-1}x$

(C) $\frac{5\pi}{6}-\sin^{-1}x$ (D) $\frac{\pi}{6}+\sin^{-1}x$

Formula used: Put $x=\sin\theta$.

Solution: Argument = $\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}=\sin\left(\theta+\frac{\pi}{6}\right)$.

For $\theta\in\left(-\frac{\pi}{6},\frac{\pi}{4}\right)$, $\theta+\frac{\pi}{6}\in\left(0,\frac{5\pi}{12}\right)$ is principal: value $\frac{\pi}{6}+\sin^{-1}x$.

Ans: (D) $\frac{\pi}{6}+\sin^{-1}x$

43. [Medium] If $(\sin^{-1}x)^2-(\cos^{-1}x)^2=a;\ 0<x<1,\ a\neq0$, then the value of $2x^2-1$ is: **[JEE Main 2021, 27 Aug Shift 1]**

(A) $\cos\left(\frac{2a}{\pi}\right)$ (B) $\sin\left(\frac{2a}{\pi}\right)$

(C) $\cos\left(\frac{4a}{\pi}\right)$ (D) $\sin\left(\frac{4a}{\pi}\right)$

Formula used: Difference of squares with $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$.

Solution: $\frac{\pi}{2}(\sin^{-1}x-\cos^{-1}x)=a\Rightarrow 2\sin^{-1}x-\frac{\pi}{2}=\frac{2a}{\pi}$.

$2x^2-1=-\cos(2\sin^{-1}x)=\sin\left(2\sin^{-1}x-\frac{\pi}{2}\right)=\sin\frac{2a}{\pi}$.

Ans: (B) $\sin\left(\frac{2a}{\pi}\right)$

44. [Easy] If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then: **[JEE Main 2013]**

(A) $2x=3y=6z$ (B) $6x=3y=2z$ (C) $6x=4y=3z$ (D) $x=y=z$

Formula used: $2\tan^{-1}y=\tan^{-1}x+\tan^{-1}z$.

Solution: $\frac{2y}{1-y^2}=\frac{x+z}{1-xz}=\frac{2y}{1-xz}$, so $y^2=xz$ (or $y=0$).

With $2y=x+z$: $(x+z)^2=4xz\Rightarrow x=z=y$.

Ans: (D) $x=y=z$

45. [Easy] The largest interval lying in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ for which the function, $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$, is defined, is: **[AIEEE 2007]**

(A) [−π/4, π/2) (B) [0, π/2)

(C) $[0,\pi]$ (D) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Formula used: $-1\le\frac{x}{2}-1\le1$ and $\cos x>0$.

Solution: $0\le x\le4$; within $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$: [0, π/2).

Ans: (B) [0, π/2)

46. [Easy] The trigonometric equation $\sin^{-1}x=2\sin^{-1}a$, has a solution for: **[AIEEE 2003]**

(A) all real values (B) $|a| \le \frac{1}{\sqrt{2}}$

(C) $|a| \ge \frac{1}{\sqrt{2}}$ (D) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

Formula used: Need $2\sin^{-1}a \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Solution: $|\sin^{-1}a| \le \frac{\pi}{4} \Rightarrow |a| \le \frac{1}{\sqrt{2}}$.

Ans: (B) $|a| \le \frac{1}{\sqrt{2}}$

47. [N] [Medium] If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x-y)^2 + 3y^2$ is equal to ____. **[JEE Main 2025, 02 Apr Shift 2]**

Formula used: Let $\theta = \cos^{-1}\frac{x}{2}$: $\cos\theta = \frac{x}{2}$, $\sin\theta = \frac{\sqrt{4-x^2}}{2}$.

Solution: With $w = \sqrt{4-x^2}$: $y = \frac{1}{2}\cdot\frac{x}{2} - \frac{\sqrt{3}}{2}\cdot\frac{w}{2} = \frac{x-\sqrt{3}w}{4}$, so $x - y = \frac{3x+\sqrt{3}w}{4}$.

$(x-y)^2 + 3y^2 = \frac{1}{16}\left[\left(3x+\sqrt{3}w\right)^2 + 3\left(x-\sqrt{3}w\right)^2\right]$

with $w = \sqrt{4-x^2}$: $= \frac{12x^2+12w^2}{16} = 3$.

