

MATHEMATICS

CLASS X

As per the latest revised syllabus prescribed by CBSE

P.PUVANESWARA
M.Sc., B.Ed., M.Phil.,



50/31, MALAI VEETHI, VELAYUTHAMPALAYAM, KARUR D.T TAMIL NADU – 639117

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7 WAYS TO FOCUS ON STUDIES & REDUCE STRESS

Create a Simple Study Plan



Set daily targets.
A clear plan reduces
last-minute stress.

Study in Short Sessions



Try 25-30 min study,
5 min break.
Boosts focus & energy.

Keep Distractions Away



Silence your phone.
Study in a quiet place.

Practice Active Learning



Solve problems,
teach aloud,
use mind maps.

Take Care of Your Body



Sleep well, eat healthy,
stay hydrated.

Relax Your Mind Daily



Try breathing,
exercise, or meditation.

Think Positive & Avoid Comparison



Focus on your progress, not on others.
Believe in yourself!

Study Smart, Stay Calm, Succeed!

01 REAL NUMBERS

State Euclid’s Division lemma

If a and b are two positive integers, such that $a > b$ then there exists unique integers q and r such that $a = b \times q + r, 0 \leq r < b$. It is known as Euclid’s division lemma.

Example 1: Use Euclid’s algorithm to find HCF of 4052 and 12576.

Solution:

$$\begin{array}{r}
 4052 \text{) } 12576 \text{ (3} \\
 \underline{12156} \\
 420 \text{) } 4052 \text{ (9} \\
 \underline{3780} \\
 272 \text{) } 420 \text{ (1} \\
 \underline{272} \\
 148 \text{) } 272 \text{ (1} \\
 \underline{148} \\
 124 \text{) } 148 \text{ (1} \\
 \underline{124} \\
 24 \text{) } 124 \text{ (5} \\
 \underline{120} \\
 4 \text{) } 24 \text{ (6} \\
 \underline{24} \\
 \underline{0}
 \end{array}$$

By Euclid’s lemma,

$$12576 = 4052 \times 3 + 420$$

$$4052 = 420 \times 9 + 272$$

$$420 = 272 \times 1 + 148$$

$$272 = 148 \times 1 + 124$$

$$148 = 124 \times 1 + 24$$

$$124 = 24 \times 5 + 4$$

$$24 = 4 \times 6 + 0$$

The remainder is zero, At this stage, the last divisor 4 is HCF of 4052 and 12756.

Example 2: Show that every positive even integer is of the form $2q$, and that every positive odd integer is of the form $2q + 1$, where q is some integer.

Solution:

Let a be any positive integer and $b = 2$.

Therefore By Euclid’s lemma,

$$a = 2q + r, \text{ for some } q \text{ and } r \text{ takes value } 0 \text{ or } 1.$$

Therefore $0 \leq r < 2$.

If $r = 0, a = 2q$, then a is positive even integer.

If $r = 1$, then $a = 2q + 1$

Therefore a is positive odd integer.

Example 3: Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is some integer.

Solution:

Let a be any positive odd integer. and $b = 4$.

By lemma, $a = 4q + r$, for some q and r takes values $0, 1, 2, 3$.

Therefore if $r = 0$, then $a = 4q$

$$\text{if } r = 1, \text{ then } a = 4q + 1$$

$$\text{if } r = 2, \text{ then } a = 4q + 2$$

$$\text{if } r = 3, \text{ then } a = 4q + 3$$

Since a is odd integer, a cannot be $4q$ and $4q + 2$

(Since they are even).

Therefore any odd integer is of the form $4q + 1$ or $4q + 3$.

Example 4: A sweetseller has 420 Kaju bartis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least are of the tray. What is the maximum number of barfis that can be place in each stack for this purpose?

Solution:

Find HCF of 420 and 130.

$$\begin{array}{r}
 130 \text{) } 420 \text{ (3} \\
 \underline{390} \\
 30 \text{) } 130 \text{ (4} \\
 \underline{120} \\
 10 \text{) } 30 \text{ (3} \\
 \underline{30} \\
 \underline{0}
 \end{array}$$

By Euclid’s lemma,

$$420 = 130 \times 3 + 30$$

$$130 = 30 \times 4 + 10$$

$$30 = 10 \times 3 + 10$$

The remainder is zero, at this stage the last divisor 10 is HCF of 420 and 130.

Therefore HCF = 10.

Hence sweetseller can make stacks of 10 for both kind of barfi.

EXERCISE 1.1

1) Use Euclid’s divisor lemma to find HCF of

i) 135 and 225

Solution:

$$\begin{array}{r} 135) 225 (1 \\ \underline{135} \\ 90) 135 (1 \\ \underline{90} \\ 45) 90 (2 \\ \underline{90} \\ 0 \end{array}$$

By Euclid’s lemma,

$$225 = 135 \times 1 + 90$$

$$135 = 90 \times 1 + 45$$

$$90 = 45 \times 2 + 0$$

The remainder is zero, at this stage the last divisor 45 is HCF of 225 and 135.

ii) 196 and 38220

$$\begin{array}{r} 196) 38220 (195 \\ \underline{196} \\ 1862 \\ \underline{1764} \\ 980 \\ \underline{980} \\ 0 \end{array}$$

By Euclid’s lemma,

$$38220 = 196 \times 195 + 0$$

The remainder is zero, at this stage the last divisor 196 is HCF of 38220 and 196.

iii) 867 and 255

$$\begin{array}{r} 255) 867 (3 \\ \underline{765} \\ 102) 255 (2 \\ \underline{204} \\ 51) 102 (2 \\ \underline{102} \\ 0 \end{array}$$

By Euclid’s lemma,

$$867 = 255 \times 3 + 102$$

$$255 = 102 \times 2 + 51$$

$$102 = 51 \times 2 + 0$$

The remainder is zero, at this stage the last divisor 51 is HCF of 867 and 255.

2) Show that any positive integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$, where q is some integer.

Solution:

Let a be any positive odd integer and $b = 6$.

Therefore By Euclid’s lemma,

$$a = 6q + r, r = 0, 1, 2, 3, 4, 5.$$

If $r = 0, a = 6q$

If $r = 1, a = 6q + 1$

If $r = 2, a = 6q + 2$

If $r = 3, a = 6q + 3$

If $r = 4, a = 6q + 4$

If $r = 5, a = 6q + 5$

Since a is odd integer, ' a ' cannot be $6q, 6q + 2, 6q + 4$ (Since they are even). Therefore any odd integer is of the form $6q + 1, 6q + 3$ or $6q + 5$.

3) An army contingent 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is maximum number of columns in which they can march?

Solution:

To find maximum number of columns, find HCF of 616 and 32.

$$\begin{array}{r} 32) 616 \text{ (19)} \\ \underline{608} \\ 8) 32 \text{ (4)} \\ \underline{32} \\ 0 \end{array}$$

By Euclid's division lemma,

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

The remainder is zero, the last divisor 8 is HCF of 616 and 32.

Hence HCF of 616 and 32 is 8.

4) Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer.

Solution:

Let a be any positive integer and $b = 3$.

Therefore By Euclid's lemma,

$$a = 3q + r, r = 0, 1, 2.$$

If $r = 0$ then $a = 3q$.

If $r = 1$ then $a = 3q + 1$.

If $r = 2$ then $a = 3q + 2$.

$$\begin{aligned} \text{Now } a = 3q \Rightarrow a^2 &= 9q^2 = 3(3q^2) \\ &= 3m \text{ (Where } m = 3q^2) \end{aligned}$$

$$\begin{aligned} \text{When } a = 3q + 1, \text{ Therefore } a^2 &= (3q + 1)^2 \\ &= 9q^2 + 6q + 1 \\ &= 3[3q^2 + 2q] + 1 \text{ is a} \end{aligned}$$

Square which is of the form $3m + 1$,

where $m = 3q^2 + 2q$.

$$\begin{aligned} \text{When } a = 3q + 2, \text{ Therefore } a^2 &= (3q + 2)^2 \\ &= 9q^2 + 12q + 4 \\ &= 9q^2 + 12q + 3 + 1 \\ &= 3[3q^2 + 4q + 1] + 1 \text{ is a} \end{aligned}$$

Square number which is of the form $3m+1$

, where $m = 3q^2 + 4q + 1$.

Hence square of any positive integer is of the form $3m$ or $3m + 1$.

Theorem: Fundamental Theorem of Arithmetic

Definition:

Every composite number can be written as a product of primes and this factorisation is expressed, apart from the order in which the prime factors occur.

Example 5: Consider the numbers 4^n , where n is a natural number, check whether there is any value of n for which 4^n ends with the digit zero.

(OR)

Prove that there is no natural number for which 4^n ends with the digit zero.

Solution:

We know that any positive integer ending with zero is divisible by 5.

So, its prime factorisation must contain the prime 5.

$$\text{Now } 4^n = (2^2)^n = 2^{2n}$$

The prime factorisation of 4 is 2.

5 does not occur in the prime factorisation of 4^n .

Therefore, 4^n does not end with the digit zero for any n .

Note:

(1) For HCF

For find the HCF of two or more numbers, take the product of all common prime factor (least power) present in the prime factorisation of given numbers.

(2) LCM

For finding LCM take the product of greatest powers of each prime factors available in the prime factorisation of given numbers.

Example 6: Find the LCM and HCF of 6 and 20 by prime factorisation method.

Solution:

$2 \overline{)6}$	$2 \overline{)20}$	$6 = 2 \times 3$
$3 \overline{)3}$	$2 \overline{)10}$	$20 = 2^2 \times 5$
1	$5 \overline{)5}$	
	1	

Therefore, HCF of 6, 20 = 2 (least power of common factor).

$$\begin{aligned} \text{LCM} &= 2^2 \times 3 \times 5 \\ &= 4 \times 3 \times 5 \\ &= 60 \quad (\text{greatest power of each prime factors}) \end{aligned}$$

Example 7: Find the HCF of 96 and 404 by prime factorisation method. Hence find their LCM.

Solution:

$2 \overline{)96}$	$2 \overline{)404}$	$96 = 2^5 \times 3$
$2 \overline{)48}$	$2 \overline{)202}$	$404 = 2^2 \times 101$
$2 \overline{)24}$	$101 \overline{)101}$	
$2 \overline{)12}$	1	
$2 \overline{)6}$		
$3 \overline{)3}$		
1		

HCF = $2^2 = 4$ (least power of common factor)

LCM = $2^5 \times 3 \times 101$ (greatest power of each prime factors)

$$\begin{aligned} &= 32 \times 3 \times 101 \\ &= 96 \times 101 \\ &= 9696 \end{aligned}$$

Note:

Product of two numbers = Their LCM x Their HCF

Example 8: Find the HCF and LCM of 6, 72, 120 using the prime factorisation method.

Solution:

$2 \overline{)6}$	$2 \overline{)72}$	$2 \overline{)120}$	$6 = 2 \times 3$
$3 \overline{)3}$	$2 \overline{)36}$	$2 \overline{)60}$	$72 = 2^3 \times 3^2$
1	$2 \overline{)18}$	$2 \overline{)30}$	$120 = 2^3 \times 3 \times 5$
	$3 \overline{)9}$	$3 \overline{)15}$	
	$3 \overline{)3}$	$5 \overline{)5}$	
	1	1	

HCF of 6, 72, 120 = 2×3

$$= 6 \quad (\text{least power of common factors})$$

LCM of 6, 72, 120

$$\begin{aligned} &= 2^3 \times 3^2 \times 5 \quad (\text{Highest power of available factors}) \\ &= 8 \times 9 \times 5 = 72 \times 5 = 360 \end{aligned}$$

EXERCISE 1.2

1) Express each number as a product of its prime factors.

(i) **140**

$$\begin{array}{r} 2 \overline{)140} \quad 140 = 2^2 \times 5 \times 7 \\ 2 \overline{)70} \\ 5 \overline{)35} \\ 7 \overline{)7} \\ 1 \end{array}$$

(ii) **156**

$$\begin{array}{r} 2 \overline{)156} \quad 156 = 2^2 \times 3 \times 13 \\ 2 \overline{)78} \\ 3 \overline{)39} \\ 13 \overline{)13} \\ 1 \end{array}$$

(iii) **3825**

$$\begin{array}{r} 5 \overline{)3825} \quad 3825 = 5^2 \times 3^2 \times 17 \\ 5 \overline{)765} \\ 3 \overline{)153} \\ 3 \overline{)51} \\ 17 \overline{)17} \\ 1 \end{array}$$

(iv) **5005**

$$\begin{array}{r} 5 \overline{)5005} \quad 5005 = 5 \times 7 \times 11 \times 13 \\ 7 \overline{)1001} \\ 11 \overline{)143} \\ 13 \overline{)13} \\ 1 \end{array}$$

(v) **7429**

$$\begin{array}{r} 17 \overline{)7429} \quad 7429 = 17 \times 19 \times 23 \\ 19 \overline{)437} \\ 23 \overline{)23} \\ 1 \end{array}$$

(i) Find the LCM and HCF of the following pairs of integers and verify that LCM x HCF = product of two numbers. (i) 26 and 91

Solution:

$2 \overline{)26}$	$7 \overline{)91}$	$26 = 2 \times 13$
$13 \overline{)13}$	$13 \overline{)13}$	$91 = 7 \times 13$
1	1	

HCF of 26, 91 = 13

LCM of 26, 91 = $2 \times 7 \times 13 = 182$

product of two numbers = 26×91

= 2366 01

$$\begin{aligned} \text{LCM} \times \text{HCF} &= 13 \times 182 \\ &= 2366 \text{ ————— } \textcircled{02} \end{aligned}$$

From $\textcircled{01}$ and $\textcircled{02}$
product of two numbers = Their LCM x HCF

(ii) 510 and 92

Solution:

$$2 \overline{)510} \quad 2 \overline{)92} \quad 510 = 2 \times 5 \times 3 \times 17$$

$$5 \overline{)255} \quad 2 \overline{)46} \quad 92 = 2^2 \times 23$$

$$3 \overline{)51} \quad 23 \overline{)23}$$

$$17 \overline{)17} \quad 1$$

1

$$\text{HCF} = 2$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 17 \times 23$$

$$= 60 \times 17 \times 23 = 60 \times 391$$

$$= 23460$$

$$\text{product of two numbers} = 510 \times 92$$

$$= 46920 \text{ ————— } \textcircled{01}$$

$$\text{LCM} \times \text{HCF} = 23460 \times 2 = 46920 \text{ ————— } \textcircled{02}$$

From $\textcircled{01}$ and $\textcircled{02}$

product of two numbers = Their LCM x HCF.

(iii) 336 and 54

Solution:

$$2 \overline{)336} \quad 2 \overline{)54} \quad 336 = 2^4 \times 3 \times 7$$

$$2 \overline{)168} \quad 3 \overline{)27} \quad 54 = 2 \times 3^3$$

$$2 \overline{)84} \quad 3 \overline{)9}$$

$$2 \overline{)42} \quad 3 \overline{)3}$$

$$3 \overline{)21} \quad 1$$

$$7 \overline{)7}$$

1

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7$$

$$= 3024 \text{ ————— } \textcircled{01}$$

$$\text{product of two numbers} = 336 \times 54$$

$$= 18144 \text{ ————— } \textcircled{02}$$

$$\text{LCM} \times \text{HCF} = 3024 \times 6 = 18144$$

From $\textcircled{01}$ and $\textcircled{02}$

Product of Two Numbers = LCM x HCF

2) Find the LCM and HCF of the following intergers by prime factorisation method.

(i) 12, 15, 21

Solution:

$$2 \overline{)12} \quad 3 \overline{)15} \quad 3 \overline{)21} \quad 12 = 2^2 \times 3$$

$$2 \overline{)6} \quad 5 \overline{)5} \quad 7 \overline{)7} \quad 15 = 3 \times 5$$

$$3 \overline{)3} \quad 1 \quad 1 \quad 21 = 3 \times 7$$

1

$$\text{HCF} = 3$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 \text{ (Highest power of available factors)}$$

$$= 420$$

(ii) 17, 23, 29

Solution:

$$17 = 17 \times 1$$

$$23 = 23 \times 1$$

$$29 = 29 \times 1$$

$$\text{HCF} = 1$$

$$\text{LCM} = 17 \times 23 \times 29$$

$$= 11339$$

(iii) 8, 9, 25

Solution:

$$8 = 2^3 \times 1$$

$$9 = 3^2 \times 1$$

$$25 = 5^2 \times 1$$

$$\text{HCF} = 1$$

$$\text{LCM} = 2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25$$

$$= 9 \times 200 = 1800$$

3) Given that HCF of 306, 657 = 9. Find LCM.

Solution:

$$\text{HCF of } 306, 657 = 9$$

$$\text{Now, LCM} = \text{product of two numbers}$$

$$\begin{array}{r} \text{HCF} \\ 34 \\ \hline = 306 \times 657 \\ \hline 9 \\ \hline = 657 \times 34 = 22338 \end{array}$$

4) Check whether 6^n can end with the digit 0 for any natural numbers n .

Solution:

We know that any positive integer ending with zero is divisible by 5.

So, its prime factorisation must contain the prime number 5.

$$\text{Now } 6^n = (2 \times 3)^n = 2^n \times 3^n$$

The prime factorisation of 6 are 2 and 3.

5 does not occur in the prime factorisation of 6^n .

Therefore, 6^n does not end with the digit zero for any n .

5) Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution:

$$7 \times 11 \times 13$$

It is product of more than two prime number.

Hence it is composite number.

Now, $7 \times 11 \times 13 + 13$ is sum of composite number and prime number.

$\therefore 7 \times 11 \times 13 + 13$ is composite number.

Hence $7 \times 11 \times 13 + 13$ is composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 7 \times 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 2 \times 1$$

It is product of more than two prime number.

Hence it to composite number.

Now, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is sum of composite number and prime number.

$\therefore 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is composite number.

Hence $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number.

6) There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 min for the same. Suppose they both starts at the same point and at the same time, and go in the same direction. After how many

minutes will they meet again at the starting point?

Solution:

Find LCM of 12, 18

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

$$\text{LCM of } 12, 18 = 2^2 \times 3^2 = 4 \times 9 = 36$$

They will meet again after 36 minutes.

Revisiting Irrational Numbers

Definition:

- 1) A number which cannot be expressed in the form of $\frac{p}{q}$, q is known as an irrational numbers.
- 2) A number which cannot be expressed either terminating form or non-terminating repeating form then it is known as irrational numbers.
- 3) Numbers in the form of non-terminating non repeating are known as irrational numbers.

Note (Theorem): Let p be a prime number. If p divides a^2 , then p divides a , where a is positive.

Theorem 1.4: Prove that $\sqrt{2}$ is an irrational number.

Proof:

Let us assume that $\sqrt{2}$ is rational number.

Therefore, $\sqrt{2} = \frac{p}{q}$ (where p and q are

coprime) squaring on both sides

$$2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2 \text{ ————— (01)}$$

$$\Rightarrow 2 \text{ divides } p^2 \Rightarrow 2 \text{ divides } p \text{ (by Theorem)}$$

$$\Rightarrow \frac{p}{2} = c \text{ (where } c \text{ is constant)}$$

$$\Rightarrow p = 2c \Rightarrow p^2 = 4c^2 \text{ ————— (02)}$$

From (01) and (02)

$$2q^2 = 4c^2$$

$$\Rightarrow q^2 = 2c^2$$

$$\Rightarrow 2 \text{ divides } q^2$$

$$\Rightarrow 2 \text{ divides } q$$

Thus **2 divides p and q** , that is 2 is common factor of p and q .

This is contradicts the assumption as there is no common factor. [Therefore p and q are co-prime is only common factor is 1].

Thus $\sqrt{2}$ is not rational.

Hence $\sqrt{2}$ is irrational.

Example 9: Prove that $\sqrt{3}$ is irrational

Proof:

Let us assume that $\sqrt{3}$ is rational.

Therefore $\sqrt{3} = \frac{p}{q}$ (Where p and q are coprime)

Squaring on both sides

$$3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2 \text{ ————— (01)}$$

$$\Rightarrow 3 \text{ divides } p^2$$

$$\Rightarrow \mathbf{3 \text{ divides } p} \Rightarrow \frac{p}{3} = c \Rightarrow p = 3c$$

$$\Rightarrow p^2 = 9c^2 \text{ ————— (02)}$$

From (01) and (02)

$$3q^2 = 9c^2$$

$$\Rightarrow q^2 = 3c^2$$

$$\Rightarrow 3 \text{ divides } q^2 \Rightarrow \mathbf{3 \text{ divides } q}$$

Thus **3 divides p and q.**

That is 3 is common factor of p and q.

This is contradicts that the assumption as there is no common factor except 1. (Since p and q are coprime)

Thus $\sqrt{3}$ is not rational.

Hence $\sqrt{3}$ is irrational number.

Example 10: Show that $5 - \sqrt{3}$ is irrational.

Solution:

Assume that $5 - \sqrt{3}$ is rational

Therefore $5 - \sqrt{3} = \frac{p}{q} \Rightarrow 5 - \frac{p}{q} = \sqrt{3}$

$$\Rightarrow \frac{5q - p}{q} = \sqrt{3} \text{ ————— (01)}$$

Since p and q are integers.

Therefore $\frac{5q - p}{q}$ is a rational number.

Therefore $\sqrt{3}$ is also a rational number (From (01))

But $\sqrt{3}$ is irrational

This shows that our assumption is incorrect.

Hence $5 - \sqrt{3}$ is irrational number.

Example 11: Show that $3\sqrt{2}$ is irrational.

Solution:

Let us assume that $3\sqrt{2}$ is rational.

Therefore $3\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2} = \frac{p}{3q} \text{ ————— (01)}$

p and q are integers.

Therefore $\frac{p}{3q}$ is rational number.

Therefore $\sqrt{2}$ is also rational number (From (01))

But $\sqrt{2}$ is irrational.

This shows that our assumption is incorrect.

Hence $3\sqrt{2}$ is irrational.

EXERCISE 1.3

1) Prove that $\sqrt{5}$ is irrational.

Solution:

Let us assume that $\sqrt{5}$ is rational.

Therefore $\sqrt{5} = \frac{p}{q}$ (Where p and q are coprime)

Squaring on both sides

$$5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2 \text{ ————— (01)}$$

$$\Rightarrow 5 \text{ divides } p^2$$

$$\Rightarrow \mathbf{5 \text{ divides } p}$$

$$\Rightarrow \frac{p}{5} = c \Rightarrow p = 5c \text{ ————— (02)}$$

Therefore $p^2 = 25c^2$

From (01) and (02)

$$5q^2 = 25c^2$$

$$\Rightarrow q^2 = 5c^2$$

$$\Rightarrow 5 \text{ divides } q^2$$

$$\Rightarrow \mathbf{5 \text{ divides } q}$$

Therefore **5 divides p and q.**

That is 5 is common factor of p and q.

This is contradicts the assumption as there is no common factor except 1.

(Therefore p and q are coprime)

Thus $\sqrt{5}$ is not rational.

Hence $\sqrt{5}$ is irrational number.

2) Prove that $3 + 2\sqrt{5}$ is irrational.

Solution:

Let us assume that $3 + 2\sqrt{5}$ is rational.

Therefore $3 + 2\sqrt{5} = \frac{p}{q}$ (Where p and q are

coprime)

$$3 - \frac{p}{q} = 2\sqrt{5}$$

$$\Rightarrow \frac{3q-p}{q} = 2\sqrt{5}$$

$$\Rightarrow \sqrt{5} = \frac{3q-p}{2q} \text{ ————— } \textcircled{01}$$

Here 2, 3, p, q are integers.

Therefore $\frac{3q-p}{2q}$ is a rational number.

Therefore $\sqrt{5}$ is rational number. (From $\textcircled{01}$)

But $\sqrt{5}$ is irrational number.

This shows that our assumption is incorrect.

Hence $3 + 2\sqrt{5}$ is irrational.

3) Prove that the following are irrational.

(i) $\frac{1}{\sqrt{2}}$

Solution:

Let us assume that $\frac{1}{\sqrt{2}}$ is rational.

$$\text{Therefore } \frac{1}{\sqrt{2}} = \frac{p}{q} \Rightarrow \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \sqrt{2} = \frac{2p}{q} \text{ ————— } \textcircled{01}$$

Here 2, p, q are integers.

Therefore $\frac{2p}{q}$ is rational number.

Therefore $\sqrt{2}$ is rational number (From $\textcircled{01}$)

But $\sqrt{2}$ is irrational number.

This shows that our assumption is incorrect.

Hence $\frac{1}{\sqrt{2}}$ is irrational numbers.

(ii) $7\sqrt{5}$

Solution:

Let us assume that $7\sqrt{5}$ is rational.

$$\text{Therefore } 7\sqrt{5} = \frac{p}{q} \Rightarrow \sqrt{5} = \frac{p}{7q} \text{ ————— } \textcircled{01}$$

Here 7, p, q are integers.

Therefore $\frac{p}{7q}$ is rational.

Therefore $\sqrt{5}$ is also rational (From $\textcircled{01}$)

But $\sqrt{5}$ is irrational.

This shows that our assumption is incorrect.

Hence $7\sqrt{5}$ is rational numbers.

(iii) $6 + \sqrt{2}$

Solution:

Let us assume that $6 + \sqrt{2}$ is rational number.

$$\text{Therefore } 6 + \sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2} = \frac{p}{q} - 6$$

$$\Rightarrow \sqrt{2} = \frac{p-6q}{q} \text{ ————— } \textcircled{01}$$

Here 6, p, q are integers.

Therefore $\frac{p-6q}{q}$ is a rational number.

Therefore $\sqrt{2}$ is rational number. (From $\textcircled{01}$)

But $\sqrt{2}$ is irrational number.

This shows that our assumption is incorrect.

Hence $6 + \sqrt{2}$ is irrational number.

Rational Numbers and their decimal Expansions:

Note:

- (1) A rational number in the form of $\frac{p}{q}$ can be expressed as terminating decimal expansion only if prime factors of denominators ie q is expressible in the form of $2^m \times 5^n$.
- (2) A rational number in the form of $\frac{p}{q}$ can be expressed in the form of non-terminating repeating decimal expansion if its prime factors of denominators (ie) q has any prime number in its prime factors other than 2^S or 5^S .
- (3) If prime factors of denominators is q has 5^3 and some other number other than 2^S then also it can be expressed in the form of non-terminating repeating decimal expansion.