Ans: 3

E. PRACTICE QUESTIONS

Principal Values

1. [Easy] $\sin^{-1}\frac{1}{2}$ equals:

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $-\frac{\pi}{6}$

2. [Medium] $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ equals:

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $\frac{7\pi}{6}$

3. [Easy] $\tan^{-1}\sqrt{3}$ equals:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) $\frac{2\pi}{3}$

4. [Easy] $\cot^{-1}(-1)$ equals:

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

5. [Medium] $\sec^{-1}(-2)$ equals:

(A) $-\frac{\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

6. [Medium] $\text{cosec}^{-1}\left(\sqrt{2}\right)$ equals:

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

7. [Easy] $\sin^{-1}(-1)$ equals:

(A) $\frac{3\pi}{2}$ (B) π (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

8. [Easy] $\cos^{-1}(-1)$ equals:

(A) $-\pi$ (B) π (C) 0 (D) $\frac{\pi}{2}$

9. [Hard] $\tan^{-1}1 + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ equals:

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) $\frac{5\pi}{4}$

10. [Medium] $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$ equals:

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

11. [Medium] The principal range of $\sec^{-1}x$ is:

(A) $[0, \pi]$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(C) $(0, \pi)$ (D) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

12. [Hard] Let $\alpha = \tan\left(\frac{5\pi}{16}\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)$ and $\beta = \cos\left(\sin^{-1}\left(\frac{4}{5}\right) + \sec^{-1}\left(\frac{5}{3}\right)\right)$, where the inverse trigonometric functions take principal values. Then, the equation whose roots are α and β is: **[JEE Main 2022, 30 Jun Shift 2]**

(A) $15x^2 - 8x - 7 = 0$ (B) $5x^2 - 12x + 7 = 0$ (C) $25x^2 - 18x - 7 = 0$ (D) $25x^2 - 32x + 7 = 0$

13. [Hard] If we consider only the principal values of the inverse trigonometric functions then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is: **[IIT-JEE 1994]**

(A) $\frac{\sqrt{29}}{3}$ (B) $\frac{29}{3}$ (C) $\frac{\sqrt{3}}{29}$ (D) $\frac{3}{29}$

Domain and Range

14. [Easy] The domain of $\sin^{-1}(3x)$ is:

(A) [−3, 3] (B) [−1, 1]

(C) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (D) [0, 1/3]

15. [Medium] The domain of $\cos^{-1}\frac{x}{2}$ is:

(A) [−1, 1] (B) [−2, 2]

(C) [0, 2] (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

16. [Easy] The domain of $\sec^{-1}x$ is:

(A) [−1, 1] (B) $\mathbb{R} - (-1,1)$ (C) $\mathbb{R}$ (D) $\mathbb{R} - \{0\}$

17. [Easy] The domain of $\sin^{-1}\sqrt{x-1}$ is:

(A) [1, 2] (B) [0, 1] (C) [1, ∞) (D) [−1, 1]

18. [Medium] The range of $\tan^{-1}x$ is:

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $(0, \pi)$

(C) $\mathbb{R}$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

19. [Medium] The range of $\sin^{-1}x + \cos^{-1}x$ is:

(A) $[0, \pi]$ (B) $\left[0, \frac{\pi}{2}\right]$

(C) $[-\pi, \pi]$ (D) $\left\{\frac{\pi}{2}\right\}$

20. [Easy] The greatest value of $\cos^{-1}x$ is:

(A) $\frac{\pi}{2}$ (B) 2π (C) 1 (D) π

21. [Medium] The minimum value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ is:

(A) 0 (B) $\frac{\pi^2}{8}$ (C) $\frac{\pi^2}{4}$ (D) $\frac{\pi^2}{16}$

22. [Hard] Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2)$ is: **[JEE Main 2025, 22 Jan Shift 1]**

(A) $22\pi^2$ (B) $24\pi^2$ (C) $18\pi^2$ (D) $31\pi^2$

23. [Hard] If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is [−α, β) − {γ}, then $\alpha + \beta + \gamma$ is equal to: **[JEE Main 2024, 30 Jan Shift 1]**