EXERCISE 1.4

1) With actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion

(i) $\frac{13}{3125}$

$$\begin{array}{r} 5 \overline{)3125} \\ \underline{5 \overline{)625}} \\ 5 \overline{)125} \\ \underline{5 \overline{)25}} \\ 5 \overline{)5} \\ \underline{5} \\ 1 \end{array}$$

$$\frac{13}{3125} = \frac{13}{5^5 \times 2^0}$$

The factors of denominator 3125 are $2^0 \times 5^5$.

Therefore $\frac{13}{3125}$ is a terminating decimal.

(ii) $\frac{7}{8} = \frac{7}{2^3 \times 5^0}$

The factors of denominator are $2^3 \times 5^0$.

Therefore $\frac{7}{8}$ is a terminating decimal.

(iii) $\frac{64}{455}$

$$\begin{array}{r} 5 \overline{)455} \\ \underline{7 \overline{)91}} \\ 13 \overline{)13} \\ \underline{1} \end{array}$$

The factors of denominator 455 are $5 \times 7 \times 13$ is not of the form $2^m \times 5^n$.

Therefore $\frac{64}{455}$ is non-terminating repeating decimal.

(iv) $\frac{15}{1600}$

$$\begin{array}{r} 2 \overline{)1600} \\ \underline{2 \overline{)800}} \\ 2 \overline{)400} \\ \underline{2 \overline{)200}} \\ 2 \overline{)100} \\ \underline{2 \overline{)50}} \\ 5 \overline{)25} \\ \underline{5 \overline{)5}} \\ 1 \end{array}$$

The factors of denominator 1600 are $2^6 \times 5^2$.

This is in the form $2^m \times 5^n$.

Therefore $\frac{15}{1600}$ is a terminating decimal.

(v) $\frac{29}{343}$

The factors of denominator 343 are 7^3 .

Which is not in the form of $2^m \times 5^n$

Therefore $\frac{29}{343}$ is non terminating repeating decimal.

(vi) $\frac{23}{2^3 \times 5^2}$

The factors of denominator is in the form of $2^m \times 5^n$.

Therefore $\frac{23}{2^3 \times 5^2}$ is terminating decimal.

(vii) $\frac{129}{2^2 \times 5^2 \times 7^5}$

The factors of denominator is not in the form of $2^m \times 5^n$.

Therefore it is non terminating repeating decimal.

(viii) $\frac{6}{15}$

$$\frac{6}{15} = \frac{6}{3 \times 5}$$

The factors of denominator is not in the form of $2^m \times 5^n$.

Therefore $\frac{6}{15}$ is non terminating repeating decimal.

(ix) $\frac{35}{50}$

$$= \frac{35}{5^2 \times 2}$$

The factors of denominators are $2^1 \times 5^2$.

Which is in the form of $2^m \times 5^n$.

Therefore $\frac{35}{5^2 \times 2}$ is terminating decimal.

(x) $\frac{77}{210}$

The factors of denominators are $2 \times 3 \times 5 \times 7$.

Which is not in the form of $2^m \times 5^n$.

Therefore $\frac{77}{210}$ is non-terminating repeating decimal.

2) The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational and of the form p/q then what we can say about the prime factors of q ?

(i) 43.123456789

We know that any rational has either terminating or non-terminating repeating decimal expansion. Here 43.123456789 has terminating decimal expansion.

Therefore $43.123456789 = \frac{43123456789}{1000000000} = \frac{p}{q}$

Here $q = 1000000000 = 10^9$
 $= (2 \times 5)^9$
 $= 2^9 \times 5^9$

The factors of q are $2^9 \times 5^9$

(ii) 0.120120012000120000.....

Here 0.120120012000120000 is non terminating, non-repeating decimal expansion.

So it is an irrational number.

(iii) $\overline{43.123456789}$

Here $\overline{43.123456789}$ is non terminating repeating decimal expansion. So it is a rational number.

Let $x = 43.12395678910$ ———— (01)

on multiplying 1000000000 on both sides.

Therefore $1000000000 x =$

$43123456789.\overline{123456789}$ ———— (02)

$2 \Rightarrow 1000000000 x = 43123456789.\overline{123456789}$

$1 \Rightarrow \underline{\hspace{10em} x = \hspace{10em} 43.123456789}$

$2 - 1 \Rightarrow 999999999 x = 43123456746$

$x = \frac{43123456746}{999999999} = \frac{p}{q}$ form

where $q = 999999999$

02 POLYNOMIALS

Definition:

A polynomial is in one variable x , is an algebraic expression of the form.

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where n is positive integer and a_0, a_1, a_2, \dots are constant and these are known as coefficient of polynomials and $a \neq 0$

Degree of a Polynomial

Highest power of $p(x)$ is called the degree of the polynomial.

Types of polynomial:

a) Zero polynomial:

A polynomial of degree zero, is called zero polynomial (OR)

A polynomial which contains only constant term is called a zero polynomial.

Note that zero polynomial is also known as a constant polynomial.

b) Linear polynomial:

A polynomial of degree one, is called linear polynomial.

c) Quadratic polynomial:

A polynomial of degree two, is called Quadratic polynomial.

d) Cubic polynomial:

A polynomial of degree three, is called Cubic polynomial.

e) Biquadratic polynomial:

A polynomial of degree four, is called Biquadratic polynomial.

Values of a polynomial at a given point:

If $p(x)$ is a polynomial in x and α is a real number, then the value obtained by putting $x = \alpha$ in $p(x)$ is called the value of $p(x)$ at $x = \alpha$.

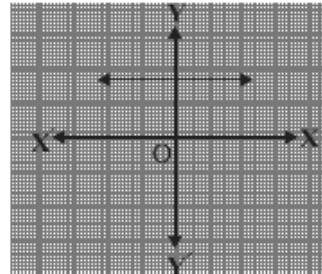
Zero of a polynomial:

A real number k is said to be a zero of a polynomial $t(x)$ is $f(k) = 0$.

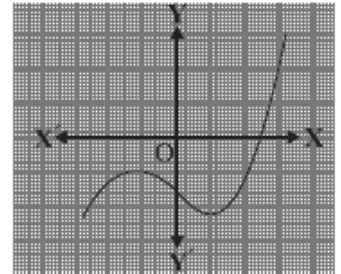
EXERCISE 2.1

Note:

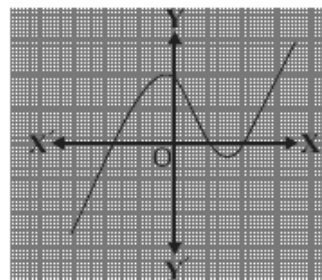
The number of points at which the graph of $p(x)$ intersects the x – axis, gives the number of zeros and their x – coordinates gives the zeros.



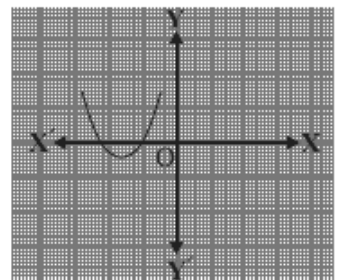
(i)



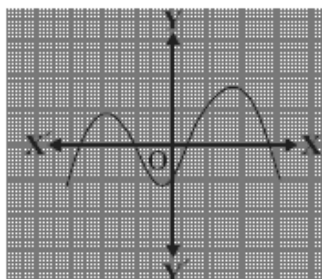
(ii)



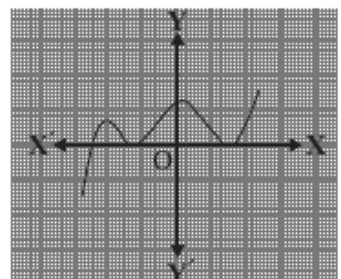
(iii)



(iv)



(v)



(vi)

Answer:

- (i) The graph does not cut the x – axis at any point. So, it has no zero.
- (ii) Here the graph cut the x – axis at one point. So, it has one zero.
- (iii) The graph cut the x – axis at three distinct points. So, it has three zeros.
- (iv) The graph cut the x – axis at two distinct points. So, it has two zeros.
- (v) The graph cut the x – axis at four distinct points. So, it has four zeros.
- (vi) The graph cut the x – axis at five distinct points. So, it has five zeros.

Relation between zeros and coefficients of polynomial:

❖ For quadratic polynomial:

In general, let α and β be the zeros of the polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$ then,

$$\begin{aligned} \text{sum of zeros } \alpha + \beta &= -\frac{b}{a} \\ &= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} \end{aligned}$$

$$\begin{aligned} \text{product of zeros } \alpha \beta &= \frac{c}{a} \\ &= \frac{\text{constant}}{\text{coefficient of } x^2} \end{aligned}$$

❖ For cubic polynomial:

In general, let α, β and γ be the zeros of the cubic polynomial.

$p(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, then

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{constant term}}{\text{coefficient of } x^3}$$

Example 2: Find the zeros of the quadratic polynomial $x^2+7x+10$, and verify the relationship between the zeros and the coefficients.

Solution:

Let $p(x) = x^2+7x+10$,

To find zeros of polynomial, $p(x) = 0$

$$(i.e) x^2+7x+10 = 0$$

$$(x+2)(x+5) = 0$$

$$\Rightarrow x+2 = 0$$

$$x = -2$$

$$x+5 = 0$$

$$x = -5$$

Therefore the zeros of the given polynomial are $\alpha = -5$, $\beta = -2$.

Comparing the given polynomial $x^2+7x+10$ with ax^2+bx+c .

Here $a = 1$, $b = 7$, $c = 10$

$$\text{Sum of zeros} = -\frac{b}{a} = -\frac{7}{1} = -7$$

$$\alpha + \beta = -5 - 2 = -7$$

$$\text{product of zeros} = \frac{c}{a} = \frac{10}{1} = 10$$

$$(i.e) \alpha \beta = (-2) (-5) = 10$$

Hence verified.

Example 3: Find the zeros of the polynomial $x^2 - 3$ and verify the relation between the zeros and the coefficients.

Answer:

Let $p(x) = x^2 - 3$

To find zeros of $p(x)$, $p(x) = 0$

$$x^2 - 3 = 0$$

$$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

Therefore the zeros of $p(x)$ are $\sqrt{3}$ and $-\sqrt{3}$

On verification, compare $p(x)$ with ax^2+bx+c

Here $a = 1$, $b = 0$, $c = -3$

$$\text{Sum of the zeros} = -\frac{b}{a} = -\frac{0}{1} = 0$$

$$\alpha + \beta = \sqrt{3} - \sqrt{3} = 0$$

$$\text{Product of zeros} = \frac{c}{a} = -\frac{3}{1} = -3$$

$$\alpha \beta = (\sqrt{3}) (-\sqrt{3}) = -3$$

Hence verified.

Formation of Quadratic and Cubic Polynomial:

- (i) If α and β are the zeros of a quadratic polynomial, Then the quadratic polynomial will be $x^2 - (\text{sum of zeros})x + \text{product of zeros}$ is $x^2 - (\alpha + \beta)x + \alpha \beta$.
- (ii) If α_1, β and γ are the zeros of cubic polynomial, then the cubic polynomial will be $x^3 - (\text{sum of zeros}) x^2 + (\text{sum of the product of two zeros at a time})x - \text{product of zeros}$.
 $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma$

Example 4: Find a quadratic polynomial the sum and product of whose zeros are -3 and 2 respectively.

Solution:

$$\text{Sum of zeros } \alpha + \beta = -3$$

$$\text{product of zeros } \alpha \beta = 2$$

Therefore the required polynomial is

$$= x^2 - (\alpha + \beta)x + \alpha \beta = x^2 - (-3)x + 2$$

$$= x^2 + 3x + 2$$

Example 5: Verify that 3, -1, $-\frac{1}{3}$ are the zeros of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$ and then verify relationship between the zeros and the coefficients.

Solution:

$$p(x) = 3x^3 - 5x^2 - 11x - 3$$

$$p(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3$$

$$= 3(27) - 5(9) - 33 - 3 = 81 - 45 - 36$$

$$p(3) = 0$$

$$p(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$$

$$= -3 - 5 + 11 - 3 = 11 - 11$$

$$p(-1) = 0$$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 - 11\left(-\frac{1}{3}\right) - 3$$

$$= 3\left(-\frac{1}{27}\right) - 5\left(\frac{1}{9}\right) + \frac{11}{3} - 3$$

$$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3 = \frac{-1-5+33-27}{9}$$

$$= \frac{33-23}{9} = 0$$

Therefore 3, -1, $-\frac{1}{3}$ are zeros of $p(x)$.

On comparing $p(x)$ with $ax^3 + bx^2 + cx + d$

Here a = 3, b = -5, c = -11, d = -3

$$\text{Sum of the zeros } \alpha + \beta + \gamma = -\frac{b}{a} = \frac{-(-5)}{3} = \frac{5}{3}$$

$$\text{But } \alpha + \beta + \gamma = 3 - 1 - \frac{1}{3} = \frac{9-3-1}{3}$$

$$= \frac{9-4}{3} = \frac{5}{3}$$

Sum of product of zeros taken two at a time

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{11}{3}$$

$$\text{But } \alpha\beta + \beta\gamma + \alpha\gamma = 3(-1) + (-1)\left(-\frac{1}{3}\right) + 3\left(-\frac{1}{3}\right)$$

$$= -3 + \frac{1}{3} - 1 = \frac{-9+1-3}{3} = -\frac{11}{3}$$

$$\text{product of zeros} = \alpha\beta\gamma = -\frac{d}{a} = \frac{+3}{3} = +1$$

$$\alpha\beta\gamma = 3(-1)\left(-\frac{1}{3}\right) = 1$$

Hence verified.

EXERCISE 2.2

1) Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.

(i) $x^2 - 2x - 8$

To find zeros, $p(x) = 0$

$$x^2 - 2x - 8 = 0$$

$$(x+2)(x-4) = 0$$

$$x+2 = 0 \quad | \quad x-4 = 0$$

$$x = -2 \quad | \quad x = 4$$

Therefore zeros of $p(x)$ are -2 and 4 on comparing $p(x)$ with $ax^2 + bx + c$

Here $a = 1, b = -2, c = -8$

$$\text{Sum of zeros} = -\frac{b}{a} = \frac{-(-2)}{1} = 2$$

$$\text{But, } \alpha + \beta = -2 + 4 = 2$$

$$\text{Product of zeros} = \frac{c}{a} = -\frac{8}{1} = -8$$

$$\alpha\beta = -2(4) = -8$$

Hence verified.

(ii) $4s^2 - 4s + 1$

Let $p(x) = 4s^2 - 4s + 1$

$$= (2s-1)(2s-1)$$

To find zeros, $p(x) = 0$

$$(2s-1)(2s-1) = 0$$

$$2s-1 = 0 \quad | \quad 2s-1 = 0$$

$$s = \frac{1}{2} \quad | \quad s = \frac{1}{2}$$

Hence zeros of polynomial are $\frac{1}{2}, \frac{1}{2}$.

$$\text{Sum of zeros} = -\frac{b}{a} = \frac{-(-4)}{4} = 1$$

$$\text{But } \alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1$$

$$\text{Product of zeros} = \frac{c}{a} = \frac{1}{4}$$

$$\text{But } \alpha\beta = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

Hence verified.

(iii) $6x^2-3-7x$

Let, $p(x) = 6x^2-7x-3$

To find zeros, $p(x) = 0$

$$6x^2-7x-3 = 0$$

$$(3x+1)(2x-3) = 0$$

$$3x+1 = 0 \quad | \quad 2x-3 = 0$$

$$3x = -1 \quad | \quad x = \frac{3}{2}$$

$$x = -\frac{1}{3}$$

$$\alpha = -\frac{1}{3} \quad \beta = \frac{3}{2}$$

on comparing $p(x)$ with ax^2+bx+c

Here $a = 6, b = -7, c = -3$

sum of the zeros = $-\frac{b}{a} = \frac{7}{6}$

But, $\alpha + \beta = -\frac{1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$

Product of zeros = $\frac{c}{a} = -\frac{3}{6} = -\frac{1}{2}$

But $\alpha \beta = -\frac{1}{3}(\frac{3}{2}) = -\frac{1}{2}$

Hence verified.

(iv) $4u^2+8u$

Let $p(x) = 4u^2+8u$

To find zeros, $p(x) = 0$

$$4u^2+8u = 0$$

$$4u(u+2) = 0$$

$$\Rightarrow u = 0 \text{ or } u+2 = 0$$

$$\Rightarrow u = 0 \text{ or } u = -2$$

Hence zeros of $p(x)$ are $\alpha = 0, \beta = -2$

on comparing $p(x)$ with ax^2+bx+c

Here $a = 4, b = 8, c = 0$

Sum of the zeros = $-\frac{b}{a} = -\frac{8}{4} = -2$

But $\alpha + \beta = 0 + (-2) = -2$

Product of zeros = $\frac{c}{a} = \frac{0}{4} = 0$

But $\alpha \beta = 0(-2) = 0$

Hence verified.

(vi) $3x^2-x-4$

Let $p(x) = 3x^2-x-4$

To find zeros, $p(x) = 0$

$$3x^2-x-4 = 0$$

$$(x+1)(3x-4) = 0$$

$$x+1 = 0 \quad | \quad 3x-4 = 0$$

$$x = -1 \quad | \quad x = \frac{4}{3}$$

Hence zeros of $p(x)$ are $\alpha = -1, \beta = \frac{4}{3}$

on comparing $p(x)$ with ax^2+bx+c

Here $a = 3, b = -1, c = -4$

Sum of the zeros = $-\frac{b}{a} = \frac{1}{3}$

But $\alpha + \beta = -1 + \frac{4}{3} = \frac{-3+4}{3} = \frac{1}{3}$

Product of zeros = $\frac{c}{a} = -\frac{4}{3}$

But, $\alpha \beta = -1(\frac{4}{3}) = -\frac{4}{3}$

Hence verified.

2) Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$

Solution:

Given, sum of zeros $\alpha + \beta = \frac{1}{4}$

product of zeros $\alpha \beta = -1$

Therefore the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha \beta$$

$$x^2 - (\frac{1}{4})x - 1 \Rightarrow \frac{4x^2-x-4}{4}$$

The other polynomial is $k(\frac{4x^2-x-4}{4})$

if $k = 4$, then the polynomial is $4x^2 - x - 4$

(ii) $\sqrt{2}, \frac{1}{3}$

Solution:

Given, sum of zeros $\alpha + \beta = \sqrt{2}$

product of zeros $\alpha \beta = \frac{1}{3}$

Therefore the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha \beta$$

$$x^2 - \sqrt{2}x + \frac{1}{3} \Rightarrow \frac{3x^2-3\sqrt{2}x+1}{3}$$

The other polynomial is $k(\frac{3x^2-3\sqrt{2}x+1}{3})$

if $k = 3$, then the polynomial is $3x^2 - 3\sqrt{2}x + 1$

(iii) $0, \sqrt{5}$ **Solution:**

Let α and β are zeros of the required polynomial

Given, sum of zeros $\alpha + \beta = 0$

product of zeros $\alpha \beta = \sqrt{5}$

Therefore the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha \beta$$

$$x^2 - 0x + \sqrt{5} \Rightarrow x^2 + \sqrt{5}$$

(iv) 1, 1**Solution:**

Let α and β are zeros of the required polynomial

Given, sum of zeros $\alpha + \beta = 1$

product of zeros $\alpha \beta = 1$

Therefore the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha \beta$$

$$x^2 - 1x + 1 \Rightarrow x^2 - x + 1$$

(v) $-\frac{1}{4}, \frac{1}{4}$ **Solution:**

Let α and β are zeros of the required polynomial

Given, Sum of zeros $\alpha + \beta = -\frac{1}{4}$

Product of zeros $\alpha \beta = \frac{1}{4}$

Therefore the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha \beta$$

$$x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4} \Rightarrow \frac{4x^2 + x + 1}{4}$$

The other polynomial is $k\left(\frac{4x^2 + x + 1}{4}\right)$

If $k = 4$, Then the polynomial is $4x^2 + x + 1$

(vi) 4, 1**Solution:**

Let α and β are zeros of the required polynomial

Given, sum of zeros $\alpha + \beta = 4$

product of zeros $\alpha \beta = 1$

Therefore the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha \beta$$

$$x^2 - 4x + 1$$

03 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Graphical Method of solution of a pair of linear equations

Example 4: Check graphically whether the pair of equations $x+3y = 6$ and $2x-3y = 12$ is consistent. If so, solve them graphically.

Solution:

$$x+3y = 6$$

$$3y = 6 - x$$

$$y = \frac{6-x}{3}$$

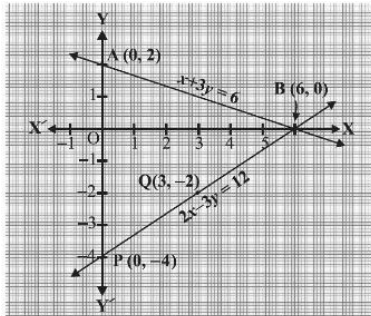
$$2x - 3y = 12$$

$$3y = 2x - 12$$

$$y = \frac{2x-12}{3}$$

x	0	6
$y = \frac{6-x}{3}$	2	0
x	0	3
$y = \frac{2x-12}{3}$	-4	-2

Plot the points A(0, 2), B(6, 0) and P(0, -4) and Q(3, -2)



Clearly, the two lines intersect at the point (6, 0). Therefore unique solution exist and system is consistent.

Example 5: Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solution.

Solution:

$$5x - 8y + 1 = 0 \quad \text{and} \quad 3x - \frac{24}{5}y + \frac{3}{5} = 0$$

$$5x - 8y + 1 = 0 \quad \text{--- (01)}$$

$$3x - \frac{24}{5}y + \frac{3}{5} = 0 \quad \text{--- (02)}$$

Multiply (02) by $\frac{5}{3}$

$$3 \times \frac{5}{3}x - \frac{24}{5} \times \frac{5}{3}y + \frac{3}{5} \times \frac{5}{3} = 0$$

$$5x - 8y + 1 = 0$$

$$5x - 8y + 1 = 0$$

From this equation (01) and (02) are same.

Hence the given lines are coincident.

Therefore the equations have infinitely many solutions.

Example 6: Champa went to a “sale” to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, “The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased”. Help her friends to find how many pants and skirts champa bought.

Solution:

Let x denote the number of pants and y denote the number of skirts.

By data, $y = 2x - 2$ and $y = 4x - 4$

$$y = 2x - 2$$

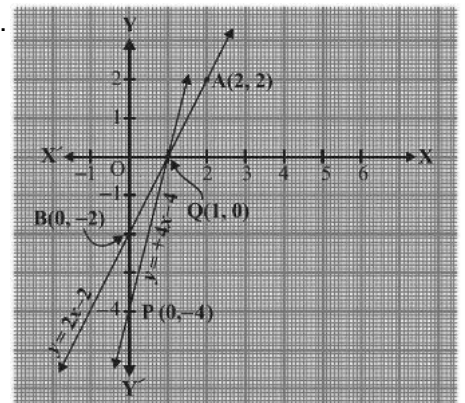
$$y = 4x - 4$$

x	0	2
y	-2	2

x	0	1
y	-4	0

From the graph the two lines intersect at (1, 0).

So, $x = 1, y = 0$.



Hence the number pants she purchased is 1 and she did not buy any skirts.

EXERCISE 3.2

1) Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of class x took part in Mathematic quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Solution:

let x denote the number of boys and y denote the number of girls.

By data, $x + y = 10$ and $y = x + 4$

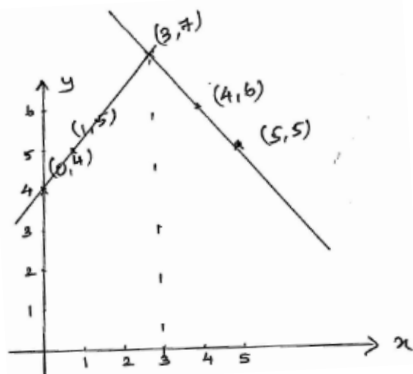
$$y = 10 - x \quad y = x + 4$$

x	0	1
y	4	5

x	5	4
y	5	6

From the graph the two lines intersect at $(3, 7)$.

So, $x = 3, y = 7$.



Hence the required number of boys is 3 and girls is 7.

(ii) 5 pencils and 7 pens together cost ₹50, where as 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and that of one pen.

Solution:

let x denote the cost of pencil (one) and y denote the cost of one pen.

By data, $5x + 7y = ₹50$ and 01

$$7x + 5y = ₹46 \quad \text{---} \quad \text{span style="border: 1px solid black; border-radius: 50%; padding: 2px;">02$$

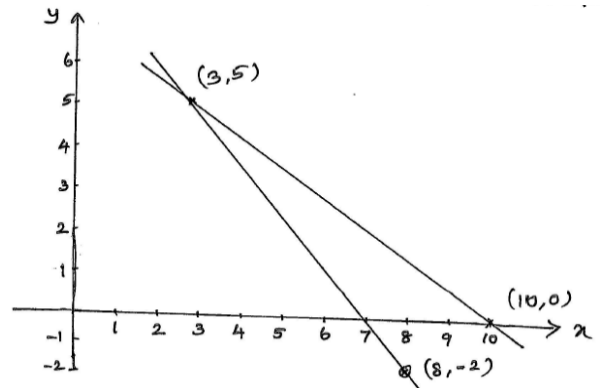
$$5x + 7y = ₹50 \quad 7x + 5y = 46$$

$$7y = 50 - 5x \quad 5y = 46 - 7x$$

$$y = \frac{50 - 5x}{7} \quad y = \frac{46 - 7x}{5}$$

x	10	3
y	0	5

x	8	3
y	-2	5



From the graph $x = 3, y = 5$ is the required solution.

Hence the cost of one pencil is ₹3 and cost of one pen is ₹5.