(A) 9 (B) 8 (C) 11 (D) 12

24. [Medium] The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is: **[JEE Main 2022, 29 Jul Shift 2]**

(A) [1, ∞) (B) [−1, 2] (C) [−1, ∞) (D) (−∞, 2]

25. [Hard] Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of $\tan(M-m)$ is equal to: **[JEE Main 2021, 27 Aug Shift 2]**

(A) $2 - \sqrt{3}$ (B) $3 - 2\sqrt{2}$ (C) $3 + 2\sqrt{2}$ (D) $2 + \sqrt{3}$

Composition

26. [Easy] $\sin(\sin^{-1}0.3)$ equals:

(A) $\sin 0.3$ (B) 0.7 (C) $\sin^{-1}0.3$ (D) 0.3

27. [Medium] $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ equals:

(A) $\frac{2\pi}{5}$ (B) $\frac{3\pi}{5}$ (C) $-\frac{2\pi}{5}$ (D) $\frac{\pi}{5}$

28. [Medium] $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$ equals:

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

29. [Medium] $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ equals:

(A) $\frac{7\pi}{6}$ (B) $-\frac{\pi}{6}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

30. [Easy] $\sin^{-1}(\sin 2)$ equals:

(A) 2 (B) $2 - \pi$ (C) $\frac{\pi}{2} - 2$ (D) $\pi - 2$

31. [Easy] $\cos^{-1}(\cos 4)$ equals:

(A) 4 (B) $2\pi - 4$ (C) $4 - \pi$ (D) $\pi - 4$

32. [Easy] $\sin\left(\cos^{-1}\frac{4}{5}\right)$ equals:

(A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{3}{4}$ (D) $-\frac{3}{5}$

33. [Medium] $\tan\left(\sin^{-1}\frac{5}{13}\right)$ equals:

(A) $\frac{12}{5}$ (B) $\frac{5}{13}$ (C) $\frac{12}{13}$ (D) $\frac{5}{12}$

34. [Medium] $\cos(\tan^{-1}x)$ equals:

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}$

(C) $\frac{1}{1+x^2}$ (D) $\frac{1}{\sqrt{1+x^2}}$

35. [Easy] $\sin(2\cos^{-1}x)$ equals:

(A) $1 - 2x^2$ (B) $2x\sqrt{1-x^2}$ (C) $2x^2 - 1$ (D) $x\sqrt{1-x^2}$

36. [Easy] $\cos\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$ equals:

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) −1

37. [Easy] $\cot(\tan^{-1}a + \cot^{-1}a)$ equals:

(A) 0 (B) 1 (C) a (D) $\frac{1}{a}$

38. [Easy] $\cos\left(\cos^{-1}(-0.5)\right)$ equals:

(A) −0.5 (B) 0.5 (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{3}$

39. [Medium] $\tan^{-1}\frac{1}{\sqrt{3}} + \cot^{-1}\frac{1}{\sqrt{3}}$ equals:

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) π (D) $\frac{\pi}{2}$

40. [Medium] $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$ equals:

(A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

41. [Hard] The value of $\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right)$ $-\cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$ is equal to: **[JEE Main 2025, 08 Apr Shift 2]**

(A) $\pi+\frac{3}{2}$ (B) $\pi-\frac{5}{4}$ (C) $\pi-\frac{3}{2}$ (D) $\pi+\frac{5}{2}$

42. [Hard] $2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ is equal to: **[JEE Main 2020, 03 Sep Shift 1]**

(A) $\frac{\pi}{2}$ (B) $\frac{5\pi}{4}$ (C) $\frac{3\pi}{2}$ (D) $\frac{7\pi}{4}$

43. [Hard] The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to: **[JEE Main 2019, 12 Apr Shift 1]**

(A) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ (B) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$

(C) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$ (D) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$

44. [Hard] If $f(x)=2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $x>1$, then $f(5)$ is equal to: **[JEE Main 2015 (Online)]**

(A) $\frac{\pi}{2}$ (B) π

(C) $4\tan^{-1}(5)$ (D) $\tan^{-1}\left(\frac{65}{156}\right)$

Properties (JEE Extension)

45. [Easy] $\sin^{-1}x+\cos^{-1}x$ $(|x|\le 1)$ equals:

(A) π (B) $\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{4}$

46. [Easy] $\cos^{-1}(-x)$ equals:

(A) $\pi-\cos^{-1}x$ (B) $-\cos^{-1}x$

(C) $\cos^{-1}x$ (D) $\frac{\pi}{2}-\cos^{-1}x$

47. [Easy] $\tan^{-1}(-x)$ equals:

(A) $\pi-\tan^{-1}x$ (B) $\tan^{-1}x$

(C) $\frac{\pi}{2}-\tan^{-1}x$ (D) $-\tan^{-1}x$

48. [Hard] $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}$ equals:

(A) $\tan^{-1}\frac{9}{35}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1}\frac{1}{2}$ (D) $\tan^{-1}2$

49. [Medium] $\tan^{-1}3-\tan^{-1}\frac{1}{2}$ equals:

(A) $\frac{\pi}{3}$ (B) $\tan^{-1}\frac{5}{2}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

50. [Medium] $2\tan^{-1}\frac{1}{2}$ equals:

(A) $\tan^{-1}\frac{4}{3}$ (B) $\tan^{-1}1$ (C) $\tan^{-1}\frac{3}{4}$ (D) $\tan^{-1}2$

51. [Medium] $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$ equals:

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{4}$ (C) 2π (D) π

52. [Medium] $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}$ equals:

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $\sin^{-1}\frac{7}{5}$

53. [Hard] $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}$ equals:

(A) $\frac{\pi}{4}$ (B) $\tan^{-1}\frac{3}{4}$ (C) $\frac{\pi}{2}$ (D) $\tan^{-1}\frac{27}{11}$

54. [Easy] $\tan^{-1}x+\tan^{-1}\frac{1}{x}$ for $x>0$ equals:

(A) π (B) 0 (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

55. [Easy] $\tan^{-1}x+\tan^{-1}\frac{1}{x}$ for $x<0$ equals:

(A) $\frac{\pi}{2}$ (B) 0 (C) $-\frac{\pi}{2}$ (D) π

56. [Medium] $\sin^{-1}\frac{2x}{1+x^2}$ for $|x|\le 1$ equals:

(A) $\tan^{-1}x$ (B) $2\sin^{-1}x$ (C) $2\tan^{-1}x$ (D) $\cos^{-1}x$

57. [Medium] $\cos^{-1}\frac{1-x^2}{1+x^2}$ for $x\ge 0$ equals:

(A) $\tan^{-1}x$ (B) $-2\tan^{-1}x$ (C) $2\tan^{-1}x$ (D) $2\cot^{-1}x$

58. [Hard] $\tan^{-1}\frac{\cos x}{1+\sin x}$, $-\frac{\pi}{2}<x<\frac{\pi}{2}$, equals:

(A) $\frac{\pi}{4}-\frac{x}{2}$ (B) $\frac{\pi}{4}+\frac{x}{2}$

(C) $\frac{x}{2}$ (D) $\frac{\pi}{2}-x$

59. [Medium] $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, $0<x<\pi$, equals:

(A) x (B) $\frac{\pi}{2}-\frac{x}{2}$

(C) $\frac{x}{2}$ (D) $2x$

60. [Medium] $\sum_{n=1}^{\infty}\tan^{-1}\frac{1}{1+n+n^2}$ equals:

(A) $\frac{\pi}{2}$ (B) π (C) $\tan^{-1}2$ (D) $\frac{\pi}{4}$

61. [Medium] $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18$ equals:

(A) $\frac{\pi}{4}$ (B) $\cot^{-1}3$ (C) $\cot^{-1}33$ (D) $\cot^{-1}2$

62. [Hard] The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $|x|<\frac{1}{2}$, $x\neq 0$, is equal to: **[JEE Main 2017 (Online)]**

(A) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$ (B) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

(C) $\frac{\pi}{4}-\cos^{-1}x^2$ (D) $\frac{\pi}{4}+\cos^{-1}x^2$

63. [Hard] If $0<x<1$, then $\sqrt{1+x^2}[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1]^{1/2}$ = **[AIEEE 2008]**