2) On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

(i) $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$

Here $a_1 = 5, b_1 = -4, c_1 = 8$ and

$$a_2 = 7, b_2 = 6, c_2 = -9$$

$$\text{Now } \frac{a_1}{a_2} = \frac{5}{7} \text{ and } \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore given lines are intersecting lines.

(ii) $9x + 3y + 12 = 0$ and

$$18x + 6y + 24 = 0$$

Here $a_1 = 9, b_1 = 3, c_1 = 12$ and

$$a_2 = 18, b_2 = 6, c_2 = 24$$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2};$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore given lines are coincident lines.

(iii) $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$

Here $a_1 = 6, b_1 = -3, c_1 = 10$ and

$$a_2 = 2, b_2 = -1, c_2 = 9$$

$$\frac{a_1}{a_2} = \frac{6}{2} = 3; \frac{b_1}{b_2} = \frac{-3}{-1} = 3; \frac{c_1}{c_2} = \frac{10}{9}$$

Now $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Therefore given lines are parallel.

3) On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent or inconsistent.

(i) $3x + 2y = 5, 2x - 3y = 7$ (or)
 $3x + 2y - 5 = 0$ and $2x - 3y - 7 = 0$

Note:

For consistent $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

For inconsistent $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Here $a_1 = 3, b_1 = 2, c_1 = -5$ and

$$a_2 = 2, b_2 = -3, c_2 = -7$$

$$\frac{a_1}{a_2} = \frac{3}{2}; \frac{b_1}{b_2} = \frac{2}{-3}; \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$$

Now $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore given lines are consistent.

(ii) $2x - 3y = 8, 4x - 6y = 9$

$$2x - 3y - 8 = 0; 4x - 6y - 9 = 0$$

Here $a_1 = 2, b_1 = -3, c_1 = -8$ and

$$a_2 = 4, b_2 = -6, c_2 = -9$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-8}{-9}$$

Now $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore given lines are inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

$$\frac{3}{2}x + \frac{5}{3}y - 7 = 0; 9x - 10y - 14 = 0$$

Here $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7$ and

$$a_2 = 9, b_2 = -10, c_2 = -14$$

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{2 \times 9} = \frac{1}{2 \times 3} = \frac{1}{6};$$

$$\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{5}{3(-10)} = \frac{5}{-30} = \frac{1}{-6};$$

$$\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$$

Now $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore given lines are consistent.

(iv) $5x - 3y = 11, -10x + 6y = -22$

$$5x - 3y - 11 = 0; -10x + 6y + 22 = 0$$

Here $a_1 = 5, b_1 = -3, c_1 = -11$ and

$$a_2 = -10, b_2 = 6, c_2 = 22$$

$$\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2};$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2};$$

$$\frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$$

Now $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = -\frac{1}{2}$

Therefore given lines are consistent.

(v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

$$\frac{4}{3}x + 2y - 8 = 0; 2x + 3y - 12 = 0$$

Here $a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8$ and

$$a_2 = 2, b_2 = 3, c_2 = -12$$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{3 \times 2} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{3}$$

$$\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

Now $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore given lines are consistent.

4) Which of the following pair of linear equations are consistent / inconsistent? If consistent, obtain the solution graphically.

(i) $x + y = 5$ and $2x + 2y = 10$

Solution:

$$x + y = 5 \Rightarrow x + y - 5 = 0 \text{ ————— (01)}$$

$$2x + 2y = 10 \Rightarrow$$

$$2x + 2y - 10 = 0 \text{ ————— (02)}$$

Here $a_1 = 1, b_1 = 1, c_1 = -5$ and

$$a_2 = 2, b_2 = 2, c_2 = -10$$

$$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Hence the given equations are consistent.

$$x + y = 5$$

$$2x + 2y = 10$$

$$y = 5 - x$$

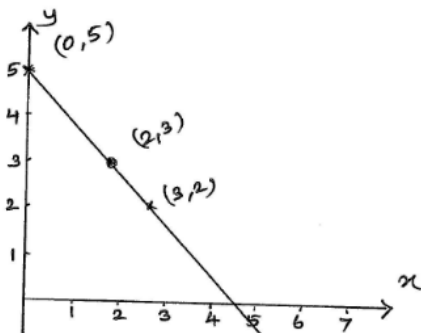
$$x + y = 5$$

$$y = 5 - x$$

x	0	3
y	5	2

x	0	1
y	-2	0

From the graph the two lines are coincident.



Therefore the equations have infinitely many solutions.

(ii) $x - y = 8$ and $3x - 3y = 16$

Solution:

$$x - y = 8$$

$$\Rightarrow x - y - 8 = 0 \text{ ————— (01)}$$

$$3x - 3y = 16$$

$$\Rightarrow 3x - 3y - 16 = 0 \text{ ————— (02)}$$

Here $a_1 = 1, b_1 = -1, c_1 = -8$ and

$$a_2 = 3, b_2 = -3, c_2 = -16$$

$$\frac{a_1}{a_2} = \frac{1}{3}; \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}; \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore the pair of linear equations is inconsistent.

(iii) $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$

Solution:

Here $a_1 = 2, b_1 = 1, c_1 = -6$ and

$$a_2 = 4, b_2 = -2, c_2 = -4$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{1}{-2}; \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence given equations consistent and have unique solution.

$$2x + y - 6 = 0$$

$$4x - 2y - 4 = 0$$

$$y = 6 - 2x$$

$$2y = 4x - 4$$

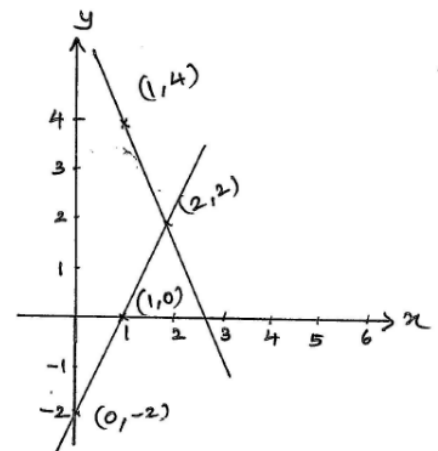
$$y = \frac{4(x-1)}{2}$$

$$y = 2(x - 1)$$

$$= 2x - 2$$

x	0	2
y	5	3

x	1	2
y	4	2



The lines are intersect at (2, 2).

Hence the solutions is $x = 2, y = 2$.

(iv) $2x - 2y - 2 = 0$ and $4x - 4y - 5 = 0$

Solution:

Here $a_1 = 2, b_1 = -2, c_1 = -2$ and

$$a_2 = 4, b_2 = -4, c_2 = -5$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence given equations are inconsistent.

- 5) Half the perimeter of a rectangular garden, whose length is 4 more than its width is 36m. Find the dimension of the garden.

Solution:

x	2	4	0
y	3	0	6

Let the length of

the garden be x m and its breadth be y m.

Perimeter of the garden = $2(x + y)$

Half the Perimeter = $x + y$

Given that, $x + y = 36$ ———— (01)

Also given, length is 4 more than its width.

$x = y + 4$ ———— (02)

$x - y = 4$ ———— (02)

$x + y = 36$ ———— (01)

$1 + 2 \Rightarrow 2x = 40$

$$x = \frac{40}{2} = 20$$

$$x = 20$$

sub $x = 20$, in 2

$$20 - y = 4 \Rightarrow y = 20 - 4$$

$$y = 16m$$

Hence length of the garden $x = 20m$

Breadth of the garden $y = 16m$.

- 6) Given the linear equation $2x + 3y - 8 = 0$ write another linear equation in two variables such that the geometrical representation of the pair so formed is
- intersecting lines
 - parallel lines
 - coincident lines

(i) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore any intersecting lines may be

$$\text{taken as } 5x + 9y - 7 = 0 \text{ or}$$

$$3x + 5y + 9 = 0.$$

(ii) For parallel lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore any parallel lines to

$$2x + 3y - 8 = 0 \text{ is } 6x + 9y + 7 = 0 \text{ or}$$

$$4x + 6y - 9 = 0$$

(iii) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore any line coincident to

$$2x + 3y - 8 \text{ is } 4x + 6y - 16 = 0 \text{ or}$$

$$8x + 12y - 32 = 0.$$

- 7) Draw the graphs of the equations

$$x - y + 1 = 0 \text{ and } 3x + 2y - 12 = 0.$$

Determine the coordinates of the vertices of the triangle formed by these lines and the x - axis and shade the triangular region.

Solution:

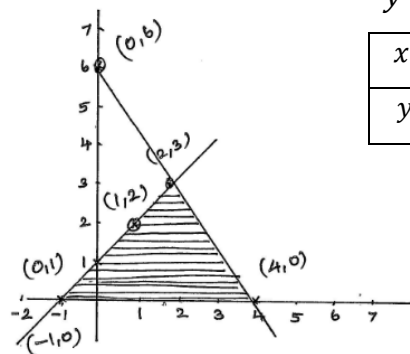
$$x - y + 1 = 0$$

$$3x + 2y - 12 = 0$$

$$y = x + 1$$

$$2y = -3x + 12$$

$$y = \frac{-3x + 12}{2}$$



x	0	1	-1
y	1	2	0

From the graph two lines intersect at $(2, 3)$.

Hence $x = 2, y = 3$ is the solution of the given pair of lines.

Algebraic Method of solving a pair of linear equations:

Substitution Method:

Example 7: Solve the following pair of equations by substitution method.

$$7x - 15y = 2 \text{ and } x + 2y = 3$$

Solution:

$$7x - 15y = 2 \text{ ———— (01)}$$

$$x + 2y = 3 \text{ ———— (02)}$$

From 2

$$x = 3 - 2y \text{ ———— (03)}$$

substitute 3 in 1

$$7(3 - 2y) - 15y = 2$$

$$21 - 14y - 15y = 2$$

$$21 - 29y = 2 \Rightarrow 29y = 21 - 2$$

$$29y = 19$$

$$y = \frac{19}{29}$$

substitute $y = \frac{19}{29}$ in 3

$$x = 3 - 2\left(\frac{19}{29}\right) = 3 - \frac{38}{29} = \frac{87-38}{29} = \frac{49}{29}$$

Therefore the solution is $x = \frac{49}{29}$ and $y = \frac{19}{29}$

Example 8: Aftab tells his daughter. “Seven years ago, I was seven times as old as you were then, Also, three years from now, I shall be three times as old as you will be. Represent this in substitution method.

Solution:

Let the father’s present age be x and daughter present age be y years.

Seven years ago father’s age = $x - 7$

His daughter’s age = $y - 7$

By data, $x - 7 = 7(y - 7)$

$$x - 7 = 7y - 49$$

$$x - 7y = 7 - 49$$

$$x - 7y = -42 \text{ ————— (01)}$$

3 years hence, father’s age = $x + 3$ years

Daughter’s age = $y + 3$

By data, $x + 3 = 3(y + 3)$

$$x + 3 = 3y + 9$$

$$x - 3y = 9 - 3$$

$$x - 3y = 6 \text{ ————— (02)}$$

Therefore Algebraic representation is

$$x - 7y = -42 \text{ ————— (01)}$$

$$x - 3y = 6 \text{ ————— (02)}$$

$$2 \Rightarrow x = 6 + 3y \text{ ————— (03)}$$

substitute (03) in (01)

$$6 + 3y - 7y = -42$$

$$6 - 4y = -42$$

$$4y = 42 + 6$$

$$4y = 48$$

$$y = \frac{48}{4} = 12$$

Therefore Aftab present age is 42 and His daughter’s present age is 12.

Example 9: Romila went to a stationary shop and purchased 2 pencil and 3 erasers for ₹9. Her friend sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for ₹18. Represent this in substitution method.

Solution:

Let the cost of one pencil be ₹ x and the cost of one eraser be ₹ y .

Given that Romila purchased 2 pencil and 3 eraser for ₹9.

$$\text{Therefore } 2x + 3y = 9 \text{ ————— (01)}$$

Also, given that sonali purchased 4 pencils and 6 erasers for ₹18.

$$\text{Therefore } 4x + 6y = 18 \text{ ————— (02)}$$

$$\text{From 1 } 2x = 9 - 3y$$

$$x = \frac{9-3y}{2} \text{ ————— (03)}$$

substitute (03) in (02)

$$4\left(\frac{9-3y}{2}\right) + 6y = 18$$

$$2(9-3y) + 6y = 18$$

$$18 - 6y + 6y = 18$$

$$18 = 18$$

This given that both the given equations are same.

Therefore the given equations have infinitely many solution. We cannot find the cost of pencil and eraser uniquely.

Example 10: Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ Represent this by substitution method.

Solution:

$$x + 2y - 4 = 0 \text{ ————— (01)}$$

$$2x + 4y - 12 = 0 \text{ ————— (02)}$$

$$\text{From 1 } x = 4 - 2y \text{ ————— (03)}$$

Substitute (03) in (02)

$$2(4 - 2y) + 4y - 12 = 0$$

$$8 - 4y + 4y - 12 = 0$$

$$8 - 12 = 0 \Rightarrow -4 = 0$$

which is a false statement.

So, the two rails will not cross each other.

EXERCISE 3.3

1. Solve the following pair of linear equation by the substitution method.

(i) $x + y = 14$ and $x - y = 4$

Solution:

$$x + y = 14 \text{ ————— (01)}$$

$$x - y = 4 \text{ ————— (02)}$$

From (02) $x = y + 4$ ————— (03)

substitute (03) in (01)

$$y + 4 + y = 14$$

$$2y = 14 - 4 = 10$$

$$2y = 10 \Rightarrow y = \frac{10}{2} = 5$$

$$y = 5$$

substitute $y = 5$ in (03)

$$x = 5 + 4 = 9$$

Therefore the solution is $x=9$ and $y=5$

(ii) $s - t = 3$, $\frac{s}{3} + \frac{t}{2} = 6$

Solution:

$$s - t = 3 \text{ ————— (01) and}$$

$$\frac{s}{3} + \frac{t}{2} = 6 \text{ ————— (02)}$$

$$\Rightarrow \frac{2s + 3t}{6} = 6 \text{ ————— (03)}$$

$$\Rightarrow 2s + 3t = 36$$

From 1 $s = 3 + t$

substitute 3 in 2

$$2(3 + t) + 3t = 36$$

$$6 + 2t + 3t = 36$$

$$5t = 30$$

$$t = \frac{30}{5} = 6$$

substitute $t = 6$ in 3

$$s = 3 + 6 = 9$$

$$s=9$$

Hence the solution is $t=6$, $s=9$

(iii) $3x - y = 3$ and $9x - 3y = 9$

Solution:

$$3x - y = 3 \text{ ————— (01)}$$

$$9x - 3y = 9 \text{ ————— (02)}$$

substitute (01) in (02)

$$9x - 3(3x - 3) = 9$$

$$9x - 9x + 9 = 9$$

$$9 = 9$$

This shows that the given equations has infinitely many solutions.

(iv) $0.2x + 0.3y = 1.3$ and $0.4x + 0.5y = 2.3$

Solution:

$$0.2x + 0.3y = 1.3 \text{ ————— (01)}$$

$$1 \times 10 \Rightarrow 2x + 3y = 13 \text{ ————— (02)}$$

$$0.4x + 0.5y = 2.3 \text{ ————— (03)}$$

$$(03) \times 10 \Rightarrow 4x + 5y = 23 \text{ ————— (04)}$$

From (02) $2x + 3y = 13$

$$x = \frac{13-3y}{2} \text{ ————— (05)}$$

Substitute (05) in (04)

$$4\left(\frac{13-3y}{2}\right) + 5y = 23$$

$$26 - 6y + 5y = 23$$

$$26 - y = 23 \Rightarrow y = 26 - 23$$

$$y = 3$$

substitute $y = 3$ in (05)

$$x = \frac{13-3(3)}{2} = \frac{13-9}{2} = \frac{4}{2} = 2$$

Hence the solution is $x = 2$, $y = 3$.

(v) $\sqrt{2}x + \sqrt{3}y = 0$ and $\sqrt{3}x - \sqrt{8}y = 0$

Solution:

$$\sqrt{2}x + \sqrt{3}y = 0 \text{ ————— (01)}$$

$$\sqrt{3}x - \sqrt{8}y = 0 \text{ ————— (02)}$$

From 2 $\sqrt{3}x = \sqrt{8}y$

$$x = \frac{\sqrt{8}}{\sqrt{3}}y \text{ ————— (03)}$$

Substitute (03) in (01)

$$\sqrt{2}\left(\frac{\sqrt{8}}{\sqrt{3}}y\right) + \sqrt{3}y = 0$$

$$\frac{\sqrt{16}y}{\sqrt{3}} + \sqrt{3}y = 0$$

$$\frac{4y}{\sqrt{3}} + \sqrt{3}y = 0$$

$$\frac{4y + 3y}{\sqrt{3}} = 0 \Rightarrow 7y = 0 \Rightarrow y = 0$$

substitute $y = 0$ in

$$x = \frac{\sqrt{8}}{\sqrt{3}}(0) = 0$$

Hence the solution is $x = 0$ and $y = 0$.

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$ and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Solution:

$$\frac{3x}{2} - \frac{5y}{3} = -2 \qquad \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{9x-10y}{6} = -2 \qquad \frac{2x+3y}{6} = \frac{13}{6}$$

$$9x - 10y = -12 \text{ ————— (01)}$$

$$2x + 3y = 13 \text{ ————— (02)}$$

From 2 $2x = 13 - 3y$

$$x = \frac{13-3y}{2} \text{ ————— (03)}$$

substitute 3 in 1

$$9\left(\frac{13-3y}{2}\right) - 10y = -12$$

$$\frac{117-27y}{2} - 10y = -12$$

$$\frac{117-27y-20y}{2} = -12$$

$$117 - 47y = -24$$

$$-47y = -24 - 117$$

$$-47y = -141$$

$$y = \frac{141}{47} = 3$$

$$y = 3$$

substitute $y = 3$ in 3

$$x = \frac{13 - 3(3)}{2}$$

$$x = \frac{13 - 9}{2}$$

$$x = \frac{4}{2} = 2$$

Hence the solution is $x=2$, $y=3$.

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Solution:

$$2x + 3y = 11 \text{ ————— (01)}$$

$$2x - 4y = -24 \text{ ————— (02)}$$

From (02) $2x = 4y - 24$

$$x = \frac{4y-24}{2} = \frac{2(2y-12)}{2} = 2y-12 \text{ ————— (03)}$$

Substitute $x = 2y - 12$ in (01)

$$2(2y - 12) + 3y = 11$$

$$4y - 24 + 3y = 11$$

$$7y = 11 + 24$$

$$7y = 35$$

$$y = \frac{35}{7}$$

$$y = 5$$

substitute $y = 5$ in (03)

$$x = 2(5) - 12$$

$$x = 10 - 12 = -2$$

Hence the solution is $x = -2$ and $y = 5$

Now $y = mx + 3$ ————— (04)

substitute $x = -2$ and $y = 5$ in

$$5 = m(-2) + 3$$

$$5 - 3 = -2m$$

$$2 = -2m$$

$$m = -1$$

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Solution:

Let the two nos be x and $y(x>y)$

By data, $x - y = 26$ ————— (01)

and $x = 3y$ ————— (02)

substitute (02) in (01)

$$3y - y = 26$$

$$2y = 26$$

$$y = \frac{26}{2} = 13$$

substitute $y = 13$ in

$$x = 3(13) = 39$$

Hence the two numbers are 39 and 13.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Solution:

Let the supplementary angles be x° and

y° ($x^\circ > y^\circ$)

By data, $x^\circ + y^\circ = 180^\circ$ ——— (01)

and $x^\circ - y^\circ = 18^\circ$ ——— (02)

From (02) $x^\circ = y^\circ + 18^\circ$ ——— (03)

substitute (03) in (01)

$$y + 18^\circ + y^\circ = 180^\circ$$

$$2y^\circ = 180^\circ - 18^\circ$$

$$2y^\circ = 162^\circ$$

$$y^\circ = \frac{162}{2} = 81^\circ$$

$$y = 81^\circ$$

substitute $y = 81^\circ$ in (01)

$$x = 81^\circ + 18^\circ = 99^\circ$$

Hence the required angles are 99° and 81° .

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.

Solution:

let the cost of one bat be ₹ x and the cost of one ball be ₹ y .

By data, $7x + 6y = 3800$ ——— (01)

$3x + 5y = 1750$ ——— (02)

From 2

$$3x = 1750 - 5y$$

$$x = \frac{1750 - 5y}{3}$$
 ——— (03)

substitute (03) in (01)

$$7\left(\frac{1750 - 5y}{3}\right) + 6y = 3800$$

$$\frac{12250 - 35y + 18y}{3} = 3800$$

$$12250 - 17y = 3800 \times 3$$

$$12250 - 17y = 11400$$

$$17y = 12250 - 11400$$

$$17y = 850 \Rightarrow y = \frac{850}{17} = 50$$

substitute $y = 50$ in 3

$$x = \frac{1750 - 5(50)}{3}$$

$$x = \frac{1750 - 250}{3}$$

$$x = \frac{1500}{3} = 500$$

Hence the cost of one bat is ₹500 and cost of one ball is ₹50.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Solution:

Let the fixed charge be ₹ x and the charge per kilometre be ₹ y .

By data, $x + 10y = 105$ ——— (01)

$x + 15y = 155$ ——— (02)

From 1 $x = 105 - 10y$ ——— (03)

substitute (03) in (02)

$$105 - 10y + 15y = 155$$

$$5y = 155 - 105$$

$$5y = 50 \Rightarrow y = \frac{50}{5} = 10$$

substitute $y = 10$ in (01)

$$x = 105 - 10(10)$$

$$x = 105 - 100$$

$$x = 5$$

Hence fixed charge is ₹5 and charge per km is ₹10.

Hence the amount paid by the person for 25 km is $5 + 25(10) = 5 + 250 = ₹255$

- (v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

Solution:

Let the fraction be $\frac{x}{y}$

condition I, 2 is added to both numerator and

denominator, the new fraction is $\frac{x+2}{y+2}$

By data, $\frac{x+2}{y+2} = \frac{9}{11}$

$$\Rightarrow 11(x+2) = 9(y+2)$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = 18 - 22$$

$$11x - 9y = -4 \quad \text{--- (01)}$$

condition II, 3 is added to both numerator

and denominator, the new fraction is $\frac{x+3}{y+3}$.

By data, $\frac{x+3}{y+3} = \frac{5}{6}$

$$\Rightarrow 6(x+3) = 5(y+3)$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = 15 - 18$$

$$6x - 5y = -3 \quad \text{--- (02)}$$

From (02) $6x - 5y = -3$

$$x = \frac{5y-3}{6} \quad \text{--- (03)}$$

substitute (03) in (01)

$$11\left(\frac{5y-3}{6}\right) - 9y = -4$$

$$\frac{55y - 33 - 54y}{6} = -4$$

$$y - 33 = -24$$

$$y = 33 - 24$$

$$y = 9$$

substitute $y = 9$ in (03)

$$x = \frac{5(9)-3}{6} = \frac{45-3}{6} = \frac{42}{6} = 7$$

Therefore the fraction is $\frac{x}{y} = \frac{7}{9}$.

- (vi) Five year hence, the age of Jacob will be three times that of his son. Five years ago Jacob's age was seven times that of his son. What are their present ages?

Solution:

Let the present age of Jacob be x years and the present age of Jacob's son be y years.

5 years hence (After 5 years)

Jacob's age = $x + 5$

son's age = $y + 5$

By data, $x + 5 = 3(y + 5)$

$$x + 5 = 3y + 15$$

$$x - 3y = 15 - 5$$

$$x - 3y = 10 \quad \text{--- (01)}$$

Five years ago, Jacob's age = $x - 5$

son's age = $y - 5$

By data, $x - 5 = 7(y - 5)$

$$x - 5 = 7y - 35$$

$$x - 7y = -35 + 5$$

$$x - 7y = -30$$

$$x = 7y - 30 \quad \text{--- (02)}$$

From 2 $x = 7y - 30$ --- (03)

substitute (03) in (01)

$$7y - 30 - 3y = 10$$

$$4y = 10 + 30$$

$$4y = 40 \Rightarrow y = \frac{40}{4} = 10$$

substitute $y = 10$ in 3

$$x = 7(10) - 30 = 70 - 30 = 40$$

Hence the present age of Jacob is 40 years and his son's present age is 10 years.

Elimination Method:

Example 8: The ratio of incomes of two persons is 9:7 and the ratio of expenditure is 4:3. If each of them manages to save ₹2000 per month, find their monthly incomes.

Solution:

Ratio of incomes of two persons = 9:7

Therefore income of 1st person = $9x$

Income of 2nd person = $7x$

Ratio of their expenditures = 4:3

Therefore Expenditure of 1st person = 4y

Expenditure of 2nd person = 3y

By data Their savins are ₹2000 per month

$$9x - 4y = 2000 \text{ ————— (01)}$$

$$7x - 3y = 2000 \text{ ————— (02)}$$

$$(01) \times (03) \Rightarrow 27x - 12y = 6000 \text{ ————— (03)}$$

$$(02) \times (04) \Rightarrow 28x - 12y = 8000 \text{ ————— (04)}$$

$$(04) - (03) \Rightarrow x = 2000$$

substitute $x = 2000$ in (01)

$$9(2000) - 4y = 2000$$

$$18000 - 4y = 2000$$

$$4y = 18000 - 2000 = 16000$$

$$y = \frac{16000}{4} = 4000$$

Therefore monthly income of 1st person = $9x =$

$$9 \times 2000 = ₹18000$$

Monthly income of 2nd person = $7x = 7 \times$

$$2000 = ₹14000$$

Example 9: Use elimination method to find all possible solutions of the following pair of linear equations:- $2x + 3y = 8, 4x + 6y = 7$

Solution:

$$2x + 3y = 8 \text{ ————— (01)}$$

$$4x + 6y = 7 \text{ ————— (02)}$$

$$(01) \times (02) \Rightarrow 4x + 6y = 16 \text{ ————— (03)}$$

$$4x + 6y = 7 \text{ ————— (02)}$$

$$(03) - (02) \Rightarrow 0 = 9$$

Which is false statement.

Hence, the given equation has no solution.

Example 10: The sum of two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Solution:

Let the ten's and unit's digits of the original number, be x and y .