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) x

(C) $x\sqrt{1+x^2}$ (D) $\sqrt{1+x^2}$

64. [Medium] If $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$, then $4x^2-4xy\cos\alpha+y^2$ is equal to: **[AIEEE 2005]**

(A) 4 (B) $2\sin2\alpha$ (C) $-4\sin^2\alpha$ (D) $4\sin^2\alpha$

65. [Hard] If $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}-\cdots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}-\cdots\right)=\frac{\pi}{2}$ for $0<|x|<\sqrt{2}$, then x equals: **[IIT-JEE 2001]**

(A) $\frac{1}{2}$ (B) 1 (C) $-\frac{1}{2}$ (D) −1

66. [N] [Medium] Let $S=\{x:\cos^{-1}x=\pi+\sin^{-1}x+\sin^{-1}(2x+1)\}$. Then $\sum_{x\in S}(2x-1)^2$ is equal to ____. **[JEE Main 2025, 29 Jan Shift 2]**

Equations

67. [Easy] The solution of $\tan^{-1}x=\frac{\pi}{3}$ is:

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) $\frac{\pi}{3}$

68. [Medium] The solution of $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$ is:

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 1

69. [Hard] The solution of $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$ in $\left(0,\frac{1}{2}\right)$ is:

(A) $\frac{1}{3}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

70. [Medium] The solution of $2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}$ $(a,x>0)$ is:

(A) $x=\frac{1}{a}$ (B) $x=a$ (C) $x=2a$ (D) $x=a^2$

71. [Easy] The number of solutions of $\sin^{-1}x=2$ is:

(A) 1 (B) 0 (C) 2 (D) infinitely many

72. [Easy] If $\tan^{-1}x=\cot^{-1}x$, then x is:

(A) 1 (B) 0 (C) −1 (D) $\sqrt{3}$

73. [Medium] If $\sin^{-1}\frac{x}{5}+\text{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then x is:

(A) 4 (B) 3 (C) 5 (D) 1

74. [Medium] If $\cos^{-1}x>\sin^{-1}x$, then:

(A) $\frac{1}{\sqrt{2}}<x\leq1$ (B) $0\leq x\leq1$

(C) $x>0$ (D) $-1\leq x<\frac{1}{\sqrt{2}}$

75. [Easy] If $3\tan^{-1}x+\cot^{-1}x=\pi$, then x equals:

(A) 1 (B) −1 (C) 0 (D) $\sqrt{3}$

76. [Hard] Let $0<\alpha<1$, $\beta=\frac{1}{3\alpha}$ and $\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)=\frac{\pi}{4}$. Then $6(\alpha+\beta)$ is equal to: **[JEE Main 2026, 06 Apr Shift 1]**

(A) 6 (B) 7 (C) 8 (D) 9

77. [Hard] The number of solutions of $\tan^{-1}4x+\tan^{-1}6x=\frac{\pi}{6}$, where $-\frac{1}{2\sqrt{6}}<x<\frac{1}{2\sqrt{6}}$, is equal to: **[JEE Main 2026, 22 Jan Shift 1]**

(A) 3 (B) 0 (C) 2 (D) 1

Answer Key with Hints

1. (B) – Principal range.

2. (A) – $\pi-\frac{\pi}{6}$.

3. (B) – Standard value.

4. (C) – Range $(0,\pi)$.

5. (C) – $\cos^{-1}\left(-\frac{1}{2}\right)$.

6. (C) – $\sin^{-1}\frac{1}{\sqrt{2}}$.

7. (C) – Endpoint of range.

8. (B) – Endpoint of range.

9. (A) – $\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$.

10. (C) – $\frac{\pi}{3}+\frac{\pi}{3}$.

11. (D) – Definition.

12. (C) – $\sin(2\theta)=2\sin\theta\cos\theta$.

13. (D) – $\cos^{-1}\frac{1}{5\sqrt{2}}=\tan^{-1}7$, $\sin^{-1}\frac{4}{\sqrt{17}}=\tan^{-1}4$.

14. (C) – $|3x|\leq1$.

15. (B) – $\left|\frac{x}{2}\right|\leq1$.

16. (B) – $|x|\geq1$.

17. (A) – $0\leq x-1\leq1$.

18. (D) – Asymptotes excluded.

19. (D) – Constant.

20. (D) – At $x = -1$.