Therefore the original number is of the form

$$10x + y.$$

When the digits are reversed, the reversed number is of the form $10y + x$.

By data, $(10x + y) + (10y + x) = 66$

$$11x + 11y = 66$$

$$11(x + y) = 66$$

$$x + y = 6 \text{ ————— (01)}$$

Also given that the digits differ by 2.

$$\text{Therefore } x - y = 2 \text{ ————— (02)}$$

$$\frac{x + y = 6 \text{ ————— (01)}}{x + y = 6}$$

$$(01) + (02) \quad 2x = 8$$

$$x = 4$$

substitute $x = 4$ in 1 $4 + y = 6$

$$y = 6 - 4 = 2$$

Therefore the original number is

$$10x + y = 10(4) + 2 = 42$$

The reversed number is $10y + x = 10(2) + 4 = 24$

Hence two such numbers are 42 and 24.

EXERCISE 3.4

1) Solve the following pair of linear equations by the elimination method and the substitution method.

(i) $x + y = 5, 2x - 3y = 4$

Elimination method:

$$x + y = 5 \text{ ————— (01)}$$

$$2x - 3y = 4 \text{ ————— (02)}$$

$$(01) \times (03) \Rightarrow 3x + 3y = 15 \text{ ————— (03)}$$

$$2x - 3y = 4 \text{ ————— (02)}$$

$$(02) + (03) \Rightarrow 5x = 19$$

$$x = \frac{19}{5}$$

substitute $x = \frac{19}{5}$ in (01)

$$\frac{19}{5} + y = 5 \Rightarrow y = 5 - \frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$$

Therefore the solution is $x = \frac{19}{5}, y = \frac{6}{5}$

Substitution method:

$$x + y = 5 \text{ ————— (01)}$$

$$2x - 3y = 4 \text{ ————— (02)}$$

$$\text{From 1 } x = 5 - y \text{ ————— (03)}$$

substitute (03) in (02)

$$2(5 - y) - 3y = 4$$

$$10 - 2y - 3y = 4$$

$$10 - 5y = 4 \Rightarrow 5y = 10 - 4$$

$$5y = 6$$

$$y = \frac{6}{5}$$

substitute $y = \frac{6}{5}$ in (01)

$$x + \frac{6}{5} = 5 \Rightarrow x = 5 - \frac{6}{5}$$

$$x = \frac{25-6}{5} = \frac{19}{5}$$

$$x = \frac{19}{5}$$

Hence the solution is $x = \frac{19}{5}, y = \frac{6}{5}$.

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

Elimination method:-

$$3x + 4y = 10 \text{ ————— (01)}$$

$$2x - 2y = 2 \text{ ————— (02)}$$

$$\text{Equation (02) } \times 2 \Rightarrow 4x - 4y = 4 \text{ ————— (03)}$$

$$3x + 4y = 10 \text{ ————— (01)}$$

$$(01) + (03) \Rightarrow 7x = 14$$

$$x = \frac{14}{7} = 2$$

$$x = 2$$

substitute $x = 2$ in (01)

$$3(2) + 4y = 10$$

$$6 + 4y = 10$$

$$4y = 10 - 6 = 4$$

$$y = \frac{4}{4} = 1$$

$$y = 1$$

Hence the solution is $x = 2, y = 1$

Substitution method:

$$3x + 4y = 10 \text{ ————— (01)}$$

$$2x - 2y = 2 \text{ ————— (02)}$$

$$\text{From 2 } 2x = 2 + 2y$$

$$x = \frac{2(1+y)}{2} = 1 + y \text{ ————— (03)}$$

substitute $x = 1 + y$ in (01)

$$3(1 + y) + 4y = 10$$

$$3 + 3y + 4y = 10$$

$$7y = 10 - 3$$

$$7y = 7$$

$$y = \frac{7}{7} = 1$$

$$y = 1$$

substitute $y = 1$ in 3

$$\text{Therefore } x = 1 + 1 = 2$$

$$x = 2$$

Hence $x = 2, y = 1$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Solution:

Elimination method:

$$3x - 5y = 4 \text{ ————— (01)}$$

$$9x - 2y = 7 \text{ ————— (02)}$$

$$(01) \times (03) \Rightarrow 9x - 15y = 12 \text{ ————— (03)}$$

$$(02) \Rightarrow 9x - 2y = 7 \text{ ————— (02)}$$

$$(03) - (02) \Rightarrow -13y = 5$$

$$y = \frac{-5}{13}$$

substitute $y = \frac{-5}{13}$ in (01)

$$3x - 5\left(\frac{-5}{13}\right) = 4$$

$$3x + \frac{25}{13} = 4$$

$$3x = 4 - \frac{25}{13}$$

$$3x = \frac{52-25}{13}$$

$$x = \frac{27}{3 \times 13} = \frac{9}{13}$$

$$x = \frac{9}{13}$$

Substitution method:

$$3x - 5y = 4 \text{ ————— (01)}$$

$$9x - 2y = 7 \text{ ————— (02)}$$

$$\text{From 1 } 3x = 5y + 4$$

$$x = \frac{5y+4}{3} \text{ ————— (03)}$$

substitute 3 in 2

$$9\left(\frac{5y+4}{3}\right) - 2y = 7$$

$$15y + 12 - 2y = 7$$

$$13y + 12 = 7$$

$$13y = 7 - 12$$

$$13y = -5$$

$$y = \frac{-5}{13}$$

substitute $y = \frac{-5}{13}$ in 3

$$x = \frac{5\left(\frac{-5}{13}\right)+4}{3}$$

$$x = \frac{-25+52}{13 \times 3}$$

$$x = \frac{27}{13 \times 3}$$

$$x = \frac{9}{13}$$

Therefore $x = \frac{9}{13}, y = \frac{-5}{13}$

2) Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method.

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Solution: Let the fraction be $\frac{x}{y}$.

1st condition:

If we add 1 to the numerator and subtract 1 from denominator, then the new fraction is

$$\frac{x+1}{y-1}$$

$$\text{By data, } \frac{x+1}{y-1} = 1 \Rightarrow x + 1 = y - 1$$

$$\Rightarrow x - y = -1 - 1$$

$$\Rightarrow x - y = -2 \text{ ————— (01)}$$

2nd condition:

1 is added to the denominator, the fraction is

$$\frac{x}{y+1}$$

$$\text{By data, } \frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x = y + 1$$

$$2x - y = 1 \text{ ————— (02)}$$

$$2x - y = 1 \text{ ————— (02)}$$

$$x - y = -2 \text{ ————— (01)}$$

$$2 - 1 \Rightarrow \frac{x = 3}{x = 3}$$

substitute $x = 3$ in 1

$$3 - y = -2$$

$$3 + 2 = y$$

Therefore $y = 5$

Therefore fraction is $\frac{x}{y} = \frac{3}{5}$

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu.

Solution:

Let the present age of Nuri be x years and the present age of Sonu be y years.

5 years ago,

$$\text{Nuri's age} = x - 5$$

$$\text{Sonu's age} = y - 5$$

$$\text{By data, } x - 5 = 3(y - 5)$$

$$x - 5 = 3y - 15$$

$$x - 3y = -15 + 5$$

$$x - 3y = -10 \text{ ————— (01)}$$

$$\text{After 10 years, Nuri's age} = x + 10$$

$$\text{Sonu's age} = y + 10$$

$$\text{By data, } x + 10 = 2(y + 10)$$

$$x + 10 = 2y + 20$$

$$x - 2y = 20 - 10$$

$$x - 2y = 10 \text{ ————— (02)}$$

$$x - 3y = -10 \text{ ————— (01)}$$

$$2 - 1 \Rightarrow \frac{y = 20}{y = 20}$$

substitute $y = 20$ in 1

$$x - 3(20) = -10$$

$$x - 60 = -10$$

$$x = 60 - 10$$

$$x = 50$$

Therefore Nuri's present age $x = 50$ years.

Sonu's present age $y = 20$ years.

(iii) The sum of the digits of two digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Solution:

Let x be the 10's place of two digit number and y be the unit place of two digit number.

Therefore the original number is of the form $10x + y$.

When the number is reversed, the reversed number is of the form $10y + x$.

By data,

$$9(10x + y) = 2(10y + x)$$

$$90x + 9y = 20y + 2x$$

$$90x - 2x = 20y - 9y$$

$$88x = 11y$$

$$x = \frac{11y}{88} = \frac{y}{8}$$

Also given that sum of the digit is 9

$$x + y = 9 \text{ ————— (01)}$$

substitute $x = \frac{y}{8}$ in 1

$$\frac{y}{8} + y = 9 \Rightarrow \frac{y+8y}{8} = 9$$

$$9y = 72$$

$$y = \frac{72}{9} = 8$$

substitute $y = 8$ in 1

$$x + 8 = 9$$

$$x = 9 - 8 = 1$$

Hence the required two digit number is

$$10x + y = 10(1) + 8 \\ = 18$$

(iv) Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena get 25 notes in all. Find how many notes of ₹50 and ₹100 she received.

Solution:

Let x denote the number of ₹50 and y denote the number of ₹100.

$$\text{By data, } 50x + 100y = 2000 \text{ ————— (01)}$$

$$(\div 50) \quad x + 2y = 40 \text{ ————— (02)}$$

$$x + 2y = 40$$

$$\underline{x + y = 25}$$

$$2 - 1 \Rightarrow \underline{y = 15}$$

substitute $y = 15$ in 1

$$x + 15 = 25$$

$$x = 25 - 15$$

$$x = 10$$

Hence, number of ₹50 notes is 10 and the number of ₹100 notes is 15.

(v) A leading library has a fixed charge for the first 3 days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for 5 days. Find the fixed charge and the charge for each extra days?

Solution:

Let x denote the fixed charge for first 3 days, and y denote, the additional charge per day.

By data,

$$\text{Saritha paid for 7 days} = ₹27$$

$$\text{(i.e) for (3 + 4) days} = ₹27$$

$$x + 4y = ₹27 \text{ ————— (01)}$$

$$\text{Susy paid for 5 days} = ₹21$$

$$\text{(i.e) 3 days + 2 days} = ₹21$$

$$x + 2y = ₹21 \text{ ————— (02)}$$

$$x + 4y = 27 \text{ ————— (01)}$$

$$\underline{x + 2y = 21 \text{ ————— (02)}}$$

$$1 - 2 \Rightarrow \underline{2y = 6}$$

$$y = \frac{6}{2} = 3$$

$$y = 3$$

substitute $y = 3$ in 2

$$x + 2(3) = 21$$

$$x + 6 = 21$$

$$x = 21 - 6$$

$$x = 15$$

Hence fixed charge for 3 days = ₹15 and additional charge for each extra days is ₹3.

Cross Multiplication Method:

Note:

Conditions for pair of linear equations

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0 \text{ are}$$

(i) unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) No solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Note 2:

For unique solution, use cross multiplication method.

$$\begin{array}{r} x \quad y \quad 1 \\ \hline b_1 \quad c_1 \quad a_1 \quad b_1 \\ \times \quad \times \quad \times \\ b_2 \quad c_2 \quad a_2 \quad b_2 \\ \hline \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \end{array}$$

Example 14: From a Bus stand in Bangalore, if we buy 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, the total cost is ₹46; but if we buy 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur the total cost is ₹74. Find the fare from Bus stand to Malleswaram and to Yeshwanthpur.

Solution:

Let x denote the fare from Bus stand to Malleswaram and y denote the frare from Bus stand to Yeshwanthpur.

By data,

$$\begin{array}{l} 2x + 3y = 46 \quad \Rightarrow 2x + 3y - 46 = 0 \quad \text{--- (01)} \\ 3x + 5y = 74 \quad \Rightarrow 3x + 5y - 74 = 0 \quad \text{--- (02)} \end{array}$$

$$\begin{array}{r} x \quad y \quad 1 \\ \hline 3 \quad -46 \quad 2 \quad 3 \\ 5 \quad -74 \quad 3 \quad 5 \\ \hline \frac{x}{3(-74) - 5(-46)} = \frac{y}{3(-46) - 2(-74)} = \frac{1}{2(5) - 3(3)} \end{array}$$

$$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$$

$$\frac{x}{8} = \frac{y}{10} = \frac{1}{1} \Rightarrow \frac{x}{8} = 1 \Rightarrow x = 8 \text{ and}$$

$$\frac{y}{10} = 1 \Rightarrow y = 10$$

Hence the fare from Bus stand in Bangalore to Malleswaram is ₹8 and to Yeshwantapur is ₹10.

Example 15: For which values of P does the pair of equations given below has unique solution?

Solution:

$$4x + py + 8 = 0 \text{ and } 2x + 2y + 2 = 0$$

Here $a_1 = 4, b_1 = p, c_1 = 8$ and

$$a_2 = 2, b_2 = 2, c_2 = 2$$

Given that pair of equations have unique solution.

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{4}{2} \neq \frac{p}{2} \Rightarrow p \neq 4$$

Therefore for all values of p , except 4, the given pair of equations will have a unique solution.

Example 16: For what value of K will the following pair of linear equations have infinitely many solutions? $kx + 3y - (k - 3) = 0$, and $12x + ky - k = 0$

Solution:

Here $a_1 = k, b_1 = 3, c_1 = -(k - 3)$ and

$$a_2 = 12, b_2 = k, c_2 = -k$$

Given that the equations have infinitely many solutions.

Therefore $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{k}{12} = \frac{3}{k} = \frac{-(k - 3)}{-k}$$

$$\text{Therefore } \frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36$$

Therefore $k = \pm 6$

$$\frac{3}{k} = \frac{-(k - 3)}{-k}$$

$$\frac{3}{k} = \frac{k - 3}{k} \Rightarrow k - 3 = 3$$

$$\Rightarrow k = 6 \text{ or}$$

$$3k = k(k - 3)$$

$$3k = k^2 - 3k \Rightarrow k^2 - 3k - 3k = 0$$

$$\Rightarrow k^2 - 6k = 0$$

$$\Rightarrow k(k - 6) = 0$$

$$\Rightarrow k = 0, k - 6 = 0$$

Therefore $k = 6$

Therefore $k = 0$ and $k = 6$

EXERCISE 3.5

1) Which of the following pairs of linear equations has unique solutions, or infinitely many solutions or no solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0, 3x - 9y - 2 = 0$

Solution:

We have $x - 3y - 3 = 0, 3x - 9y - 2 = 0$

Here $a_1 = 1, b_1 = -3, c_1 = -3$ and

$a_2 = 3, b_2 = -9, c_2 = -2$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9}, \frac{c_1}{c_2} = \frac{-3}{-2}$

Therefore $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence the given equations has no solution.

(ii) $2x + y = 5$ and $3x + 2y = 8$

Solution:

$2x + y - 5 = 0$ and $3x + 2y - 8 = 0$

Here $a_1 = 2, b_1 = 1, c_1 = -5$ and

$a_2 = 3, b_2 = 2, c_2 = -8$

Now, $\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-8}$

Therefore $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ the equations has unique

solution.

Therefore $\frac{x}{1} \quad \frac{y}{-5} \quad \frac{1}{2}$

$\frac{1}{2} \quad \frac{-5}{-8} \quad \frac{2}{3}$

$\frac{2}{2(2)-3(1)}$

$\frac{x}{-8+10} = \frac{y}{-5(3)-(-8)2} = \frac{1}{2(2)-3(1)}$

$\frac{x}{2} = \frac{y}{-15+16} = \frac{1}{4-3}$

$\frac{x}{2} = \frac{y}{1} = \frac{1}{1}$

Consider $\frac{x}{2} = 1 \Rightarrow x = 2$

Consider $\frac{y}{1} = 1 \Rightarrow y = 1$

(iii) $3x - 5y = 20, 6x - 10y = 40$

Solution:

$3x - 5y - 20 = 0$ and $6x - 10y - 40 = 0$

Here $a_1 = 3, b_1 = -5, c_1 = -20$ and

$a_2 = 6, b_2 = -10, c_2 = -40$

Now, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

Therefore $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Hence the given equation has infinitely many solutions.

(iv) $x - 3y - 7 = 0$ and $3x - 3y - 15 = 0$

Solution:

Here $a_1 = 1, b_1 = -3, c_1 = -7$ and

$a_2 = 3, b_2 = -3, c_2 = -15$

Now, $\frac{a_1}{a_2} = \frac{1}{3} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \frac{c_1}{c_2} = \frac{-7}{-15}$

Therefore $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence the given equations has a unique solutions.

$$\begin{array}{r} \frac{x}{-3(-15)-(-3)(-7)} = \frac{y}{-7(3)-(-15)(1)} = \frac{1}{1(-3)-3(-3)} \\ \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9} \\ \frac{x}{24} = \frac{y}{-6} = \frac{1}{6} \end{array}$$

Consider

$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

Consider

$\frac{x}{24} = \frac{1}{6}$

$6x = 24$

$x = \frac{24}{6} = 4$

$\frac{y}{-6} = \frac{1}{6}$

$\Rightarrow y = -1$

2) (i) For which value of a and b does the following pair of linear equations have an infinite number of solutions? $2x+3y=7, (a-b)x + (a+b)y = 3a+b-2$

Solution:

We have $2x + 3y - 7 = 0,$

$(a - b)x + (a + b)y - (3a + b - 2) = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$ and

$a_2 = a - b, b_2 = a + b, c_2 = -(3a + b - 2)$

Now $\frac{a_1}{a_2} = \frac{2}{a-b}, \frac{b_1}{b_2} = \frac{3}{a+b}, \frac{c_1}{c_2} = \frac{7}{3a+b-2}$

For infinite number of solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$

Now, $\frac{2}{a-b} = \frac{3}{a+b} \Rightarrow 2(a + b) = 3(a - b)$

$$2a + 2b = 3a - 3b$$

$$2b + 3b = 3a - 2a$$

$$a = 5b$$

$$\text{Also, } \frac{3}{a+b} = \frac{7}{3a+b-2}$$

$$\Rightarrow 3(3a + b - 2) = 7(a + b)$$

$$9a + 3b - 6 = 7a + 7b$$

$$9a - 7a + 3b - 7b - 6 = 0$$

$$2a - 4b - 6 = 0$$

$$(\div 2), \quad a - 2b - 3 = 0$$

$$a = 2b + 3 \quad \text{--- (02)}$$

$$\text{from 1 and 2 } 2b + 3 = 5b$$

$$5b - 2b = 3$$

$$3b = 3 \Rightarrow b = 1$$

substitute $b = 1$ in equation 1

$$a = 5(1) = 5$$

Therefore $a = 5, b = 1$

Hence required values of a and b are 5 and 1.

(ii) For which value of K will the following pair of linear equations have no solution?

$$3x + y = 1, (2k - 1)x + (k - 1)y = 2k + 1$$

Solution:

We have $3x + y - 1 = 0$, and

$$(2k - 1)x + (k - 1)y - (2k + 1) = 0$$

Here $a_1 = 3, b_1 = 1, c_1 = -1$ and

$$a_2 = 2k - 1, b_2 = k - 1, c_2 = -(2k + 1)2$$

$$\text{Now } \frac{a_1}{a_2} = \frac{3}{2k-1}, \frac{b_1}{b_2} = \frac{1}{k-1}$$

$$\text{For no solution, } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \Rightarrow 3(k - 1) = 2k - 1$$

$$3k - 3 = 2k - 1$$

$$3k - 2k = 3 - 1$$

$$k = 2$$

When $k = 2$ the equation has no solution.

3) Solve the following pair of linear equations by the substitution and Cross-multiplication

$$\text{methods: } 8x + 5y = 9, 3x + 2y = 4$$

Solution: (Substitution Method)

$$8x + 5y = 9 \quad \text{--- (01)}$$

$$\text{From 1 } 5y = 9 - 8x$$

$$y = \frac{9-8x}{5} \quad \text{--- (03)}$$

$$3x + 2y = 4 \quad \text{--- (02)}$$

$$3x + 2\left(\frac{9-8x}{5}\right) = 4$$

$$3x + \frac{18-16x}{5} = 4$$

$$\frac{15x-18-16x}{5} = 4$$

$$18 - x = 20$$

$$18 - 20 = x$$

$$\text{Therefore } x = -2$$

substitute 3 in 2

substitute $x = -2$ in 3

$$y = \frac{9-8(-2)}{5} = \frac{9+16}{5} = \frac{25}{5} = 5$$

$$y = 5$$

Therefore solutions are $x = -2, y = 5$.

Cross Multiplication Method:

$$8x + 5y - 9 = 0, \text{ and } 3x + 2y - 4 = 0$$

$$\begin{array}{r} x \quad y \quad 1 \\ \hline 5 \quad -9 \quad 8 \quad 5 \\ 2 \quad -4 \quad 3 \quad 2 \end{array}$$

$$\frac{x}{5(-4)-2(-9)} = \frac{y}{3(-9)-(-4)(8)} = \frac{1}{8-(2)-3(5)}$$

$$\frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$

$$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

Consider

Also

$$\Rightarrow \frac{x}{-2} = 1 \Rightarrow x = -2 \quad \Bigg| \quad \frac{y}{5} = 1 \Rightarrow y = 5$$

Hence the solutions are $x = -2, y = 5$.

4) Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:-

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day (CCE 2010).

Solution:-

Let x denote the monthly hostel charge and y denote the cost of food for one day.

For A student:

$$x + 20y = ₹1000 \quad \text{--- (01)}$$

For B student:

$$x + 26y = ₹1180 \quad \text{--- (02)}$$

$$2 - 1 \quad x + 26y = ₹1180$$

$$x + 20y = ₹1000$$

$$\hline 6y = 180$$

$$6y = 180$$

$$\Rightarrow y = \frac{180}{6} = 30$$

substitute $y = 30$ in 1

$$x + 20(30) = 1000$$

$$x + 600 = 1000$$

$$x = 1000 - 600 = 400.$$

Hence monthly fixed hostel charge is ₹400 and cost of food for one day is ₹30.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from

the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Solution:

Let the fraction be $\frac{x}{y}$.

When 1 is subtracted from the numerator, the

fraction is $\frac{x-1}{y} = \frac{1}{3}$ (Given)

$$\Rightarrow 3(x - 1) = y$$

$$\Rightarrow 3x - 3 = y$$

$$\Rightarrow 3x - y = 3 \quad \text{--- (01)}$$

When 8 is added to its denominator, the fraction is

$$\frac{x}{y+8} = \frac{1}{8} \text{ (given)}$$

$$4x = y + 8$$

$$4x - y = 8 \quad \text{--- (02)}$$

$$3x - y = 3 \quad \text{--- (01)}$$

$$2 - 1 \Rightarrow x = 5$$

substitute $x = 5$ in 1

$$3(5) - y = 3$$

$$15 - 3 = y \Rightarrow y = 12$$

Therefore the fraction is $\frac{x}{y} = \frac{5}{12}$

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then yash would have scored 50 marks. How many questions were there in the test?

Solution:

Let x denote the number of right answer and y denote the number of wrong answer.

Total number of questions is $x + y$.

In the first case:

Marks scored for x right answers = $3x$

Marks lost for y wrong answers = $y \times 1 = y$

$$\text{By data } 3x - y = 40 \quad \text{--- (01)}$$

In the second case:

Marks scored for x right answers = $4x$

Marks lost for y wrong answers = $2y$

$$\text{By data, } 4x - 2y = 50 \quad \text{--- (02)}$$

$$1 \times 2 \Rightarrow 6x - 2y = 80 \quad \text{--- (03)}$$

$$4x - 2y = 50 \quad \text{--- (02)}$$

$$3 - 2 \Rightarrow 2x = 30$$

$$x = \frac{30}{2} = 15$$

$$x = 15$$

substitute $x = 15$ in 1

$$3(15) - y = 40$$

$$45 - y = 40 \Rightarrow y = 45 - 40 = 5$$

Hence total number of questions

$$x + y = 15 + 5 = 20$$

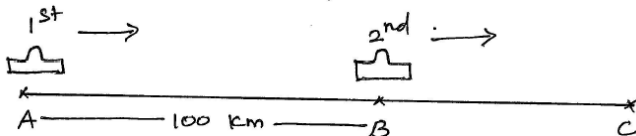
- (iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other they meet in 1 hour. What are the speeds of the cars?

Solution: (1st case)

Let speed of the 1st car = x km/hr

and speed of the 2nd car = y km/hr

1st car starts from the point A and the second car starts from the point B (same direction).



The two cars meet at C after 5 hours.

$$AC = \begin{cases} \text{The distance travelled by the} \\ \text{1st car in 5 hours} \end{cases} = 5x \text{ km}$$

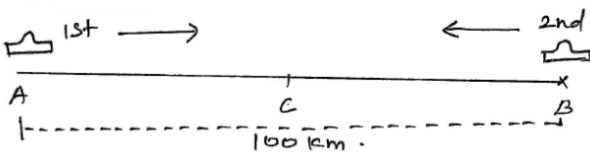
$$BC = \begin{cases} \text{The distance travelled by the} \\ \text{2nd car in 5 hours} \end{cases} = 5y \text{ km}$$

$$AC - BC = AB$$

$$5x - 5y = 100$$

$$(\div 5) \quad x - y = 20 \quad \text{--- (01)}$$

(second case)



Let two cars meet at C after one hour.

$$\text{then} \quad x + y = 100$$

$$x - y = 20$$

$$1 + 2 \quad 2x = 120$$

$$x = \frac{120}{2} = 60$$

Therefore $x = 60$

substitute $x = 60$ in 1

$$60 + y = 100$$

$$y = 100 - 60$$

$$y = 40$$

Therefore speed of the 1st car = 60 km/hr

and speed of 2nd car = 40 km/hr.

- (v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution:

Let x be the length of the rectangle and y be the breadth of the rectangle.

Therefore its area = xy .

case (i)

Area is reduced by 9 sq. units, when the length = $x - 5$ and breadth = $y + 3$

$$\text{Therefore Area} = (x - 5)(y + 3)$$

$$\text{By data, } (x - 5)(y + 3) = xy - 9$$

$$xy + 3x - 5y - 15 = xy - 9$$

$$3x - 5y = 15 - 9$$

$$3x - 5y = 6 \quad \text{--- (01)}$$

case (ii)

Area is increased by 67 sq. units when length = $(x + 3)$ and breadth $(y + 2)$

$$\text{Therefore Area} = (x + 3)(y + 2)$$

$$\text{By data, } (x + 3)(y + 2) = xy + 67$$

$$xy + 2x + 3y + 6 = xy + 67$$

$$2x + 3y = 67 - 6$$

$$2x + 3y = 61 \quad \text{--- (02)}$$

$$1 \times 2 \quad \Rightarrow 6x - 10y = 12 \quad \text{--- (03)}$$

$$2 \times 3 \quad \Rightarrow 6x + 9y = 183 \quad \text{--- (04)}$$

$$3 \times 4 \quad \Rightarrow -19y = -171$$

$$y = \frac{171}{19} = 9$$

Therefore $y = 9$

substitute $y = 9$ in 1

$$3x - 5(9) = 6$$

$$3x - 45 = 6$$

$$3x = 45 + 6$$

$$3x = 51$$

$$x = \frac{51}{3} = 17$$

$$x = 17$$

Hence length of the rectangle $x = 17$ units and breadth of the rectangle $y = 9$ units.

Equations Reducible to a pair of linear Equations in Two variables:

Example 17: Solve the pair of equations $\frac{2}{x} + \frac{3}{y} = 13$,

$$\frac{5}{x} - \frac{4}{y} = -2$$

Solution:

$$2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) = 13 \quad \text{--- (01)}$$

$$5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) = -2 \quad \text{--- (02)}$$

Let $\frac{1}{x} = a, \frac{1}{y} = b$

Therefore 1 and 2 becomes

$$2a + 3b = 13 \quad \text{--- (03)}$$

$$5a - 4b = -2 \quad \text{--- (04)}$$

$$3 \times 4 \Rightarrow 8a + 12b = 52 \quad \text{--- (05)}$$

$$4 \times 3 \Rightarrow 15a - 12b = -6 \quad \text{--- (06)}$$

$$5 + 6 \Rightarrow 23a = 46$$

$$a = \frac{46}{23} = 2$$

$a = 2$, But, $a = \frac{1}{x}$

$$\text{Hence, } \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$$

substitute $a = 2$ in 3

$$2(2) + 3b = 13$$

$$4 + 3b = 13$$

$$3b = 13 - 4$$

$$3b = 9$$

$$b = \frac{9}{3} = 3$$

$$b = 3$$

$b = 3$, But $b = \frac{1}{y}$

$$\text{Hence } \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$$

Hence the solution is $x = \frac{1}{2}, y = \frac{1}{3}$

Example 18: Solve:- $\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1$

Solution:

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \text{--- (01)}$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad \text{--- (02)}$$

Let $\frac{1}{x-1} = a$ and $\frac{1}{y-2} = b$

$$\text{Therefore } 1 \Rightarrow 5a + b = 2 \quad \text{--- (03)}$$

$$2 \Rightarrow 6a - 3b = 1 \quad \text{--- (04)}$$

$$3 \times 3 \Rightarrow 15a + 3b = 6 \quad \text{--- (05)}$$

$$6a - 3b = 1 \quad \text{--- (06)}$$

$$5 + 4 \Rightarrow 21a = 7$$

$$a = \frac{7}{21} = \frac{1}{3}$$

$$a = \frac{1}{3}$$

substitute $a = \frac{1}{3}$ in 3

$$5\left(\frac{1}{3}\right) + b = 2$$

$$b = 2 - \frac{5}{3} = \frac{6-5}{3} = \frac{1}{3}$$

$$a = \frac{1}{3}, \text{ But } a = \frac{1}{x-1}$$

$$\text{Hence, } \frac{1}{x-1} = \frac{1}{3}$$

$$x - 1 = 3$$

$$x = 4$$

Therefore $x = 4$

$$b = \frac{1}{3}, \text{ But } b = \frac{1}{y-2}$$

$$\text{Hence } \frac{1}{y-2} = \frac{1}{3}$$

$$y - 2 = 3$$

$$y = 5$$

Therefore the solution of the given equation is

$$x = 4; y = 5.$$

Example 19: A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Solution:

Let the speed of the boat in still water be x km/hr.

Let the speed of stream be y km/hr.

The speed of boat in downstream = $(x + y)$ km/hr.

Speed of boat is upstream = $(x - y)$ km/hr.

For condition I:

Distance = Speed \times Time

$$\text{Therefore time} = \frac{\text{Distance}}{\text{Speed}}$$

When boat goes 30 km upstream.

$$\text{Therefore the time taken } t_1 = \frac{30}{x-y} \text{ hours.}$$

When boat goes 44 km downstream.

$$\text{Therefore the time taken } t_2 = \frac{44}{x+y} \text{ hours.}$$

But total time taken for both upstream and down stream = 10 hours.

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \text{ ————— (01)}$$

For condition II:

When boat goes 40 km in upstream time taken

$$T_1 = \frac{40}{x-y}$$

When boat goes 55 km in down stream the time taken

$$T_2 = \frac{55}{x+y}$$

But total time taken for both Upstream and downstream

$$T_1 + T_2 = 13 \text{ hours.}$$

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \text{ ————— (02)}$$

let $\frac{1}{x-y} = a$ and $\frac{1}{x+y} = b$.

Equation 1 becomes $\Rightarrow 30a + 44b = 10$ ————— (03)

Equation 2 becomes $\Rightarrow 40a + 55b = 13$ ————— (04)

$3 \times 4 \Rightarrow 120a + 176b = 40$ ————— (05)

$4 \times 3 \Rightarrow 120a + 165b = 39$ ————— (06)

$5 - 6 \Rightarrow 11b = 1$

$$b = \frac{1}{11}$$

substitute $b = \frac{1}{11}$ in 3

$$30a + 44\left(\frac{1}{11}\right) = 10$$

$$30a + 4 = 10$$

$$30a = 10 - 4$$

$$30a = 6$$

$$a = \frac{6}{30} = \frac{1}{5}$$

But, $a = \frac{1}{x-y}$

Therefore $\frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5$ ————— (07)

$b = \frac{1}{11}$, but $b = \frac{1}{x+y}$

Therefore $\frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11$ ————— (08)

$$x - y = 5$$

$$x + y = 11 \text{ (+)}$$

$$\hline 2x = 16$$

$$x = \frac{16}{2} = 8$$

$$x = 8$$

substitutue $x = 8$ in 8

$$8 + y = 11 \Rightarrow y = 11 - 8$$

$$y = 3$$

Hence the speed of the boat in still water = 8 km/hr.

Speed of the stream water = 3 km/hr.

EXERCISE 3.6

1)Solve:-

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$; $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Solution:

$$\frac{1}{2x} + \frac{1}{3y} = 2 \text{ ————— (01)}$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6} \text{ ————— (02)}$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

Equation 1 becomes $\frac{a}{2} + \frac{b}{3} = 2$

$$\frac{3a+2b}{6} = 2$$

$$3a + 2b = 12 \text{ ————— (03)}$$

Equation 2 becomes $\frac{a}{3} + \frac{b}{2} = \frac{13}{6}$

$$\frac{2a+3b}{6} = \frac{13}{6}$$

$$\Rightarrow 2a + 3b = 13 \text{ ————— (04)}$$

$3 \times 3 \Rightarrow 9a + 6b = 36$ ————— (05)

$4 \times 2 \Rightarrow 4a + 6b = 26$ ————— (06)

$5 - 5 \Rightarrow 5a = 10$

$$a = \frac{10}{5}$$

$$a = 2$$

substitute $a = 2$ in 3

$$3(2) + 2b = 12$$

$$6 + 2b = 12$$

$$2b = 12 - 6$$

$$2b = 6$$

$$b = 3$$

Now $a = 2$ But $a = \frac{1}{x}$

Therefore $\frac{1}{x} = 2$

$$x = \frac{1}{2}$$

Now $b = 3$ But, $b = \frac{1}{y}$

$$\frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$$

Therefore the solution is $x = \frac{1}{2}, y = \frac{1}{3}$.

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$; $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Solution:

$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ ——— (01)

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$ ——— (02)

Let $\frac{1}{\sqrt{x}} = a$ and $\frac{1}{\sqrt{y}} = b$

$2a + 3b = 2$ ——— (03)

$4a - 9b = -1$ ——— (04)

$3 \times 3 \Rightarrow 6a + 9b = 6$ ——— (05)

$4a - 9b = -1$ ——— (04)

$4 + 5 \Rightarrow 10a = 5$

$a = \frac{5}{10} = \frac{1}{2}$

substitute $a = \frac{1}{2}$ in 3

$2\left(\frac{1}{2}\right) + 3b = 2$

$1 + 3b = 2$

$3b = 2 - 1$

$3b = 1$

$b = \frac{1}{3}$

$a = \frac{1}{2}$, But $\frac{1}{\sqrt{x}} = a$

Therefore $\frac{1}{\sqrt{x}} = \frac{1}{2}$

$\Rightarrow \sqrt{x} = 2$

squaring, $x = 4$

$b = \frac{1}{3}$, But $\frac{1}{\sqrt{y}} = b$

$\frac{1}{\sqrt{y}} = \frac{1}{3}$

$\Rightarrow \sqrt{y} = 3$

squaring, $y = 9$

Therefore the solution of the given equations

$x = 4, y = 9.$

(iii) $\frac{4}{x} + 3y = 14$; $\frac{3}{x} - 4y = 23$

Solution:

$\frac{4}{x} + 3y = 14$ ——— (01)

$\frac{3}{x} - 4y = 23$ ——— (02)

Put $\frac{1}{x} = a$

Therefore

Equation 1 becomes $\Rightarrow 4a + 3y = 14$ ——— (03)

Equation 2 becomes $\Rightarrow 3a - 4y = 23$ ——— (04)

$3 \times 4 \Rightarrow 16a + 12y = 56$ ——— (05)

$4 \times 3 \Rightarrow 9a - 12y = 69$ ——— (06)

$5 + 6 \Rightarrow \frac{25a}{\quad} = 125$

$a = \frac{125}{25} = 5$

$a = 5$

substitute $a = 5$ in 3

$4(5) + 3y = 14$

$20 + 3y = 14$

$3y = 14 - 20$

$3y = -6$

$y = \frac{-6}{3}$

$y = -2$

$a = 5$, But $\frac{1}{x} = a$

$\frac{1}{x} = 5$

$x = \frac{1}{5}$

Therefore $x = \frac{1}{5}, y = -2.$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$; $\frac{6}{x-1} - \frac{3}{y-2} = 1$

Solution:

$\frac{5}{x-1} + \frac{1}{y-2} = 2$ ——— (01)

$\frac{6}{x-1} - \frac{3}{y-2} = 1$ ——— (02)

Let $\frac{1}{x-1} = a, \frac{1}{y-2} = b.$

Equation 1 becomes $5a + b = 2$ ——— (03)

Equation 2 becomes $6a - 3b = 1$ ——— (04)

$3 \times 3 \Rightarrow 15a + 3b = 6$ ——— (05)

$6a - 3b = 1$ ——— (04)

$5 + 4 \Rightarrow 21a = 7$

$a = \frac{7}{21}$

$a = \frac{1}{3}$

substitute $a = \frac{1}{3}$ in 3

$5\left(\frac{1}{3}\right) + b = 2$

$\frac{5}{3} + b = 2$

$b = 2 - \frac{5}{3}$

$b = \frac{6-5}{3}$

$b = \frac{1}{3}$

$$a = \frac{1}{3}, \text{ But } \frac{1}{x-1} = a$$

$$\text{Therefore } \frac{1}{x-1} = \frac{1}{3} \Rightarrow x - 1 = 3$$

$$x = 4$$

$$b = \frac{1}{3}, \text{ But } \frac{1}{y-2} = b$$

$$\text{Therefore } \frac{1}{y-2} = \frac{1}{3} \Rightarrow y - 2 = 3$$

$$y = 5$$

Therefore solutions of the given equations are

$$x = 4, y = 5.$$

$$(v) \frac{7x-2y}{xy} = 5; \frac{8x+7y}{xy} = 15$$

Solution:

$$\frac{7x-2y}{xy} = 5$$

$$\frac{7x}{xy} - \frac{2y}{xy} = 5$$

$$\frac{7}{y} - \frac{2}{x} = 5 \text{ ————— (01)}$$

$$\frac{8x+7y}{xy} = 15$$

$$\frac{8x}{xy} + \frac{7y}{xy} = 15$$

$$\frac{8}{y} + \frac{7}{x} = 15 \text{ ————— (02)}$$

$$\text{Let } \frac{1}{x} = a; \frac{1}{y} = b$$

Therefore

$$\text{Equation 1 becomes } 7b - 2a = 5 \text{ ————— (03)}$$

$$\text{Equation 2 becomes } 8b + 7a = 15 \text{ ————— (04)}$$

$$3 \times 7 \Rightarrow 49b - 14a = 35 \text{ ————— (05)}$$

$$4 \times 2 \Rightarrow 16b + 14a = 30 \text{ ————— (06)}$$

$$5 + 6 \Rightarrow 65b = 65$$

$$b = 1$$

substitute $b = 1$ in 3

$$7(1) - 2a = 5$$

$$7 - 2a = 5$$

$$2a = 7 - 5$$

$$2a = 2$$

$$a = 1$$

$$a = 1, \text{ But } \frac{1}{x} = a$$

$$\text{Therefore, } \frac{1}{x} = 1$$

$$\Rightarrow x = 1$$

$$b = 1, \text{ But } \frac{1}{y} = b$$

$$\text{Therefore, } \frac{1}{y} = 1 \Rightarrow y = 1$$

Hence $x = 1, y = 1$.

$$(vi) 6x + 3y = 6xy \text{ and } 2x + 4y = 5xy$$

Solution:

$$6x + 3y = 6xy$$

$$\frac{6x+3y}{xy} = 6$$

$$\frac{6x}{xy} + \frac{3y}{xy} = 6$$

$$\frac{6}{y} + \frac{3}{x} = 6 \text{ ————— (01)}$$

$$2x + 4y = 5xy$$

$$\frac{2x+4y}{xy} = 5$$

$$\frac{2x}{xy} + \frac{4y}{xy} = 5$$

$$\frac{2}{y} + \frac{4}{x} = 5 \text{ ————— (02)}$$

$$\text{Let } \frac{1}{x} = a, \frac{1}{y} = b$$

Therefore

$$\text{Equation 1 becomes } 6b + 3a = 6 \text{ ————— (03)}$$

$$\text{Equation 2 becomes } 2b + 4a = 5 \text{ ————— (04)}$$

$$3 \times 4 \Rightarrow 24b + 12a = 24 \text{ ————— (05)}$$

$$4 \times 3 \Rightarrow 6b + 12a = 15 \text{ ————— (06)}$$

$$5 - 6 \Rightarrow 18b = 9$$

$$b = \frac{9}{18}$$

$$b = \frac{1}{2}$$

substitute $b = \frac{1}{2}$ in 3

$$6\left(\frac{1}{2}\right) + 3a = 6$$

$$3 + 3a = 6$$

$$3a = 6 - 3$$

$$3a = 3$$

$$a = 1$$

$$a = 1, \text{ But } \frac{1}{x} = a$$

$$\text{Therefore, } \frac{1}{x} = 1 \Rightarrow x = 1$$

$$b = \frac{1}{2}, \text{ But } \frac{1}{y} = b$$

$$\text{Therefore, } \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$$

Hence, $x = 1, y = 2$.

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$ and $\frac{15}{x+y} - \frac{5}{x-y} = -2$

Solution:

$$\frac{10}{x+y} + \frac{2}{x-y} = 4 \text{ ————— (01)}$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \text{ ————— (02)}$$

Let $\frac{1}{x+y} = a, \frac{1}{x-y} = b$

Equation 1 becomes $10a + 2b = 4$ ————— (03)

Equation 2 becomes $15a - 5b = -2$ ————— (04)

$$3 \times 5 \Rightarrow 50a + 10b = 20 \text{ ————— (05)}$$

$$4 \times 2 \Rightarrow 30a - 10b = -4 \text{ ————— (06)}$$

$$5 + 6 \Rightarrow 80a = 16$$

$$a = \frac{16}{80} = \frac{1}{5}$$

substitute $a = \frac{1}{5}$ in 3

$$10\left(\frac{1}{5}\right) + 2b = 4$$

$$2 + 2b = 4$$

$$2b = 4 - 2$$

$$2b = 2$$

$$b = 1$$

$a = \frac{1}{5}$, But, $\frac{1}{x+y} = a$

Therefore, $\frac{1}{x+y} = \frac{1}{5}$

$$\Rightarrow x + y = 5 \text{ ————— (07)}$$

$b = 1$, But, $\frac{1}{x-y} = b$

Therefore, $\frac{1}{x-y} = 1 \Rightarrow x - y = 1$ ————— (08)

$$x + y = 5 \text{ ————— (07)}$$

$$x - y = 1 \text{ ————— (08)}$$

$$7 + 8 \Rightarrow 2x = 6$$

$$x = \frac{6}{2} = 3$$

substitute $x = 3$ in 7

$$3 + y = 5$$

$$y = 5 - 3$$

$$y = 2$$

Hence, $x = 3, y = 2$.

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$ and $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

Solution:

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} \text{ ————— (01)}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8} \text{ ————— (02)}$$

Let $\frac{1}{3x+y} = a$ and $\frac{1}{3x-y} = b$

Equation 1 becomes $a + b = \frac{3}{4}$

$$\Rightarrow 4a + 4b = 3 \text{ ————— (03)}$$

Equation 2 becomes $\frac{a}{2} - \frac{b}{2} = -\frac{1}{8}$

$$\frac{a-b}{2} = -\frac{1}{8} \Rightarrow \frac{8(a-b)}{2} = -1$$

$$4(a - b) = -1$$

$$4a - 4b = -1 \text{ ————— (04)}$$

$$4a + 4b = 3 \text{ ————— (03)}$$

$$4a - 4b = -1 \text{ ————— (04)}$$

$$8a = 2$$

$$a = \frac{2}{8}$$

$$a = \frac{1}{4}$$

substitute $a = \frac{1}{4}$ in 3

$$4\left(\frac{1}{4}\right) + 4b = 3$$

$$1 + 4b = 3$$

$$4b = 3 - 1$$

$$4b = 2$$

$$b = \frac{2}{4}$$

$$b = \frac{1}{2}$$

$a = \frac{1}{4}$ But, $\frac{1}{3x+y} = a$

Therefore, $\frac{1}{3x+y} = \frac{1}{4} \Rightarrow 3x + y = 4$ ————— (05)

$b = \frac{1}{2}$ But, $\frac{1}{3x-y} = b$

Therefore, $\frac{1}{3x-y} = \frac{1}{2} \Rightarrow 3x - y = 2$ ————— (06)

$$3x + y = 4 \text{ ————— (05)}$$

$$3x - y = 2 \text{ ————— (06)}$$

$$6x = 6$$

$$x = 1$$

substitute $x = 1$ in 5

$$3(1) + y = 4$$

$$y = 4 - 3$$

$$y = 1$$

Hence, $x = 1, y = 1$.

2) Formulate the following problems as a pair of equations, and hence find their solutions:-

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of current.

Solution:

Let speed of Ritu in still water = x km/hr.

and speed of current = y km/hr.

Therefore downstream speed = $(x + y)$ km/hr.

and Upstream speed = $(x - y)$ km/hr.

case (i):

Ritu can row downstream, 20 km in 2 hours.

Then time taken $t_1 = \frac{20}{x+y}$ hours.

Therefore $\frac{20}{x+y} = 2 \Rightarrow x + y = \frac{20}{2}$

$$x + y = 10 \text{ ————— (01)}$$

case (ii):

Ritu can row upstream 4 km in 2 hours.

Therefore time taken = $\frac{4}{x-y}$.

$$\frac{4}{x-y} = 2 \Rightarrow x - y = \frac{4}{2}$$

$$x - y = 2 \text{ ————— (02)}$$

$$1+2 \Rightarrow 2x = 12$$

$$x = 6$$

substitute $x = 6$ in 1

$$6 + y = 10$$

$$y = 10 - 6$$

$$y = 4$$

Therefore Hence, speed of Ritu is still water = **6 km/hr.**

Speed of current = **4 km/hr.**

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 women alone to finish the work, and also that taken by 1 man alone.

Solution:

Let work of one woman in one day = $\frac{1}{x}$.

Work of one men in one day = $\frac{1}{y}$.

Case (i): Therefore work of 2 women and 5 men

in one day = $\frac{2}{x} + \frac{5}{y} = \frac{5x+2y}{xy}$

Therefore number of days required to complete

the work by 2 women and 5 men = $\frac{xy}{5x+2y}$.

By data, $\frac{xy}{5x+2y} = 4$ (Given)

Therefore $\frac{5x+2y}{xy} = \frac{1}{4}$

$$\frac{5x}{xy} + \frac{2y}{xy} = \frac{1}{4}$$

$$\frac{5}{y} + \frac{2}{x} = \frac{1}{4} \text{ ————— (01)}$$

case (ii):

Now work of 3 women and 6 men one day work =

$$\frac{3}{x} + \frac{6}{y} = \frac{6x+3y}{xy}$$

Therefore number of days required to complete

the work by 3 women and 6 men = $\frac{xy}{6x+3y}$.

By data, $\frac{xy}{6x+3y} = 3$ (Given)

Therefore $\frac{6x+3y}{xy} = \frac{1}{3}$

$$\frac{6x}{xy} + \frac{3y}{xy} = \frac{1}{3}$$

$$\frac{6}{y} + \frac{3}{x} = \frac{1}{3} \text{ ————— (02)}$$

let $\frac{1}{x} = a, \frac{1}{y} = b$

Equation 1 becomes $5b + 2a = \frac{1}{4}$

$$20b + 8a = 1 \text{ ————— (03)}$$

Equation 2 becomes $6b + 3a = \frac{1}{3}$

$$18b + 9a = 1 \text{ ————— (04)}$$

$$3 \times 9 \Rightarrow 180b + 72a = 9 \text{ ————— (05)}$$

$$4 \times 8 \Rightarrow 144b + 72a = 8 \text{ ————— (06)}$$

$$5 - 6 \Rightarrow \frac{36b}{36b} = 1$$

$$b = \frac{1}{36}$$

substitute $b = \frac{1}{36}$ in 3

$$20 \left(\frac{1}{36} \right) + 8a = 1$$

$$\frac{5}{9} + 8a = 1$$

$$8a = 1 - \frac{5}{9}$$

$$8a = \frac{4}{9}$$

$$a = \frac{4}{9 \times 8} = \frac{1}{18}$$

$$a = \frac{1}{18} \text{ But, } \frac{1}{x} = a \qquad b = \frac{1}{36} \text{ But, } \frac{1}{y} = b$$

$$\text{Therefore } \frac{1}{x} = \frac{1}{18}$$

$$\Rightarrow x = 18$$

$$\text{Therefore } \frac{1}{y} = \frac{1}{36}$$

$$\Rightarrow y = 36$$

Hence one woman can finish the work in 18 days. One man can finish the work in 36 days.

- (iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solution:

Let the speed of the train be x km/hr and the speed of the bus be y km/hr.

Total distance travelled by her = 300 km.

case (i): Roohi travels 60 km by train and remaining $(300-60) = 240$ km by bus in 4 hours.

$$\text{Therefore } \frac{60}{x} + \frac{240}{y} = 4$$

$$(\div 4) \quad \frac{15}{x} + \frac{60}{y} = 1 \quad \text{--- (01)}$$

case (ii): She travels 100 km by train and remaining $(300-100) = 200$ km by bus in 4 hours and 10 min.

$$\text{Therefore } \frac{100}{x} + \frac{200}{y} = 4 \text{ hours} + 10 \text{ min}$$

[Because 60 min = 1 hour

$$10 \text{ min} = \frac{1}{6} \text{ hour}]$$

$$\frac{100}{x} + \frac{200}{y} = \left(4 + \frac{1}{6}\right) \text{ hours.}$$

$$\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\left(\div \frac{25}{6}\right) \quad \frac{100}{\frac{25}{6}x} + \frac{200}{\frac{25}{6}y} = 1$$

$$\frac{6 \times 100}{25x} + \frac{6 \times 200}{25y} = 1$$

$$\frac{24}{x} + \frac{48}{y} = 1 \quad \text{--- (02)}$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$$\text{Equation 1 becomes } 15a + 60b = 1 \quad \text{--- (03)}$$

$$\text{Equation 2 becomes } 24a + 48b = 1 \quad \text{--- (04)}$$

$$3 \times 8 \Rightarrow 120a + 480b = 8 \quad \text{--- (05)}$$

$$4 \times 5 \Rightarrow \underline{120a + 240b = 5} \quad \text{--- (06)}$$

$$5 - 6 \Rightarrow 240b = 3$$

$$b = \frac{3}{240} = \frac{1}{80}$$

substitute $b = \frac{1}{80}$ in 3

$$15a + 60b = 1$$

$$15a + 60\left(\frac{1}{80}\right) = 1$$

$$15a + \frac{3}{4} = 1$$

$$15a = 1 - \frac{3}{4} = \frac{1}{4}$$

$$a = \frac{1}{4 \times 15} = \frac{1}{60}$$

$$a = \frac{1}{60}$$

$$a = \frac{1}{60} \text{ But, } \frac{1}{x} = a$$

$$\text{Therefore, } \frac{1}{60} = \frac{1}{x} \Rightarrow x = 60$$

$$b = \frac{1}{80}$$

$$b = \frac{1}{80} \text{ But, } b = \frac{1}{y}$$

$$\text{Therefore, } \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$$

Therefore speed of the train is 60 km/hr. and speed of the bus is 80 km/hr.

04 QUADRATIC EQUATIONS

Note:

$ax^2 + bx + c = 0$ is called the standard form of a quadratic Equation.

Example 2: Check whether the following are quadratic equations:-

(i) $(x - 2)^2 + 1 = 2x - 3$

Solution:

$$(x - 2)^2 + 1 = 2x - 3$$

$$x^2 + 4 - 4x + 1 - 2x + 3 = 0$$

$$x^2 - 6x + 8 = 0$$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is a quadratic equation.