21. (B) – At $\sin^{-1}x = \frac{\pi}{4}$.

22. (A) – $\text{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$; $t = \sec^{-1}x \in [0,\pi] - \left\{\frac{\pi}{2}\right\}$.

23. (C) – Three conditions: $|2-|x|\ | \le 4$, $3 - x > 0$, $\log_e(3-x) \neq 0$.

24. (C) – Denominator > 0 always.

25. (B) – $\sin x + \cos x \in \left[1, \sqrt{2}\right]$ on this interval.

26. (D) – Inverse then function.

27. (A) – $\pi - \frac{3\pi}{5}$.

28. (A) – $2\pi - \frac{5\pi}{3}$.

29. (C) – Subtract π.

30. (D) – $2 \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

31. (B) – $4 \in [\pi, 2\pi]$.

32. (A) – 3-4-5 triangle.

33. (D) – 5-12-13 triangle.

34. (D) – $\sec^2 = 1 + \tan^2$.

35. (B) – $2\sin\theta\cos\theta$.

36. (C) – Sum is $\frac{\pi}{2}$.

37. (A) – $\cot\frac{\pi}{2}$.

38. (A) – Inverse then function.

39. (D) – $\frac{\pi}{6} + \frac{\pi}{3}$.

40. (B) – Already in the principal range.

41. (B) – $\frac{\sec\theta - 1}{\tan\theta} = \tan\frac{\theta}{2}$ and $\frac{\sec\theta + 1}{\tan\theta} = \cot\frac{\theta}{2}$ (watch signs of sec).

42. (C) – Convert to $\tan^{-1}$: $\frac{4}{3}, \frac{5}{12}, \frac{16}{63}$.

43. (A) – $\sin(A - B)$.

44. (B) – For $x > 1$: $\sin^{-1}\frac{2x}{1+x^2} = \pi - 2\tan^{-1}x$.

45. (B) – Complementary.

46. (A) – Negative argument rule.

47. (D) – Odd function.

48. (C) – $\frac{2/11 + 7/24}{1 - 14/264} = \frac{125/264}{250/264}$.

49. (D) – $\frac{3 - 1/2}{1 + 3/2} = 1$.

50. (A) – $\frac{1}{1 - 1/4}$.

51. (D) – $\tan^{-1}2 + \tan^{-1}3 = \frac{3\pi}{4}$.

52. (A) – $\sin^{-1}\frac{4}{5} = \cos^{-1}\frac{3}{5}$.

53. (D) – $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5}$: $\frac{27/20}{11/20}$.

54. (C) – Complementary.

55. (C) – Both terms negative.

56. (C) – Put $x = \tan\theta$.

57. (C) – Put $x = \tan\theta$.

58. (A) – Half-angle form.

59. (C) – $\tan\frac{x}{2}$.

60. (D) – $\frac{\pi}{2} - \frac{\pi}{4}$.

61. (B) – Convert to $\tan^{-1}$ and add.

62. (B) – Put $x^2 = \cos 2\theta$.

63. (C) – $\cos(\cot^{-1}x) = \frac{x}{\sqrt{1+x^2}}$, $\sin(\cot^{-1}x) = \frac{1}{\sqrt{1+x^2}}$.

64. (D) – Take cos of both sides.

65. (B) – $\sin^{-1}A + \cos^{-1}B = \frac{\pi}{2}$ requires $A = B$.

66. 5 – $\cos^{-1}x - \sin^{-1}x = \frac{\pi}{2} - 2\sin^{-1}x$.

67. (A) – $\tan\frac{\pi}{3}$.

68. (B) – Add the complementary relation: $\sin^{-1}x = \frac{\pi}{3}$.

69. (C) – $\frac{2x}{2-x^2} = \frac{8}{31}$.

70. (B) – Same form.

71. (B) – $2 > \frac{\pi}{2}$.

72. (A) – Both $\frac{\pi}{4}$.

73. (B) – $\text{cosec}^{-1}\frac{5}{4} = \sin^{-1}\frac{4}{5}$, complement $\cos^{-1}\frac{4}{5}$.

74. (D) – $\sin^{-1}x < \frac{\pi}{4}$.