(ii) $x(x + 1) + 8 = (x + 2)(x - 2)$

Solution:

$$x(x + 1) + 8 = (x + 2)(x - 2)$$

$$x^2 + x + 8 = x^2 - 4$$

$$x^2 + x + 8 - x^2 + 4 = 0 \Rightarrow x + 12 = 0$$

This is not in the form of quadratic equation $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

(iii) $x(2x + 3) = x^2 + 1$

Solution:

$$x(2x + 3) = x^2 + 1$$

$$2x^2 + 3x - x^2 - 1 = 0 \Rightarrow x^2 + 3x - 1 = 0$$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is a quadratic equation.

(iv) $(x + 2)^3 = x^3 - 4$

Solution:

$$(x + 2)^3 = x^3 - 4$$

$$x^3 + 2^3 + 3x^2(2) + 3(x)(2^2) - x^3 + 4 = 0$$

$$8 + 6x^2 + 12x + 4 = 0$$

$$6x^2 + 12x + 12 = 0$$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is a quadratic equation.

EXERCISE 4.1

1) Check whether the following are quadratic equations:-

(i) $(x + 1)^2 = 2(x - 3)$

Solution:

$$(x + 1)^2 = 2(x - 3)$$

$$x^2 + 2x + 1 = 2x - 6$$

$$x^2 + 2x + 1 - 2x + 6 = 0$$

$$x^2 + 7 = 0 \Rightarrow x^2 + 0x + 7 = 0$$

This is in the form $ax^2 + bx + c = 0$.

Therefore the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

Solution:

$$x^2 - 2x = (-2)(3 - x)$$

$$x^2 - 2x = -6 + 2x$$

$$x^2 - 2x + 6 - 2x = 0 \Rightarrow x^2 - 4x + 6 = 0$$

This is in the form $ax^2 + bx + c = 0$.

Therefore the given equation is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

Solution:

$$(x - 2)(x + 1) = (x - 1)(x + 3)$$

$$x^2 + x - 2x - 2 = x^2 + 3x - x - 3$$

$$x^2 - x - 2 = x^2 + 2x - 3$$

$$x^2 - x - 2 - x^2 - 2x + 3 = 0$$

$$-3x + 1 = 0$$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

Solution:

$$(x - 3)(2x + 1) = x(x + 5)$$

$$2x^2 + x - 6x - 3 = x^2 + 5x$$

$$2x^2 - 5x - 3 - x^2 - 5x = 0$$

$$x^2 - 10x - 3 = 0$$

This is in the form of $ax^2 + bx + c = 0$.

Therefore the given equation is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

Solution:

$$(2x - 1)(x - 3) = (x + 5)(x - 1)$$

$$2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$$

$$2x^2 - 7x + 3 = x^2 + 4x - 5$$

$$x^2 - 11x + 8 = 0$$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x - 2)^2$

Solution:

$$x^2 + 3x + 1 = (x - 2)^2$$

$$x^2 + 3x + 1 = x^2 - 4x + 4$$

$$x^2 + 3x + 1 - x^2 + 4x - 4 = 0$$

$$7x - 3 = 0$$

This is not in the form of

$$ax^2 + bx + c = 0.$$

Therefore the given equation is not a quadratic equation.

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

Solution:

$$(x + 2)^3 = 2x(x^2 - 1)$$

$$x^3 + 2^3 + 3x^2(2) + 3x(2^2) = 2x^3 - 2x$$

$$x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$$

$$[(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2]$$

$$-2x^3 + 6x^2 + 14x + 8 = 0$$

This is not in the form $ax^2 + bx + c = 0$.

Therefore the given equation is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Solution:

$$x^3 - 4x^2 - x + 1 = (x - 2)^3$$

$$x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3x^2(2) + 3x(2)$$

$$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$$

$$2x^2 - 13x + 9 = 0$$

This is in the form of $ax^2 + bx + c = 0$.

Therefore the given equation is a quadratic equation.

2) Represent the following situation is in the form of quadratic equations:-

(i) **The area of a rectangular plot is $528 m^2$. The length of the plot (in meters) is one more than twice its breadth. We need to find the length and breadth of the plot.**

Solution:

Let breadth of plot be x m.

Given, length of the plot is one more than twice its breadth.

(i.e) Length = $(2x + 1)m$.

$$\begin{aligned} \text{Area of the plot} &= \text{length} \times \text{breadth} \\ &= (2x + 1)x \\ &= 528 \end{aligned}$$

(According to the Question)

$$(2x + 1)x = 528$$

$$2x^2 + x - 528 = 0$$

which is the required quadratic equation.

(ii) **The product of two consecutive positive integers is 306. We need to find the integer.**

Solution:

Let the two consecutive positive integers be x and $x + 1$.

By data, $x(x + 1) = 306$

$$x^2 + x - 306 = 0$$

Which is the required quadratic equation.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Solution:

Let the present age of Rohan = x years.

His mother's age = $x + 26$ (condition given)

3 years hence,

Rohan's age = $(x+3)$ years.

His mother's age = $(x + 3 + 26)$ years.
= $(x + 29)$ years.

By data, $(x + 3)(x + 29) = 360$

$$x^2 + 29x + 3x + 87 = 360$$

$$x^2 + 32x + 87 - 360 = 0$$

$$x^2 + 32x - 273 = 0$$

Which is the required quadratic equation.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution:

Let the speed of the train be x km/hr.

The distance travelled by the train = 480 km

Time taken for travelling 480 km = $\frac{480}{x}$ hours.

[Distance = speed x time]

$$\left[\text{Time} = \frac{\text{Distance}}{\text{speed}} \right]$$

If the speed had been 8 km/hr less,

$(x - 8)$ km/hr, then

time taken for travelling 480 km = $\frac{480}{x-8}$ hours.

$$\text{By data, } \frac{480}{x-8} - \frac{480}{x} = 3$$

$$\frac{480x - 480(x - 8)}{x(x - 8)} = 3$$

$$\frac{480[x - (x - 8)]}{x(x - 8)} = 3$$

$$480[x - x + 8] = 3x(x - 8)$$

$$480(8) = 3x^2 - 24x$$

$$3840 = 3x^2 - 24x$$

$$3x^2 - 24x - 3840 = 0$$

$$x^2 - 8x - 1280 = 0$$

Which is the required quadratic equation.

Solution of a quadratic Equation by Factorisation

Example 3: Find the roots of the equation $2x^2 - 5x + 3 = 0$ by factorisation.

Solution:

$$2x^2 - 5x + 3 = 0$$

$$(x - 1)(2x - 3) = 0$$

$$\Rightarrow x - 1 = 0 \quad \text{or} \quad 2x - 3 = 0$$

$$x = 1 \quad \text{or} \quad 2x = 3$$

$$x = \frac{3}{2}$$

Therefore solutions of the equation are

$$x = 1, \quad x = \frac{3}{2}$$

Example 4: Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Solution:

$$6x^2 - x - 2 = 0$$

$$(2x + 1)(3x - 2) = 0$$

$$\Rightarrow 2x + 1 = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$2x = -1 \quad \text{or} \quad 3x = 2$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = \frac{2}{3}$$

Therefore Roots of the given equation are

$$x = -\frac{1}{2}, \quad x = \frac{2}{3}$$

Example 5: Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

Solution:

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow \sqrt{3}x - \sqrt{2} = 0$$

$$\sqrt{3}x - \sqrt{2} = 0$$

$$\sqrt{3}x = \sqrt{2} \Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} \text{ (twice)}$$

Therefore the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$ are

$$\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$$

Example 6: Find the dimension of the prayer hall having a carpet area of 300 square metre with its length one metre more than twice its breadth. What should be the length and breadth of the Hall?

Solution:

Let breadth of the hall = x m.

Therefore length of the hall = $2x + 1$

Area of the Hall = $300m^2$

By Data,

$$x(2x + 1) = 300$$

$$2x^2 + x - 300 = 0$$

$$(x - 12)(2x + 25) = 0$$

$$x - 12 = 0$$

$$x = 12$$

$$2x + 25 = 0$$

$$2x = -25$$

$$x = \frac{-25}{2}$$

x cannot be negative

Therefore $x = 12$

Hence, the breadth of the hall = 12 m.

length of the hall = $2x + 1$

$$= 2(12) + 1$$

$$= 24 + 1 = 25 \text{ m.}$$

EXERCISE 4.2

1) Find the roots of the following quadratic equations by factorisation:-

(i) $x^2 - 3x - 10 = 0$

Solution:

$$x^2 - 3x - 10 = 0$$

$$(x + 2)(x - 5) = 0$$

$$\Rightarrow x + 2 = 0 \quad x - 5 = 0$$

$$x = -2 \quad x = 5$$

Therefore roots of the equation are

$$x = -2, x = 5.$$

(ii) $2x^2 + x - 6 = 0$

Solution:

$$2x^2 + x - 6 = 0$$

$$(x + 2)(2x - 3) = 0$$

$$x + 2 = 0 \quad 2x - 3 = 0$$

$$x = -2 \quad 2x = 3 \Rightarrow x = \frac{3}{2}$$

Therefore solutions are $x = -2, \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Solution:

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$$

$$x + \sqrt{2} = 0$$

$$x = -\sqrt{2}$$

$$\sqrt{2}x + 5 = 0$$

$$\sqrt{2}x = -5$$

$$x = \frac{-5}{\sqrt{2}}$$

Therefore roots of the equations are

$$x = -\sqrt{2}, \frac{-5}{\sqrt{2}}$$

(iv) $2x^2 - x + \frac{1}{8} = 0$

Solution:

$$\frac{16x^2 - 8x + 1}{8} = 0$$

$$\Rightarrow 6x^2 - 8x + 1 = 0$$

$$(4x - 1)(4x - 1) = 0$$

$$4x - 1 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$4x - 1 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

Therefore Roots are $x = \frac{1}{4}, \frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$

Solution:

$$100x^2 - 20x + 1 = 0$$

$$(10x - 1)(10x - 1) = 0$$

$$10x - 1 = 0$$

$$10x = 1$$

$$x = \frac{1}{10}$$

$$10x - 1 = 0$$

$$10x = 1$$

$$x = \frac{1}{10}$$

Therefore the roots are $x = \frac{1}{10}, \frac{1}{10}$

2) John and Jivanthi together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Solution:

Let the number marbles John had be x .

Number of marbles Jivanthi had $45 - x$.

Given both of them lost 5 marbles.

Therefore now number of marbles John had $x - 5$

and Number of marbles Jivanthi had $45 - x - 5$.

$$= 40 - x$$

Therefore their product = 124

$$(x - 5)(40 - x) = 124$$

$$40x - x^2 - 200 + 5x = 124$$

$$-x^2 + 45x - 200 - 124 = 0$$

$$-x^2 + 45x - 324 = 0$$

$$x^2 - 45x + 324 = 0$$

$$(x - 36)(x - 9) = 0$$

$$x - 36 = 0 \qquad x - 9 = 0$$

$$x = 36 \qquad x = 9$$

Hence, Number of marbles they have

$x = 36, 9$ or $x = 9, 36$.

3) Find two numbers whose sum is 27 and product is 182.

Solution:

Let one number be x .

Therefore the other number is $27 - x$

[Therefore sum of two number = 27]

Given that their product = 182

$$x(27 - x) = 182$$

$$27x - x^2 = 182$$

$$x^2 - 27x + 182 = 0$$

$$(x - 13)(x - 14) = 0$$

$$x - 13 = 0 \qquad x - 14 = 0$$

$$x = 13 \qquad x = 14$$

Therefore the two numbers are $x = 13, 14$.

4) Find the consecutive positive integers sum of whose squares is 365.

Solution:

Let two consecutive positive integers be $x, x + 1$.

$$\text{By data, } x^2 + (x + 1)^2 = 365$$

$$x^2 + x^2 + 2x + 1 = 365$$

$$2x^2 + 2x + 1 - 365 = 0$$

$$2x^2 + 2x - 364 = 0$$

$$x^2 + x - 182 = 0$$

$$(x - 13)(x + 14) = 0$$

$$x - 13 = 0 \qquad x + 14 = 0$$

$$x = 13 \qquad x = -14$$

But x cannot be negative

Therefore $x = 13$.

Hence the two consecutive positive integers are **13 and 14**.

5) The altitude of a right triangle is 7 less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Let the base of the right triangle be x .

By data, altitude $AB = x - 7$

Hypotenuse $AC = 13$ cm.

By Theorem, $AC^2 = AB^2 + BC^2$

$$B^2 = (x - 7)^2 + x^2$$

$$169 = x^2 + 49 - 14x + x^2$$

$$169 = 2x^2 - 14x + 49$$

$$\text{Therefore } 2x^2 - 14x + 49 - 169 = 0$$

$$2x^2 - 14x - 120 = 0$$

$$(\div 2) \qquad x^2 - 7x - 60 = 0$$

$$(x + 5)(x - 12) = 0$$

$$\Rightarrow x + 5 = 0 \qquad x - 12 = 0$$

$$x = -5 \qquad x = 12$$

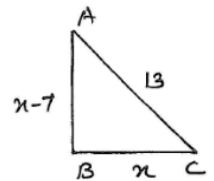
x cannot be negative

Therefore $x = 12$

side $AB = 12 - 7 = 5$ and

side $BC = x = 12$

Therefore the other two sides are **12, 5**.



6) A cottage industry produces a certain number of pottery articles in a day. If was observed on a particular day that the cost of production of each article was (₹ in) 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.

Solution:

Let the number of pottery articles produced on a particular day be x .

By data, cost of production of each article

$$= ₹(2x + 3).$$

Total cost of production = Number of articles \times Cost of each articles.

$$= x(2x + 3)$$

$$= 2x^2 + 3x$$

But total cost = ₹90.

$$\text{Therefore } 2x^2 + 3x = 90$$

$$2x^2 + 3x - 90 = 0$$

$$(2x + 15)(x - 6) = 0$$

$$\Rightarrow 2x + 15 = 0 \qquad x - 6 = 0$$

$$2x = -15 \qquad x = 6$$

$$x = \frac{-15}{2}$$

But x cannot be negative

Therefore $x = 6$

(i.e) number of articles produced in a day = 6

Cost of each article = $2x + 3$

$$= 2(6) + 3 = 12 + 3 = \text{₹}15$$

Solution of a quadratic equation by completing the square.

Example 7: Solve the equation $2x^2 - 5x + 3 = 0$ by the method of completing square.

Solution:

$$2x^2 - 5x + 3 = 0$$

$$(\div 2) \quad x^2 - \frac{5}{2}x + \frac{3}{2} = 0$$

[convert the 2nd term into in the form of 2ab]

$$x^2 - 2\left(\frac{5}{4}\right)x + \frac{3}{2} = 0$$

$$x^2 - 2\left(\frac{5}{4}\right)x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 + \frac{3}{2} = 0$$

$$\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 + \frac{3}{2} = 0$$

$$\left(x - \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{3}{2} = 0$$

$$\left(x - \frac{5}{4}\right)^2 - \frac{25+24}{16} = 0$$

$$\left(x - \frac{5}{4}\right)^2 - \frac{1}{16} = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$$

$$\text{Therefore } x - \frac{5}{4} = \pm \frac{1}{4}$$

$$x = \frac{5}{4} \pm \frac{1}{4} = \left(\frac{5+1}{4}, \frac{5-1}{4}\right) = \left(\frac{6}{4}, \frac{4}{4}\right) = \left(\frac{3}{2}, 1\right)$$

$$\text{Therefore } x = \frac{3}{2} \text{ or } x = 1$$

Example 8: Find the roots of the equation $5x^2 - 6x - 2 = 0$ by the method of completing square.

Solution:

$$5x^2 - 6x - 2 = 0$$

$$(\div 5) \quad x^2 - \frac{6}{5}x - \frac{2}{5} = 0$$

[convert the 2nd term in the form of 2ab]

$$x^2 - 2\left(\frac{3}{5}\right)x - \frac{2}{5} = 0$$

$$x^2 - 2\left(\frac{3}{5}\right)x + \left(\frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 - \frac{2}{5} = 0$$

$$\left(x - \frac{3}{5}\right)^2 - \frac{9}{25} - \frac{2}{5} = 0$$

$$\left(x - \frac{3}{5}\right)^2 - \frac{9-10}{25} = 0$$

$$\left(x - \frac{3}{5}\right)^2 - \frac{19}{25} = 0$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$\text{Therefore } x - \frac{3}{5} = \pm \frac{\sqrt{19}}{5}$$

$$x = \frac{3}{5} \pm \frac{\sqrt{19}}{5}$$

Therefore the roots are $\frac{3+\sqrt{19}}{5}$ or $\frac{3-\sqrt{19}}{5}$

Example 9: Find the roots of $4x^2 + 3x + 5 = 0$ by the method of completing the square.

Solution:

$$4x^2 + 3x + 5 = 0$$

$$(2x)^2 + 2\left(\frac{3}{4}\right)(2x) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + 5 = 0$$

$$\Rightarrow \left(2x + \frac{3}{4}\right)^2 - \frac{9}{16} + 5 = 0$$

$$\left(2x + \frac{3}{4}\right)^2 - \frac{9+80}{16} = 0$$

$$\left(2x + \frac{3}{4}\right)^2 + \frac{71}{16} = 0$$

$$\left(2x + \frac{3}{4}\right)^2 = -\frac{71}{16} < 0$$

But $\left(2x + \frac{3}{4}\right)^2$ cannot be negative for any real value of x .

Therefore the given equation has no real value.

Therefore the given equation has no real roots.

Note: The roots of the quadratic equation

$$ax^2 + bx + c = 0 \text{ is } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 10: The area of a rectangular plot is 528 m^2 . The length of the plot is one more than twice its breadth. Find the length and breadth by using quadratic formula.

Solution:

Let the breadth of the plot be $x \text{ m}$.

Therefore the length = $(2x + 1) \text{ m}$ (by condition).

Therefore Area of the plot = $x(2x + 1)$

$$x(2x + 1) = 528 \text{ (given)}$$

$$2x^2 + x - 528 = 0$$

Here $a = 2, b = 1, c = -528$

$$\text{Therefore } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1 - 4(2)(-528)}}{2(2)} \\ &= \frac{-1 \pm \sqrt{1 + 4224}}{4} = \frac{-1 \pm \sqrt{4225}}{4} \\ &= \frac{-1 \pm 65}{4} \\ &= \frac{64}{4}, \frac{-66}{4} \\ &= 16, \frac{-33}{2} \end{aligned}$$

But x cannot be negative. Therefore $x = 16$.

Breadth of the plot $x = 16 \text{ m}$ and length of the plot

$$2x + 1 = 2(16) + 1 = 33 \text{ m}.$$

Example 11: Find two consecutive odd positive integers, sum of whose squares is 290.

Solution:

Let the smaller of the two consecutive odd positive integer be x . The next odd positive integer will be $x + 2$.

$$\text{By data, } x^2 + (x + 2)^2 = 290$$

$$x^2 + x^2 + 4x + 4 = 290$$

$$2x^2 + 4x + 4 - 290 = 0$$

$$2x^2 + 4x - 286 = 0$$

$$x^2 + 2x - 143 = 0 \quad (\div 2)$$

Here $a = 1, b = 2, c = -143$

$$\text{Therefore } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-143)}}{2(1)}$$

$$\begin{aligned} &= \frac{-2 \pm \sqrt{4 + 572}}{2} = \frac{-2 \pm \sqrt{576}}{2} \\ &= \frac{-2 \pm 24}{2} = \frac{-2 - 24}{4}, \frac{-2 + 24}{2} \\ &= \frac{-26}{2}, \frac{22}{2} \end{aligned}$$

$$x = -13, 11$$

But x cannot be negative. Therefore $x = 11$.

Therefore the two consecutive odd integers are 11 and 13.

Example 12: A rectangular park is to be designed whose breadth is 3m less than its length. Its area is to be 4 square metres more than the area of the park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12m. Find the length and breadth.

Solution:

Let the length of the rectangular park be $x \text{ m}$.

Therefore its breadth = $(x - 3) \text{ m}$ [given]

Now its area = $\text{length} \times \text{breadth}$

$$= x(x - 3)$$

$$= x^2 - 3x$$

Given that the base of the isosceles triangle is breadth of the park and height of the triangle is 12m.

Therefore Area of the isosceles triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (x - 3) \times 12$$

$$= 6(x - 3)$$

$$= 6x - 18$$

According to the problem

$$x^2 - 3x = 6x - 18 + 4$$

$$x^2 - 3x - 6x + 18 - 4 = 0$$

$$x^2 - 9x + 14 = 0$$

$$(x - 7)(x - 2) = 0 \Rightarrow x = 7 \text{ or } x = 2$$

But $x = 2$ is not possible.

Since breadth = $2 - 3 = -1$ not possible.

Therefore $x = 7$.

Hence length of the park = 7 m

and its breadth = $x - 3$

$$= 7 - 3 = 4 \text{ m}.$$

Example 13: Find the roots of the following quadratic equations, if they exist, using the quadratic formula.

(i) $3x^2 - 5x + 2 = 0$

Solution:

Compare this equation with $ax^2 + bx + c = 0$

Here $a = 3, b = -5, c = 2$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{5 \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)} \\ &= \frac{-5 \pm \sqrt{25 - 24}}{6} \\ &= \frac{5 \pm \sqrt{1}}{6} \\ &= \frac{5 \pm 1}{6} = \left(\frac{5+1}{6}, \frac{5-1}{6}\right) \\ &= \left(\frac{6}{6}, \frac{4}{6}\right) \\ &= \left(1, \frac{2}{3}\right) \end{aligned}$$

Therefore the roots are $1, \frac{2}{3}$.

(ii) $x^2 + 4x + 5 = 0$

Solution:

Compare this equation with $ax^2 + bx + c = 0$

Here $a = 1, b = 4, c = 5$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{16 - 20}}{2} \\ &= \frac{-4 \pm \sqrt{-4}}{2} \end{aligned}$$

$\sqrt{-4}$ is not a real value.

Therefore the given equation has no real value.

(iii) $2x^2 - 2\sqrt{2}x + 1 = 0$

Solution:

Compare this equation with $ax^2 + bx + c = 0$

Here $a = 2, b = -2\sqrt{2}, c = 1$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2\sqrt{2} \pm \sqrt{(-2\sqrt{2})^2 - 4(2)(1)}}{2(2)} \\ &= \frac{2\sqrt{2} \pm \sqrt{8 - 8}}{4} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \end{aligned}$$

Therefore the roots are $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

Example 14: Find the roots of the following equations:-

(i) $x + \frac{1}{x} = 3, x \neq 0$

Solution:

$$x + \frac{1}{x} = 3$$

$$\frac{x^2 + 1}{x} = 3 \Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

Here $a = 1, b = -3, c = 1$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(1)(1)}}{2(1)} \\ &= \frac{3 \pm \sqrt{9 - 4}}{2} \\ &= \frac{3 \pm \sqrt{5}}{2} \end{aligned}$$

Therefore the roots are $\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$.

(ii) $\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$.

Solution:

$$\frac{1}{x} - \frac{1}{x-2} = 3 \Rightarrow \frac{x-2-x}{x(x-2)} = 3$$

$$\frac{-2}{x^2 - 2x} = 3$$

$$\Rightarrow -2 = 3(x^2 - 2x)$$

$$-2 = 3x^2 - 6x$$

$$3x^2 - 6x + 2 = 0$$

Here $a = 3, b = -6, c = 2$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(3)(2)}}{2(3)} \\ &= \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6} \\ &= \frac{6 \pm \sqrt{4 \times 3}}{6} = \frac{6 \pm 2\sqrt{3}}{6} \\ &= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} \end{aligned}$$

Therefore the roots are $\frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$.

Example 15: A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution:

Let the speed of the stream be x km/hr.

Therefore speed of Upstream = $(18 - x)$ km/hr and

Speed of downstream = $(18 + x)$ km/hr.

$$\begin{aligned} \text{Now time taken to go Upstream} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{24}{18-x} \text{ hours.} \end{aligned}$$

$$\text{Time taken to go downstream} = \frac{24}{18+x} \text{ hours.}$$

$$\text{By data, } \frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\frac{24(18+x) - 24(18-x)}{(18-x)(18+x)} = 1$$

$$\Rightarrow 24[18 + x - 18 + x] = 18^2 - x^2$$

$$2x(24) = 324 - x^2$$

$$48x = 324 - x^2$$

$$\text{Therefore } x^2 + 48x - 324 = 0$$

Here $a = 1, b = 48, c = -324$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-48 \pm \sqrt{48^2 - 4(1)(-324)}}{2(1)}$$

$$= \frac{-48 \pm \sqrt{2304 + 1296}}{2} = \frac{-48 \pm \sqrt{3600}}{2}$$

$$= \frac{-48 \pm 60}{2} = \frac{-48 + 60}{2}, \frac{-48 - 60}{2}$$

$$= \frac{12}{2}, \frac{-108}{2} = (6, -54)$$

Speed of the stream cannot be negative.

Therefore $x = 6$.

Hence the speed of the stream = 6 km/hr.

EXERCISE 4.3

1) Find the roots of the following quadratic equations if they exist, by the method of completing the square.

(i) $2x^2 - 7x + 3 = 0$

Solution:

$$2x^2 - 7x + 3 = 0$$

$$(\div 2) \quad x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$x^2 - 2\left(\frac{7}{4}\right)x + \frac{3}{2} = 0$$

$$x^2 - 2\left(\frac{7}{4}\right)x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 + \frac{24 - 49}{16} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 + \left(\frac{-25}{16}\right) = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \Rightarrow x - \frac{7}{4} = \sqrt{\frac{25}{16}}$$

$$x - \frac{7}{4} = \pm \frac{5}{4}$$

$$\text{Therefore } x = \frac{7}{4} \pm \frac{5}{4}$$

$$x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$x = 3 \text{ or } \frac{1}{2}$$

Therefore the roots are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

Solution:

$$2x^2 + x - 4 = 0$$

$$(\div 2) \quad x^2 + \frac{x}{2} - \frac{4}{2} = 0$$

$$x^2 + 2\left(\frac{x}{4}\right) - 2 = 0$$

$$x^2 + 2\left(\frac{1}{4}\right)x - 2 = 0$$

$$x^2 + 2\left(\frac{1}{4}\right)x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{1-32}{16} = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{33}{16} = 0 \Rightarrow \left(x + \frac{1}{4}\right)^2$$

$$= \frac{33}{16}$$

$$\Rightarrow x + \frac{1}{4} = \pm \sqrt{\frac{33}{16}}$$

$$x = -\frac{1}{4} \pm \frac{\sqrt{33}}{4}$$

$$\text{Therefore } x = \frac{\sqrt{33}-1}{4} \text{ or } \frac{-1-\sqrt{33}}{4}$$

Therefore the roots are $\frac{\sqrt{33}-1}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Solution:

$$4x^2 + 4\sqrt{3}x + 3 = 0$$

$$(\div 4) \quad x^2 + \sqrt{3}x + \frac{3}{4} = 0$$

$$x^2 + 2\left(\frac{\sqrt{3}}{2}\right)x + \frac{3}{4} = 0$$

$$x^2 + 2\left(\frac{\sqrt{3}}{2}\right)x + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$$

$$\left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0$$

$$\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$$x + \frac{\sqrt{3}}{2} = 0$$

$$\Rightarrow x = -\frac{\sqrt{3}}{2}$$

Therefore the roots are $-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

Solution:

$$2x^2 + x + 4 = 0$$

$$(\div 2) x^2 + \frac{x}{2} + 2 = 0$$

$$x^2 + 2\left(\frac{1}{4}\right)x + 2 = 0$$

$$x^2 + 2\left(\frac{1}{4}\right)x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$$

$$\left(x + \frac{1}{4}\right)^2 - \frac{1}{16} + 2 = 0$$

$$\left(x + \frac{1}{4}\right)^2 + \frac{32-1}{16} = 0$$

$$\left(x + \frac{1}{4}\right)^2 + \frac{31}{16} = 0$$

$$\left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$$

Taking square root on both side.

$$x + \frac{1}{4} = \sqrt{\frac{-31}{16}}$$

But $\sqrt{\frac{-31}{16}}$ is not possible. Because square of real number cannot be negative. Therefore the roots are not real value.

2) Find the roots of the following quadratic equation by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

Solution:

Compare the given equation with

$$ax^2 + bx + c = 0$$

Here $a = 2, b = -7, c = 3$.

$$\text{Now, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$= \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4} \Rightarrow \left(\frac{7+5}{4}, \frac{7-5}{4}\right)$$

$$= \left(\frac{12}{4}, \frac{2}{4}\right) = \left(3, \frac{1}{2}\right)$$

Therefore the roots are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

Solution:

Compare the given equation with

$$ax^2 + bx + c = 0$$

Here $a = 2, b = 1, c = -4$.

$$\text{Now, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1+32}}{4} = \frac{-1 \pm \sqrt{33}}{4}$$

$$\Rightarrow \left(\frac{\sqrt{33}-1}{4}, \frac{-\sqrt{33}-1}{4}\right)$$

Therefore the roots are $\frac{\sqrt{33}-1}{4}$ and $\frac{-\sqrt{33}-1}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Solution:

Compare the given equation with

$$ax^2 + bx + c = 0$$

Here $a = 4, b = 4\sqrt{3}, c = 3$.

$$\text{Therefore } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2(4)}$$

$$= \frac{-4\sqrt{3} \pm \sqrt{48-48}}{8} = \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$$

Therefore the roots are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

Solution:

Compare the given equation with

$$ax^2 + bx + c = 0$$

Here $a = 2, b = 1, c = 4$.

$$\text{Therefore } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(4)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1-32}}{4} = \frac{-1 \pm \sqrt{-31}}{4}$$

Now $\sqrt{-31}$ is not possible.

Because square of a real number cannot be negative.

Therefore Roots does not exists.

3) Find the roots of the following equations

(i) $x - \frac{1}{x} = 3, x \neq 0$

Solution:

$$x - \frac{1}{x} = 3$$

$$\Rightarrow \frac{x^2 - 1}{x} = 3$$

$$\Rightarrow x^2 - 1 = 3x$$

$$\Rightarrow x^2 - 3x - 1 = 0$$

Here $a = 1, b = -3, c = -1$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} = \frac{3 \pm \sqrt{9+4}}{2} \\ &= \frac{3 \pm \sqrt{13}}{2} = \left(\frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2} \right) \end{aligned}$$

Therefore the roots are $\frac{3 + \sqrt{13}}{2}$, and $\frac{3 - \sqrt{13}}{2}$.

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$.

Solution:

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{x-7-x-4}{x^2-7x+4x-28} = \frac{11}{30}$$

$$\frac{-11}{x^2-3x-28} = \frac{11}{30}$$

$$\Rightarrow -11(30) = 11(x^2 - 3x - 28)$$

$$(\div 11) \quad -30 = x^2 - 3x - 28$$

$$\Rightarrow x^2 - 3x - 28 + 30 = 0$$

$$x^2 - 3x + 2 = 0$$

Here $a = 1, b = -3, c = 2$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{+3 \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)} \\ &= \frac{3 \pm \sqrt{9-8}}{2} \\ &= \frac{3 \pm \sqrt{1}}{2} = \left(\frac{3+1}{2}, \frac{3-1}{2} \right) = \left(\frac{4}{2}, \frac{2}{2} \right) \\ &= (2, 1) \end{aligned}$$

Therefore the roots are 2 and 1.

4) The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Solution:

Let the present age of Rehman be x years.

3 years ago, Rehman's age = $(x - 3)$ years.

5 years ago, Rehman's age = $(x + 5)$ years.

By condition,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$$

$$\Rightarrow 3(2x + 2) = x^2 + 2x - 15$$

$$\Rightarrow 6x + 6 = x^2 + 2x - 15$$

$$\Rightarrow x^2 + 2x - 15 - 6x - 6 = 0$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

Here $a = 1, b = -4, c = -21$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{4 \pm \sqrt{16 - 4(1)(-21)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16+84}}{2} = \frac{4 \pm \sqrt{100}}{2} \\ &= \frac{4 \pm 10}{2} = \frac{4+10}{2} \text{ or } \frac{4-10}{2} \\ &= \frac{14}{2} \text{ or } \frac{-6}{2} \\ &= 7 \text{ or } -3 \end{aligned}$$

Therefore the roots are 7 and -3.

5) In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:

Let Shefali's maths mark be x .

Given that sum of Maths and English marks is 30.

Therefore Shefali's English mark = $30 - x$.

By condition, $(\text{Maths Mark} + 2) \times$

$$(\text{English Mark} - 3) = 210$$

$$(x + 2)(30 - x - 3) = 210$$

$$\Rightarrow (x + 2)(27 - x) = 210$$

$$\Rightarrow 27x - x^2 + 54 - 2x = 210$$

$$25x - x^2 + 54 - 210 = 0$$

$$25x - x^2 - 156 = 0$$

$$\text{Therefore } x^2 - 25x + 156 = 0$$

Here $a = 1, b = -25, c = 156$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{25 \pm \sqrt{625 - 4(1)(156)}}{2(1)} \\ &= \frac{25 \pm \sqrt{625 - 624}}{2} \\ &= \frac{25 \pm \sqrt{1}}{2} \Rightarrow \frac{25 \pm 1}{2} \\ &\Rightarrow \frac{25+1}{2} \text{ or } \frac{25-1}{2} \\ &\Rightarrow \frac{26}{2} \text{ or } \frac{24}{2} \\ &= (13 \text{ or } 12) \end{aligned}$$

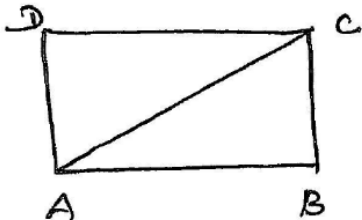
When $x = 12$ marks in Maths, then the marks in English is $30 - x = 30 - 12 = 18$.

When $x = 13$ marks in Maths, then the marks in English is $30 - x = 30 - 13 = 17$.

6) The diagonals of a rectangle field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, then find the sides of the field.

Solution: Let ABCD be the rectangular field.

The shorter side of the field is $BC = x$ m.



Given that the longer side AB is 30m more than the shorter side.

$$\text{Therefore } AB = (x + 30)m$$

Also given that the diagonal of the field AC is 60m more than the shorter side.

$$\text{Therefore } AC = (x + 60)m.$$

In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

(By pythagoras theorem)

$$\begin{aligned} (x + 60)^2 &= (x + 30)^2 + x^2 \\ \Rightarrow x^2 + 120x + 3600 &= x^2 + 60x + 900 + x^2 \\ \Rightarrow x^2 + 120x + 3600 - 2x^2 - 60x - 900 &= 0 \end{aligned}$$

$$\Rightarrow -x^2 + 60x + 2700 = 0$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

Here $a = 1, b = -60, c = -2700$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{60 \pm \sqrt{3600 - 4(1)(-2700)}}{2(1)} \\ &= \frac{60 \pm \sqrt{3600 + 10800}}{2} = \frac{60 \pm \sqrt{14400}}{2} \\ &= \frac{60 \pm 120}{2} = \frac{60+120}{2} \text{ or } \frac{60-120}{2} \\ &= \frac{180}{2} \text{ or } \frac{-60}{2} = 90 \text{ or } -30 \end{aligned}$$

But x cannot be negative.

Therefore $x = 90$.

Hence, breadth of the rectangle $x = 90m$.
and length of the rectangle $= x + 30$
 $= 90 + 30$
 $= 120 m$.

7) The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
Solution:

Let two numbers be x and $y, x > y$.

Given that the difference of squares of two numbers is 180.

$$x^2 - y^2 = 180 \text{ ————— (01)}$$

Also it is given that the square of smaller number is 8 times the larger number.

$$y^2 = 8x \text{ ————— (02)}$$

substitute 2 in 1

$$x^2 - 8x = 180 \Rightarrow x^2 - 8x - 180 = 0$$

$$(x - 18)(x + 10) = 0 \text{ (by factorisation)}$$

$$\Rightarrow x - 18 = 0 \text{ or } x + 10 = 0$$

$$\Rightarrow x = 18 \text{ or } x = -10$$

$$\text{When } x = 18, \text{ equation 2 } \Rightarrow y^2 = 8(18)$$

$$y^2 = 144 \Rightarrow y = \pm 12$$

$$\text{When } x = -10, 2 \Rightarrow y^2 = 8(-10)$$

$$= -80 \text{ which is not possible.}$$

Therefore square of a number cannot be negative. Hence the required numbers are 18 and 12 or 18 and -12.

8) A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

Let the speed of the train be x km/hr.

Given that distance covered by the train is 360 km.

Therefore time taken to cover the distance 360 km

$$\text{is } t = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x} \text{ hours.}$$

By condition,

the speed of the train = $(x + 5)$ km/hr.

Distance = 360 km.

Therefore time taken to cover the distance =

$$\frac{360}{x+5} \text{ hours.}$$

By condition,

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow 360 \left[\frac{1}{x} - \frac{1}{x+5} \right] = 1$$

$$\Rightarrow 360 \left[\frac{x+5-x}{x(x+5)} \right] = 1$$

$$\Rightarrow 360 \left[\frac{5}{x^2+5x} \right] = 1$$

$$\Rightarrow 360 \times 5 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 1800 = 0$$

Here $a = 1, b = 5, c = -1800$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{25 - 4(1)(-1800)}}{2(1)} \\ &= \frac{-5 \pm \sqrt{25 + 7200}}{2} = \frac{-5 \pm \sqrt{7225}}{2} \\ &= \frac{-5 \pm 85}{2} = \frac{-5 + 85}{2} \text{ or } \frac{-5 - 85}{2} \\ &= \frac{80}{2} \text{ or } \frac{-90}{2} = 40 \text{ or } -45 \end{aligned}$$

But speed $x = -45$ is not possible.

Therefore speed of the train $x = 40$ km/hr.

9) Two water taps together can fill a tank in $9\frac{3}{8}$ hrs.

The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Time taken by smaller tap to fill the tank completely = x hours.

Therefore the value of the tank filled by the smaller tap in 1 hour = $\frac{1}{x}$.

Now time taken by larger tap to fill the tank completely = $(x - 10)$ hours.

Therefore the volume of tank filled by the larger tap in 1 hour = $\frac{1}{x-10}$.

$$\begin{aligned} \text{Time taken by both the tap to fill} &= 9\frac{3}{8} \text{ hours.} \\ &= \frac{75}{8} \text{ hours.} \end{aligned}$$

Therefore tank filled by the smaller tap in $\frac{75}{8}$ hours.

$$= \frac{75}{8} \left(\frac{1}{x} \right) = \frac{75}{8x}$$

Tank filled by the larger tap is $\frac{75}{8}$ hours.

$$= \frac{75}{8} \left(\frac{1}{x-10} \right)$$

$$\text{Therefore } \frac{75}{8x} + \frac{75}{8(x-10)} = 1$$

$$\frac{75}{8} \left[\frac{1}{x} + \frac{1}{x-10} \right] = 1$$

$$\Rightarrow \frac{75}{8} \left[\frac{x-10+x}{(x)(x-10)} \right] = 1$$

$$\Rightarrow \frac{75}{8} \left[\frac{2x-10}{x^2-10x} \right] = 1$$

$$\Rightarrow 75(2x - 10) = 8(x^2 - 10x)$$

$$\Rightarrow 150x - 750 = 8x^2 - 80x$$

$$\Rightarrow 8x^2 - 80x - 150x + 750 = 0$$

$$\Rightarrow 8x^2 - 230x + 750 = 0$$

Here $a = 8, b = -230, c = 750$.

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{230 \pm \sqrt{230^2 - 4(8)(750)}}{2(8)} \\ &= \frac{230 \pm \sqrt{52900 - 24000}}{16} = \frac{230 \pm \sqrt{28900}}{16} \\ &= \frac{230 \pm 170}{16} \Rightarrow \frac{230+170}{16} \text{ or } \frac{230-170}{16} \\ &\Rightarrow \frac{400}{16} \text{ or } \frac{60}{16} \Rightarrow 25 \text{ or } \frac{15}{4} \end{aligned}$$

When $x = 25$, time taken by smaller tap = 25 hours.

Therefore time taken by larger tap

$$= x - 10 \text{ hrs} = 25 - 10 = 15 \text{ hours.}$$

when $x = \frac{15}{4}$, time taken by the smaller tap

$$= \frac{15}{4} \text{ hours.}$$

Therefore time taken by the larger tap = $x - 10$
 $= \frac{15}{4} - 10 = \frac{15-40}{4} = \frac{-25}{4}$

Which is not possible.

**Therefore time taken by smaller tap = 25 hours
 and time taken by larger tap = 15 hours.**

10) An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangaluru. If the average speed of the express train is 11 km/hr more than that of passenger train, then find the average speed of the two train.

Solution:

The average speed of the passenger train be x km/hr.

By condition, average speed of the express train = $(x + 11)$ km/hr.

Distance from Mysore to Bangaluru = 132 km.

Time taken to cover the distance 132 km by

Express train = $\frac{132}{x+11}$ hours.

By data, $\frac{132}{x} - \frac{132}{x+11} = 1$

$\Rightarrow 132 \left[\frac{1}{x} - \frac{1}{x+11} \right] = 1$

$\Rightarrow 132 \left[\frac{x+11-x}{x(x+11)} \right] = 1$

$\Rightarrow 132 \left[\frac{11}{x^2+11x} \right] = 1$

$\Rightarrow x^2 + 11x = 132 \times 11$

$\Rightarrow x^2 + 11x - 1452 = 0$

Here $a = 1, b = 11, c = -1452$.

Therefore $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{11 \pm \sqrt{11^2 - 4(1)(-1452)}}{2(1)} = \frac{-11 \pm \sqrt{121 - 5808}}{2}$
 $= \frac{-11 \pm \sqrt{5929}}{2} = \frac{-11 \pm 77}{2}$
 $= \frac{-11+77}{2}$ or $\frac{-11-77}{2} = \frac{66}{2}$ or $\frac{-88}{2}$
 $= 33$ or -44

Speed cannot be negative. Therefore $x = 33$

Hence speed of passenger train = 33 km/hr.

Speed of Express train = $x + 11$
 $= 33 + 11 = 44$ km/hr.

11) Sum of the areas of two squares is $468 m^2$. If the difference of their perimeters is 24 m, find the side of two squares.

Solution:

Length of side of first square be x m.

Therefore its perimeter = $4x$.

Given that the difference of their perimeter is 24 m.

Perimeter of 2nd square – Perimeter of the 1st square = 24

Therefore perimeter of 2nd square = $24 + 4x$
 $= 4(6 + x)$

Therefore side of 2nd square = $\frac{4(6+x)}{4}$
 $= (6 + x)m$.

Also it is given that the sum of areas of two squares is $468 m^2$.

$x^2 + (6 + x)^2 = 468$

$x^2 + 36 + x^2 + 12x - 468 = 0$

$2x^2 + 12x - 432 = 0$

$(\div 2) x^2 + 6x - 216 = 0$

Here $a = 1, b = 6, c = -216$.

Therefore $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{-6 \pm \sqrt{36 - 4(1)(-216)}}{2(1)}$
 $= \frac{-6 \pm \sqrt{36 + 864}}{2} = \frac{-6 \pm \sqrt{900}}{2}$
 $= \frac{-6 \pm 30}{2} = \frac{-6+30}{2}$ or $\frac{-6-30}{2}$
 $= \frac{24}{2}$ or $\frac{-36}{2} = 12$ or -18

$x = -18$ is not possible.

Therefore side of the 1st square = 12 m.

Side of the 2nd square = $6 + x$
 $= 6 + 12 = 18$ m.

Nature of Roots:

The roots of the equation $ax^2 + bx + c = 0$ are

given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Since, $b^2 - 4ac$ determines whether the quadratic equation $ax^2 + bx + c = 0$ real roots or not. $b^2 - 4ac$ is called the discriminant of the quadratic equation.

Note:

- 1) If $b^2 - 4ac > 0$, then the roots are real and distinct.
- 2) If $b^2 - 4ac = 0$, then the roots are real and equal.
- 3) If $b^2 - 4ac < 0$, then the roots are imaginary no real roots.

Example 16: Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Solution:

Given equation is $2x^2 - 4x + 3 = 0$

Compare the given equation with $ax^2 + bx + c = 0$

Here $a = 2, b = -4, c = 3$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (-4)^2 - 4(2)(3) \\ &= 16 - 24 = -8 < 0 \end{aligned}$$

Therefore the given equation has no real roots.

Example 17: A pole has to be erected at a point on the boundary of a circular park of diameter 13 m in such a way that the differences of its distance from two diametrically opposite fixed gates A and B on the boundary is 7 m. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Solution:

Let p be the position of the pole on the boundary.

A and B are two gates diametrically opposite.

Let the distance between

the pole and gate B

be x m. ($BP = xm$).

Also it is given that the difference of the distance of the pole from the two

gates = $AP - BP$ or $BP - AP = 7m$ (given)

$$BP - AP = 7m$$

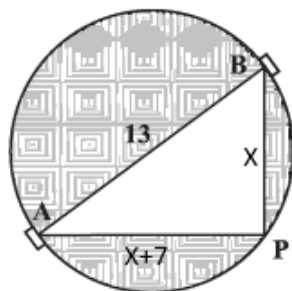
$$x - AP = 7m \text{ or } AP - x = 7m$$

Therefore $AP = (x + 7)m$

Now Diameter $AB = 13 m$

Therefore $AB^2 = AP^2 + BP^2$ (By pythagores Th)

$$13^2 = (x + 7)^2 + x^2$$



$$169 = x^2 + 14x + 49 + x^2$$

$$169 = 2x^2 + 14x + 49$$

$$2x^2 + 14x + 49 - 169 = 0$$

$$2x^2 + 14x - 120 = 0$$

$$(\div 2) \quad x^2 + 7x - 60 = 0$$

$$(x + 12)(x - 5) = 0$$

$$\Rightarrow x + 12 = 0$$

$$x - 5 = 0$$

$$x = -12$$

$$x = 5$$

But x cannot be negative

Therefore $x = 5$

distance between the pole and gate $B = 5m$.

Therefore distance between the pole and gate

$A = AP$

$$= x + 7 = 5 + 7 = 12m.$$

Therefore The pole has to be erected on the boundary at a distance of $5m$ from gate B and $12m$ from gate A .

Example 18: Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Solution:

Given equation is $3x^2 - 2x + \frac{1}{3} = 0$

Here $a = 3, b = -2, c = \frac{1}{3}$

Discriminant = $b^2 - 4ac$

$$= (-2)^2 - 4(3)(\frac{1}{3}) = 4 - 4 = 0$$

Discriminant = 0 \Rightarrow the roots are equal.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm 0}{2(3)} = \frac{2}{6} = \frac{1}{3}$$

Therefore the roots are $\frac{1}{3}$ and $\frac{1}{3}$

EXERCISE 4.4

1) Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.

(i) $2x^2 - 3x + 5 = 0$

Solution:

The given equation is $2x^2 - 3x + 5 = 0$

Here $a = 2, b = -3, c = 5$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (-3)^2 - 4(2)(5) \\ &= 9 - 40 = -31 < 0 \end{aligned}$$

Therefore the given equation has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Solution:

The given equation is $3x^2 - 4\sqrt{3}x + 4 = 0$

Here $a = 3, b = -4\sqrt{3}, c = 4$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (-4\sqrt{3})^2 - 4(3)(4) \\ &= 16 \times 3 - 48 \\ &= 48 - 48 = 0 \end{aligned}$$

Therefore roots are real and equal.

$$x = \frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2(3)} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 + 6x + 3 = 0$

Solution:

The given equation is $2x^2 + 6x + 3 = 0$

Here $a = 2, b = -6, c = 3$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= (-6)^2 - 4(2)(3) \\ &= 36 - 24 \\ &= 12 > 0 \end{aligned}$$

Therefore the roots are real and distinct.

$$\begin{aligned} \text{Now } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{12}}{2(2)} \\ &= \frac{6 \pm \sqrt{4 \times 3}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{2(3 \pm \sqrt{3})}{4} \end{aligned}$$

$$\begin{aligned} &= \frac{3 \pm \sqrt{3}}{2} \\ &= \frac{3 + \sqrt{3}}{2} \text{ or } \frac{3 - \sqrt{3}}{2} \end{aligned}$$

Therefore the roots are $\frac{3 + \sqrt{3}}{2}$ and $\frac{3 - \sqrt{3}}{2}$

2) Find the value of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

Solution:

Here $a = 2, b = k, c = 3$.

Given that they have Equal roots.

$$\begin{aligned} \text{Therefore, Discriminant} &= b^2 - 4ac = 0 \\ &= k^2 - 4(2)(3) = 0 \\ &= k^2 - 24 = 0 \\ &k^2 = 24 \end{aligned}$$

$$k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$$

(ii) $kx(x - 2) + 6 = 0$

Solution:

$$kx(x - 2) + 6 = 0$$

$$kx^2 - 2kx + 6 = 0$$

Here $a = k, b = -2k, c = 6$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac = (-2k)^2 - 4(k)(6) \\ &= 4k^2 - 24k \end{aligned}$$

Given that the two roots are equal.

$$\begin{aligned} \text{Therefore Discriminant} &= 0 \\ 4k^2 - 24k &= 0 \\ 4k(k - 6) &= 0 \\ \Rightarrow k &= 0 & k - 6 &= 0 \\ \Rightarrow k &= 0 & k &= 6 \end{aligned}$$

3) Is it possible to design a rectangular mango whose length is twice its breadth and the area is $800m^2$? If so, find its length and breadth.

Solution:

Breadth of the rectangular mango grove = xm .

Therefore length of the rectangular mango grove = $2xm$.

Area of rectangular mango grove = $800m^2$.

$$x(2x) = 800$$

$$2x^2 = 800 \Rightarrow x^2 = \frac{800}{2} = 400$$

$$x = \pm 20$$

$$x = -20 \text{ is not possible.}$$

Therefore $x = 20$

Hence, Length = $2x = 2(20) = 40m$.

Breadth = $x = 20 m$.

4) Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:

Let the age of one of two friends be x years.

Therefore the age other friend = $20 - x$

(Since, sum of their ages is 20)

4 years ago,

The age of 1st friend = $(x - 4)$ years.

and the age of 2nd friend = $20 - x - 4$

$$= (16 - x) \text{ years.}$$

By data,

$$(x - 4)(16 - x) = 48$$

$$16x - x^2 - 64 + 4x - 48 = 0$$

$$-x^2 + 20x - 112 = 0$$

$$x^2 - 20x + 112 = 0$$

Here $a = 1, b = -20, c = 112$

$$\begin{aligned} \text{Therefore } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{20 \pm \sqrt{400 - 4(1)(112)}}{2(1)} \\ &= \frac{20 \pm \sqrt{400 - 448}}{2} = \frac{20 \pm \sqrt{-48}}{2} \end{aligned}$$

Discriminant = $-48 < 0$

Therefore the roots are not real.

Hence, the situation is not possible.

5) Is it possible to design a rectangular park of perimeter 80m and area $400m^2$? If so, find its length and breadth.

Solution:

Let breadth of the park be $x m$.

Given, perimeter of the park = 80 m.

(i.e) $2(\text{Length} + \text{Breadth}) = 80 m$.

$$\text{Length} + x = \frac{80}{2} = 40 m.$$

Therefore Length = $(40 - x)m$.

Also, it is given that area of the park = $400m^2$.

(i.e) Length x Breadth = 400

$$(40 - x)x = 400$$

$$40x - x^2 - 400 = 0$$

$$x^2 - 40x + 400 = 0$$

$$(x - 20)(x - 20) = 0 \Rightarrow x = 20, 20$$

Therefore Breadth of the park $x = 20 m$.

Length of the park = $40 - x$

$$= 40 - 20 = 20 m$$

Hence, it is possible to design a rectangular park of perimeter 80 m and area $400 m^2$.

05 ARITHMETIC PROGRESSIONS (AP)

Sequence:

A list of numbers arranged in a definite order according to some definite rule, is called a sequence.

Arithmetic Progression:

An Arithmetic Progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

In general $a, a+d, a+2d, a+3d, \dots$ represent an arithmetic progression, where 'a' is the first term and d is common difference. This is called general form of an AP.