75. (A) – $2\tan^{-1}x + \frac{\pi}{2} = \pi$.

76. (B) – $\tan^{-1}u + \tan^{-1}v = \frac{\pi}{4} \Rightarrow u + v = 1 - uv$.

77. (D) – For $24x^2 < 1$: $\tan^{-1}4x + \tan^{-1}6x = \tan^{-1}\frac{10x}{1-24x^2}$.

Detailed Solutions – Additional Practice Questions

12. Formula used: $\sin(2\theta) = 2\sin\theta\cos\theta$.

Solution: $\theta = \cos^{-1}\frac{1}{\sqrt{5}}$: $\sin 2\theta = 2 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5}$; $\alpha = \tan\frac{\pi}{4} = 1$.

$\sec^{-1}\frac{5}{3} = \cos^{-1}\frac{3}{5}$: $\beta = \cos(A+B) = \frac{3}{5}\cdot\frac{3}{5} - \frac{4}{5}\cdot\frac{4}{5} = -\frac{7}{25}$.

Sum $\frac{18}{25}$, product $-\frac{7}{25}$: $25x^2 - 18x - 7 = 0$.

Ans: (C) $25x^2 - 18x - 7 = 0$

13. Formula used: $\cos^{-1}\frac{1}{5\sqrt{2}} = \tan^{-1}7$, $\sin^{-1}\frac{4}{\sqrt{17}} = \tan^{-1}4$.

Solution: $\tan = \frac{7-4}{1+28} = \frac{3}{29}$.

Ans: (D) $\frac{3}{29}$

22. Formula used: $\text{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$; $t = \sec^{-1}x \in [0,\pi] - \left\{\frac{\pi}{2}\right\}$.

Solution: $16\left(t^2 + \left(\frac{\pi}{2} - t\right)^2\right)$: minimum at $t = \frac{\pi}{4}$: $16 \cdot \frac{\pi^2}{8} = 2\pi^2$; maximum at $t = \pi$: $16\left(\pi^2 + \frac{\pi^2}{4}\right) = 20\pi^2$.

Sum = $22\pi^2$.

Ans: (A) $22\pi^2$

23. Formula used: Three conditions: $|2 - |x|\ | \le 4$, $3 - x > 0$, $\log_e(3-x) \neq 0$.

Solution: $|x| \le 6$; $x < 3$; $x \neq 2$: domain [−6, 3) − {2}.

$6 + 3 + 2 = 11$.

Ans: (C) 11

24. Formula used: Denominator > 0 always.

Solution: Upper: $x^2 - 3x + 2 \le x^2 + 2x + 7 \Rightarrow x \ge -1$. Lower: $2x^2 - x + 9 \ge 0$ always.

Domain [−1, ∞).

Ans: (C) [−1, ∞)

25. Formula used: $\sin x + \cos x \in \left[1, \sqrt{2}\right]$ on this interval.

Solution: $M = \tan^{-1}\sqrt{2}$, $m = \tan^{-1}1$.

$\tan(M-m) = \frac{\sqrt{2}-1}{1+\sqrt{2}} = 3 - 2\sqrt{2}$.

Ans: (B) $3 - 2\sqrt{2}$

41. Formula used: $\frac{\sec\theta - 1}{\tan\theta} = \tan\frac{\theta}{2}$ and $\frac{\sec\theta + 1}{\tan\theta} = \cot\frac{\theta}{2}$ (watch signs of sec).

Solution: For $\theta = 2$ (second quadrant), $\sqrt{1 + \tan^2 2} = -\sec 2$, so the first argument is $\frac{-\sec 2 - 1}{\tan 2} = -\cot 1$: $\cot^{-1}(-\cot 1) = \pi - 1$.

Second: argument = $\cot\frac{1}{4}$, giving $\frac{1}{4}$. Value = $\pi - 1 - \frac{1}{4} = \pi - \frac{5}{4}$.

Ans: (B) $\pi - \frac{5}{4}$

42. Formula used: Convert to $\tan^{-1}$: $\frac{4}{3}, \frac{5}{12}, \frac{16}{63}$.

Solution: $\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12} = \tan^{-1}\frac{63}{16}$ (product less than 1), and $\tan^{-1}\frac{63}{16} + \tan^{-1}\frac{16}{63} = \frac{\pi}{2}$.

Value = $2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$.

Ans: (C) $\frac{3\pi}{2}$

43. Formula used: $\sin(A-B)$.

Solution: $\sin(A-B) = \frac{12}{13}\cdot\frac{4}{5} - \frac{5}{13}\cdot\frac{3}{5} = \frac{33}{65}$, so $A - B = \sin^{-1}\frac{33}{65} = \cos^{-1}\frac{56}{65}$.

$= \frac{\pi}{2} - \sin^{-1}\frac{56}{65}$.

Ans: (A) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

44. Formula used: For $x > 1$: $\sin^{-1}\frac{2x}{1+x^2} = \pi - 2\tan^{-1}x$.

Solution: $f(x) = \pi$ for all $x > 1$: $f(5) = \pi$.

Ans: (B) π

62. Formula used: Put $x^2 = \cos 2\theta$.

Solution: $\sqrt{1 \pm x^2} = \sqrt{2}\cos\theta, \sqrt{2}\sin\theta$: the bracket $= \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \tan\left(\frac{\pi}{4} + \theta\right)$.

Value = $\frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$.

Ans: (B) $\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

63. Formula used: $\cos(\cot^{-1}x) = \frac{x}{\sqrt{1+x^2}}$, $\sin(\cot^{-1}x) = \frac{1}{\sqrt{1+x^2}}$.

Solution: Brace = $\frac{x^2+1}{\sqrt{1+x^2}} = \sqrt{1+x^2}$; square minus 1 = x^2.

Value = $\sqrt{1+x^2}\cdot x$.

Ans: (C) $x\sqrt{1+x^2}$

64. Formula used: Take cos of both sides.

Solution: $x\cdot\frac{y}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}=\cos\alpha$.

Isolate the root and square: $4x^2-4xy\cos\alpha+y^2=4\sin^2\alpha$.

Ans: (D) $4\sin^2\alpha$

65. Formula used: $\sin^{-1}A+\cos^{-1}B=\frac{\pi}{2}$ requires $A=B$.

Solution: G.P. sums: $A=\frac{x}{1+x/2}=\frac{2x}{2+x}$, $B=\frac{2x^2}{2+x^2}$.

$\frac{2x}{2+x}=\frac{2x^2}{2+x^2}\Rightarrow 2+x^2=x(2+x)\Rightarrow x=1$.

Ans: (B) 1

66. Formula used: $\cos^{-1}x-\sin^{-1}x=\frac{\pi}{2}-2\sin^{-1}x$.

Solution: Equation: $\frac{\pi}{2}-2\sin^{-1}x=\pi+\sin^{-1}(2x+1)\Rightarrow\sin^{-1}(2x+1)=-\frac{\pi}{2}-2\sin^{-1}x$.

Taking sin: $2x+1=-\cos(2\sin^{-1}x)=2x^2-1\Rightarrow x^2-x-1=0$; the admissible root ($|2x+1|\le 1$) is $x=\frac{1-\sqrt{5}}{2}$.

$(2x-1)^2=\left(-\sqrt{5}\right)^2=5$.

Ans: 5

76. Formula used: $\tan^{-1}u+\tan^{-1}v=\frac{\pi}{4}\Rightarrow u+v=1-uv$.

Solution: With $u=1-\alpha$, $v=1-\beta$ and $s=\alpha+\beta$, $\alpha\beta=\frac{1}{3}$: $u+v=2-s$ and $uv=1-s+\frac{1}{3}$.

$2-s=1-\left(\frac{4}{3}-s\right)\Rightarrow 2s=\frac{7}{3}\Rightarrow s=\frac{7}{6}$. So $6(\alpha+\beta)=7$.

Ans: (B) 7

77. Formula used: For $24x^2<1$: $\tan^{-1}4x+\tan^{-1}6x=\tan^{-1}\frac{10x}{1-24x^2}$.

Solution: $\frac{10x}{1-24x^2}=\frac{1}{\sqrt{3}}\Rightarrow 24x^2+10\sqrt{3}x-1=0$: roots ≈ 0.054 and -0.775.

Only 0.054 lies in the interval: 1 solution.

Ans: (D) 1