Example 1: For the AP: $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \dots$ write the first term a and the common difference d.

Solution:

The given sequence is $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \dots$

$$\text{Here } a_1 = \frac{3}{2}$$

common difference $d = a_2 - a_1$

$$= \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$$

Therefore the first term $a_1 = \frac{3}{2}$

Common difference $d = -1$

Example 2: Which of the following list of numbers form an AP? If they form an AP write the next two terms:-

Solution:

(i) 4, 10, 16, 22.....

Solution:

$$\text{Here } a_1 = 4, a_2 = 10, a_3 = 16, a_4 = 22$$

We have

$$a_2 - a_1 = 10 - 4 = 6$$

$$a_3 - a_2 = 16 - 10 = 6$$

$$a_4 - a_3 = 22 - 16 = 6$$

Here differences are equal.

Therefore the given sequence form an AP.

The next two term are $a_5 = 22 + 6 = 28$

and

$$a_6 = 28 + 6 = 34$$

(ii) 1, -1, -3, -5

Solution:

$$\text{Here } a_1 = 1, a_2 = -1, a_3 = -3, a_4 = -5$$

We have

$$a_2 - a_1 = -1 - 1 = -2$$

$$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$$

$$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$$

Here differences are equal.

Therefore the given sequence form an AP.

The next two term are $a_5 = -5 + (-2)$

$$= -5 - 2 = -7 \text{ and}$$

$$a_6 = -7 + (-2) = -9$$

(iii) -2, 2, -2, 2, -2,

$$\text{Here } a_1 = -2, a_2 = 2, a_3 = -2, a_4 = 2, \dots$$

We have

$$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$$

$$a_3 - a_2 = -2 - 2 = -4$$

$$\text{Here } a_3 - a_1 \neq a_3 - a_2$$

Therefore the given list of numbers does not form an AP.

(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3

$$a_2 - a_1 = 1 - 1 = 0$$

$$a_3 - a_2 = 1 - 1 = 0$$

$$a_4 - a_3 = 2 - 1 = 1$$

$$\text{Here } a_3 - a_2 \neq a_4 - a_3$$

Therefore the given list of numbers does not form an AP.

Note: If the difference of list of numbers are equal, then the given sequence form an AP.

EXERCISE 5.1

1) In which of the following situations, does the list of numbers involved make an AP, and why?

(i) The taxi fare after each km when the fare is ₹15 for the first km and ₹8 for each additional km.

Solution:

The fare for journey of 1st km is ₹15

The fare for journey of 2nd km is ₹15+8
= ₹23

The fare for journey of 3rd km is ₹23+8
= ₹31

Here each term is obtained by adding 8 to the preceding term.

So, it forms an A.P.

(ii) The amount of air present in a cylinder when a vaccum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Solution:

Let the amount of air present in the cylinder be y units.

By data,

Remaining air in the cylinder after using vaccum pump in the first time

$$= y - y\left(\frac{1}{4}\right) = \frac{4y-y}{4} = \frac{3y}{4}$$

Again, Remaining air in the cylinder after using vaccum pump in the second time

$$= \frac{3y}{4} - \frac{1}{4}\left(\frac{3y}{4}\right) = \frac{3y}{4} - \frac{3y}{16} = \frac{12y-3y}{16} = \frac{9y}{16}$$

Therefore the numbers are $y, \frac{3y}{4}, \frac{9y}{16}, \dots$

$$\text{Here } a_2 - a_1 = \frac{3y}{4} - y = \frac{3y-4y}{4} = \frac{-y}{4}$$

$$a_3 - a_2 = \frac{9y}{16} - \frac{3y}{4} = \frac{9y-3y}{16} = \frac{-3y}{16}$$

$$a_2 - a_1 \neq a_3 - a_2$$

Hence, it is not form an AP.

(iii) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.

Solution:

Cost of digging for the 1st metre, 2nd metre and 3rd metre are 150, (150+50), (150+50+50)

Here each term is obtained by adding 50 to the preceding terms.

So, it forms an AP.

(iv) The amount of money in the account every year, when ₹10000 is desposited at Compound interest at 8% per annum.

Solution:

Here $p = ₹10000$

$r = 8\%$ p.a CI

We know that Amount = $p\left(1 + \frac{r}{100}\right)^T$

Now, the amount in the first year

$$= 10000\left(1 + \frac{8}{100}\right)^1 = 10000\left(\frac{108}{100}\right)$$

$$= ₹10800$$

The amount in the 2nd year

$$A = 10000\left(1 + \frac{8}{100}\right)^2$$

$$= 100000\left(\frac{108}{100}\right)^2 = \frac{10000 \times 108 \times 108}{100 \times 100}$$

$$= 11664 = ₹11664$$

Therefore the numbers are 10000, 10800, 11664.....

$$\text{Here } a_2 - a_1 = 10800 - 10000 = 800$$

$$a_3 - a_2 = 11664 - 10800 = 864$$

Common difference is not equal.

Therefore it does not form an AP.

2) Write first four terms of the AP, when the first term 'a' and the common difference d are given as follows.

(i) $a = 10, d = 10$

Solution:

1st term of AP is $t_1 = a = 10$

2nd term of AP is $t_2 = a + d = 10 + 10 = 20$

3rd term of AP is $t_3 = a + 2d = 10 + 2(10)$
= $10 + 20 = 30$

4th term of AP is $t_4 = a + 3d = 10 + 3(10)$
= $10 + 30 = 40$

Therefore first four terms of the AP are 10, 20, 30, 40.

(ii) $a = -2, d = 0$

Solution:First four terms of the AP are $a, a + d, a + 2d, a + 3d$

Now, $a = -2$

$$a + d = -2 + 0 = -2$$

$$a + 2d = -2 + 2(0) = -2$$

$$a + 3d = -2 + 3(0) = -2$$

Therefore first four terms of the AP are -2, -2, -2, -2.

(iii) $a = 4, d = -3$

Solution:First four terms of the AP are $a, a + d, a + 2d, a + 3d$

Now, $a = 4$

$$a + d = 4 - 3 = 1$$

$$a + 2d = 4 + 2(0 - 3) = 4 - 6 = -2$$

$$a + 3d = 4 + 3(-3) = 4 - 9 = -5$$

Therefore first four terms of the AP are 4, 1, -2, -5.

(iv) $a = -1, d = \frac{1}{2}$

Solution:

First four terms of the AP are

$$a, a + d, a + 2d, a + 3d$$

Here, $a = -1$

$$a + d = -1 + \frac{1}{2} = \frac{-2+1}{2} = -\frac{1}{2}$$

$$a + 2d = -1 + 2\left(\frac{1}{2}\right) = -1 + 1 = 0$$

$$a + 3d = -1 + 3\left(\frac{1}{2}\right) = -1 + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$$

Therefore first four terms of the AP are $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

(v) $a = -1.25, d = -0.25$

Solution:

First four terms of the AP are

$$a, a + d, a + 2d, a + 3d$$

Now, $a = -1.25$

$$a + d = -1.25 - 0.25 = -1.50$$

$$a + 2d = -1.25 + 2(-0.25)$$

$$= -1.25 - 0.50 = -1.75$$

$$a + 3d = -1.25 + 3(-0.25)$$

$$= -1.25 - 0.75 = -2$$

Therefore first four terms of the AP are -1.25, -1.50, -1.75, -2.**3) For the following AP write the first term and the common difference.**

(i) $3, 1, -1, -3, \dots$

Solution:

Given that the list of numbers are in AP.

Here first term is $a = 3$ and the common difference is $d = a_2 - a_1$

$$= 1 - 3 = -2$$

(ii) $-5, -1, 3, 7, \dots$

Solution:

Given that the list of numbers are in AP.

Here first term is $a = -5$ and the common difference is $d = a_2 - a_1$

$$= -1 - 5 = -1 + 5 = 4$$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

Solution:

Given that the list of numbers are in AP.

Here first term is $a = \frac{1}{3}$ and the common difference is $d = a_2 - a_1$

$$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$$

(iv) $0.6, 1.7, 2.8, 3.9, \dots$

Solution:

Given that the list of numbers are in AP.

Here first term is $a = 0.6$ and the common difference is $d = a_2 - a_1$

$$= 1.7 - 0.6 = 1.1$$

4) Which of the following are Aps? If they form an AP, find the common difference and write three more terms.

(i) 2, 4, 8, 16,

Solution:

$$\text{Here } a_2 - a_1 = 4 - 2 = 2$$

$$a_3 - a_2 = 8 - 4 = 4$$

$$a_2 - a_1 \neq a_3 - a_2$$

Therefore the given list of numbers does not form an AP.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

Solution:

$$\text{Here } a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$a_2 - a_1 = a_3 - a_2$$

Therefore the given list of numbers form an AP and the common difference is $d = \frac{1}{2}$.

Therefore next three terms are

$$\frac{7}{2} + \frac{1}{2}, \left(\frac{7}{2} + \frac{1}{2}\right) + \frac{1}{2} \text{ and } \left(\frac{7}{2} + \frac{1}{2} + \frac{1}{2}\right) + \frac{1}{2}$$

$$\frac{8}{2}, \frac{9}{2}, \frac{10}{2}$$

$$4, \frac{9}{2}, 5$$

(iii) -1.2, -3.2, -5.2, -7.2, ...

Solution:

$$\text{Here } a_2 - a_1 = -3.2 - (-1.2) \\ = -3.2 + 1.2 = -2$$

$$a_3 - a_2 = -5.2 - (-3.2) \\ = -5.2 + 3.2 = -2$$

$$a_2 - a_1 = a_3 - a_2$$

Therefore the given list of numbers form an AP with common difference d is -2 .

Next three terms are

$$-7.2 + (-2) = -9.2$$

$$-9.2 + (-2) = -11.2$$

$$-11.2 + (-2) = -13.2$$

Therefore three terms are

$$-9.2, -11.2, -13.2$$

(iv) -10, -6, -2, 2,

Solution:

$$\text{Here } a_2 - a_1 = -6 - (-10) \\ = -6 + 10 = 4$$

$$a_3 - a_2 = -2 - (-6) \\ = -2 + 6 = 4$$

$$a_2 - a_1 = a_3 - a_2$$

Therefore the given list of numbers form an AP with common difference $d = 4$

Next three terms are

$$2+4 = 6$$

$$6+4 = 10$$

$$10+4 = 14$$

Therefore next three terms are 6, 10, 14.

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

Solution:

$$\text{Here } a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$$a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) \\ = 3 + 2\sqrt{2} - 3 - \sqrt{2} \\ = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

$$\text{So } a_2 - a_1 = a_3 - a_2$$

Therefore the given list of numbers form an AP and the common difference is $d = \sqrt{2}$.

Therefore next three terms are

$$3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$$

$$3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$

$$3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

Therefore the next three terms are

$$3 + 4\sqrt{2}, 3 + 5\sqrt{2}, \text{ and } 3 + 6\sqrt{2}$$

(vi) 0.2, 0.22, 0.222, 0.2222,

Solution:

$$\text{Here } a_2 - a_1 = 0.22 - 0.2 = 0.02$$

$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$

$$a_2 - a_1 \neq a_3 - a_2$$

Therefore the given list of numbers does not form an AP.

(vii) 0, -4, -8, -12, ...

Solution:

Here $a_2 - a_1 = -4 - 0 = -4$

$$a_3 - a_2 = -8 + 4 = -4$$

$$a_4 - a_3 = -12 + 8 = -4$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

Therefore the given list of numbers form an AP with common difference d is -4 .

The next three terms are

$$-12 - 4 = -16$$

$$-16 - 4 = -20$$

$$-20 - 4 = -24$$

Therefore next 3 terms are -16, -20, -24

(viii) $\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

Solution:

Here $a_2 - a_1 = -\frac{1}{2} + \frac{1}{2} = 0$

$$a_3 - a_2 = -\frac{1}{2} + \frac{1}{2} = 0$$

Now $a_2 - a_1 = a_3 - a_2 = 0$

Therefore the given list of numbers form an AP with common difference $d = 0$

Therefore $a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$

$$a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

$$a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

Therefore next 3 terms are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

(ix) 1, 3, 9, 27, ...

Solution:

Here $a_2 - a_1 = 3 - 1 = 2$

$$a_3 - a_2 = 9 - 3 = 6$$

$$a_2 - a_1 \neq a_3 - a_2$$

Therefore the given list of numbers does not form an AP.

(x) $a, 2a, 3a, 4a, \dots$

Solution:

Here $a_2 - a_1 = 2a - a = a$

$$a_3 - a_2 = 3a - 2a = a$$

Now $a_2 - a_1 = a_3 - a_2 = a$

Therefore the given list of numbers form an AP with common difference $d = a$

Therefore $a_5 = a_4 + d = 4a + a = 5a$

$$a_6 = a_5 + d = 5a + a = 6a$$

$$a_7 = a_6 + d = 6a + a = 7a$$

Therefore next 3 terms are 5a, 6a, 7a

(xi) a, a^2, a^3, a^4, \dots

Solution:

Here $a^2 - a = a(a - 1)$

$$a_2 - a_1 = a^2 - a = a(a - 1)$$

$$a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$$

$$a_2 - a_1 \neq a_3 - a_2$$

Therefore the given list of numbers does not form an AP.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

Solution:

$$\sqrt{2} = 1 \times \sqrt{2}$$

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$$

Therefore $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

$$= \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \dots$$

Here $a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$

Therefore $a_2 - a_1 = a_3 - a_2 = \sqrt{2}$

Therefore the given list of numbers form an AP with common difference $d = \sqrt{2}$

Therefore

$$a_5 = a_4 + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$$

$$a_6 = a_5 + \sqrt{2} = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$$

$$a_7 = a_6 + \sqrt{2} = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$$

Therefore next 3 terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

Solution:

Here $a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$$a_3 - a_2 = \sqrt{9} - \sqrt{6}$$

$$a_2 - a_1 \neq a_3 - a_2$$

Therefore the given list of numbers does not form an AP.

(xiv) $1^2, 3^2, 5^2, 7^2, \dots$

Solution:

Here $a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$

$$a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$$

So, $a_2 - a_1 \neq a_3 - a_2$

Therefore the given list of numbers does not form an AP.

(xv) $1^2, 5^2, 7^2, 73, \dots$

Solution:

Here $a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$

$$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$$

$$73 - 7^2 = 73 - 49 = 24$$

Now $a_2 - a_1 = a_3 - a_2 = 24$

Therefore the given list of numbers form an AP with common difference $d = 24$

Therefore $a_5 = a_4 + 24 = 73 + 24 = 97$

$$a_6 = a_5 + 24 = 97 + 24 = 121$$

$$a_7 = a_6 + 24 = 121 + 24 = 145$$

Therefore next 3 terms are 97, 121, 145

n^{th} Term of an A.P

If the first term of an A.P is 'a' and its common difference is 'd', then its n^{th} term is given by the formula

$$a_n = a + (n - 1)d$$

The n^{th} term of an AP is also called its, general term.

Note:

The n^{th} term is known as last term of an AP and it is denoted by l , which is given by $l = a + (n - 1)d$

Example 3: Find the 10th term of the AP 2, 7, 12.....

Solution:

Given AP is 2, 7, 12,

Here $a = 2,$

$$d = a_2 - a_1 = 7 - 2 = 5$$

$$n = 10$$

So, $a_{10} = a + (n - 1)d = 2 + (10 - 1)5$

$$= 2 + 9(5) = 2 + 45 = 47$$

Therefore 10th term of the given AP is 47.

Example 4: Which term of the AP 21, 18, 15, is - 81? Also, is any term 0? Give reason for your answer.

Solution:

Given AP is 21, 18, 15,

Here $a = 21, d = a_2 - a_1$

$$= 18 - 21 = -3, a_n = -8$$

$$a + (n - 1)d = -81$$

$$21 + (n - 1)(-3) = -81$$

$$-3n + 3 = -81 - 21$$

$$-3n = -81 - 21 - 3$$

$$-3n = -105$$

$$n = \frac{-105}{-3}$$

$$n = 35$$

Therefore 35th term of the given AP is -81

Suppose $a_n = 0$

$$a + (n - 1)d = 0$$

$$21 + (n - 1)(-3) = 0$$

$$(n - 1)(-3) = -21$$

$$n - 1 = \frac{-21}{-3} = 7$$

Therefore $n = 7 + 1 = 8$

So, 8th term is zero.

Example 5: Determine the AP whose 3rd term is 5 and 7th term is 9.

Solution:

Given $a_3 = 5$ and $a_7 = 9$

$$a + 2d = 5 \quad \text{--- (01)}$$

$$a + 6d = 9 \quad \text{--- (02)}$$

$$1 - 2 \Rightarrow \frac{-4d = -4}{d = 1}$$

$$d = 1$$

substitute $d = 1$ in 1

$$a + 2 = 5 \Rightarrow a = 5 - 2 = 3$$

Hence the required AP is $a, a + d, a + 2d, \dots$

$$3, 3 + 1, 3 + 2(1), \dots$$

$$3, 4, 5, \dots$$

Example 6: Check whether 301 is a term of the list of numbers 5, 11, 17, 23,

Solution:

Given list of numbers 5, 11, 17, 23,

Here $a = 5$,

$$a_2 - a_1 = 11 - 5 = 6$$

$$a_3 - a_2 = 17 - 11 = 6$$

$$\text{So, } a_2 - a_1 = a_3 - a_2 = 6$$

Here $a_2 - a_1 = a_3 - a_2$

Therefore the given list of numbers is an AP with

$a = 5$ and $d = 6$.

Let $a_n = 301$

$$a + (n - 1)d = 301$$

$$5 + (n - 1)6 = 301$$

$$(n - 1)6 = 301 - 5$$

$$(n - 1)6 = 296$$

$$\text{Therefore } n - 1 = \frac{296}{6} = 48.33$$

$$n = 49.33$$

But n is integer.

Therefore 301 is not a term in the given list of numbers.

Example 7: How many two digit numbers are divisible by 3?

Solution:

Two digit numbers which are divisible by 3 are 12, 15, 18, 21, 24,99

Here $a = 12$

$$a_2 - a_1 = 15 - 12 = 3$$

$$a_3 - a_2 = 18 - 15 = 3$$

$$\text{So, } a_2 - a_1 = a_3 - a_2 = 3$$

Therefore the given list of numbers is an AP

$$a = 12, d = 3, a_n = 99$$

$$a + (n - 1)d = 99$$

$$12 + (n - 1)d = 99$$

$$(n - 1)d = 99 - 12$$

$$(n - 1)d = 87$$

$$(n - 1)3 = 87$$

$$n - 1 = \frac{87}{3} = 29$$

$$n = 29 + 1$$

$$n = 30$$

Therefore there are 30 two digit numbers divisible by 3.

Example 8: Find the 11th term from the last term (towards the first term) of the AP 10, 7, 4, -62.

Solution:

Given AP is 10, 7, 4, -62.

Here $a = 10$,

$$d = 7 - 10 = -3$$

Let $a_n = -62$

$$a + (n - 1)d = -62$$

$$10 + (n - 1)(-3) = -62$$

$$(n - 1)(-3) = -72$$

$$(n - 1) = \frac{-72}{-3} = 24$$

$$n = 24 + 1 = 25$$

So, there are 25 terms in the given AP.

Now, the 11th term from the last term will be 15th term.

Therefore $a_{15} = a + 14d$

$$= 10 + 14(-3) = 10 - 42 = -32$$

Hence, the 11th term from last is -32.

Example 9: A sum of ₹1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interest form an AP? If so, find the interest at the end of 30 years making use of this fact.

Solution:

Here $p = ₹1000, r = 8\%$.

Therefore the interest at the end of 1st year = $\frac{pnr}{100}$

$$= \frac{1000 \times 1 \times 8}{100} = ₹80$$

$$\text{Interest at the end of 2nd year} = \frac{1000 \times 2 \times 8}{100} = ₹160$$

$$\text{Interest at the end of 3rd year} = \frac{1000 \times 3 \times 8}{100} = ₹240.$$

So, the interest at the end of 1st, 2nd, 3rd, years are 80, 160, 240,

The list of numbers is an AP with $a = 80$ and $d = 80$.

Therefore the interest at the end of 30th year is $a_{30} =$

$$a + 29d$$

$$= 80 + 29(80) = 80 + 2320 = ₹2400$$

Therefore 30th year Interest is ₹2400.

Example 10: In a flower bed, there are 23 rose plants in the first row, 21 in the second 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Solution:

Number of rose plants in the 1st, 2nd, 3rd, rows are 23, 21, 19,5

Here $a = 23$,

$$a_2 - a_1 = 21 - 23 = -2$$

$$a_3 - a_2 = 19 - 21 = -2$$

$$\text{So, } a_2 - a_1 = a_3 - a_2 = -2$$

Therefore the given list of numbers is an AP with

$$a = 23, d = -2.$$

$$\text{Let } a_n = 5$$

$$a + (n - 1)d = 5$$

$$23 + (n - 1)(-2) = 5$$

$$(n - 1)(-2) = 5 - 23$$

$$(n - 1)(-2) = -18$$

$$n - 1 = \frac{-18}{-2} = 9$$

$$n = 10$$

Hence, there are 10 rows in the flower bed.

EXERCISE 5.2

1) Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the AP.

(i) $a = 7, d = 3, n = 8, a_n = ?$

Solution:

$$a_n = a + (n - 1)d$$

$$a_8 = 7 + 7(3)$$

$$= 7 + 21 = 28$$

(ii) $a = -18, d = ?, n = 10, a_n = 0.$

Solution:

$$a_n = a + (n - 1)d \text{ and } a_{10} = 0$$

$$a_{10} = 0 \Rightarrow -18 + 9(d) = 0$$

$$9d = 18$$

$$d = \frac{18}{9}$$

$$d = 2$$

(iii) $a = ?, d = -3, n = 18, a_n = -5$

Solution:

$$a_n = -5 \Rightarrow a_{18} = -5$$

$$a + 17d = -5$$

$$a + 17(-3) = -5$$

$$a - 51 = -5$$

$$a = 51 - 5$$

$$a = 46$$

(iv) $a = -18.9, d = 2.5, n = ?, a_n = 3.6$

Solution:

$$a_n = 3.6$$

$$a + (n - 1)d = 3.6$$

$$-18.9 + (n - 1)(2.5) = 3.6$$

$$(n - 1)(2.5) = 3.6 + 18.9$$

$$(n - 1)(2.5) = 22.5$$

$$n - 1 = \frac{22.5}{2.5} = \frac{225}{25} = 9$$

$$n - 1 = 9 \Rightarrow n = 10$$

(v) $a = 3.5, d = 0, n = 105, a_n = ?$

Solution:

$$a_n = a + (n - 1)d$$

$$\text{Therefore } a_{105} = 3.5 + (n - 1)d$$

$$= 3.5 + (n - 1)(0)$$

$$= 3.5$$

$$\text{Therefore } a_{105} = 3.5$$

2) Choose the correct choice in the following and justify.

(i) 30th term of the AP: 10, 7, 4, is

a) 97 b) 77 c) -77 d) -87

Solution:

Given AP is 10, 7, 4,

$$\text{Here } a = 10, d = a_2 - a_1$$

$$= 7 - 10$$

$$= -3$$

$$a_{30} = a + 29d$$

$$= 10 + 29(-3)$$

$$= 10 - 87$$

$$= -77 \text{ Ans: (c)}$$

(ii) 11th term of the AP is $-3, -\frac{1}{2}, 2, \dots$

a) 28 b) 22 c) -38 d) $-48\frac{1}{2}$

Solution:

Given AP is $-3, -\frac{1}{2}, 2, \dots$

Here $a = -3, d = a_2 - a_1$

$$= -\frac{1}{2} + 3 = 2\frac{1}{2} = \frac{5}{2}$$

$$a_{11} = a + 10d$$

$$= -3 + 10\left(\frac{5}{2}\right)$$

$$= -3 + 25$$

$$= 22 \text{ Ans: (B)}$$

3) In the following Aps, find the missing terms in the boxes:-

(i) 2, -----, 26

Solution:

Given list of numbers is an AP.

Therefore $a_2 - a_1 = a_3 - a_2$

$$x - 2 = 26 - x$$

$$x + x = 26 + 2$$

$$2x = 28$$

$$x = 14$$

(ii) -----, 13, -----, 3

Solution:

Given that the list of numbers is an AP.

Therefore $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$$13 - x = y - 13 = 3 - y$$

Consider $y - 13 = 3 - y \Rightarrow 2y = 13 + 3$

$$2y = 16$$

$$y = 8$$

Consider, $13 - x = y - 13$

$$x = 13 - 8 + 13$$

$$x = 5 + 13 = 18$$

Therefore the first term $x = 18$ and the third term $y = 8$.

(iii) 5, --, --, $9\frac{1}{2}$

Solution:

Given that the list of numbers is an AP.

let the second and 3rd terms are x, y

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$$\text{Now, } x - 5 = y - x = 9\frac{1}{2} - y$$

$$x - 5 = y - x \quad \left| \quad y - x = \frac{19}{2} - y \right.$$

$$x + x - y = 5 \quad \left| \quad y + y - x = \frac{19}{2} \right.$$

$$2x - y = 5 \quad \left| \quad 2y - x = \frac{19}{2} \right.$$

$$4y - 2x = 19$$

$$2x - y = +5 \quad \text{--- (01)}$$

$$-2x + 4y = 19 \quad \text{--- (02)}$$

Adding Equation 1 and 2 $\Rightarrow 3y = 24$

$$y = \frac{24}{3} = 8, y = 8$$

substitute $y = 8$ in 1

$$2x - 8 = 5$$

$$2x = 8 + 5$$

$$2x = 13$$

$$x = \frac{13}{2}$$

Therefore 2nd and 3rd terms are $\frac{13}{2}, 8$.

4) Which term of the AP 3, 8, 13, 18,is 78?

Solution:

Given that the list of numbers is an AP.

Here $a = 3, d = a_2 - a_1 = 8 - 3 = 5$

Let $a_n = 78$

$$a + (n - 1)d = 78$$

$$3 + (n - 1)d = 78$$

$$(n - 1)5 = 78 - 3$$

$$(n - 1)5 = 75$$

$$n - 1 = \frac{75}{5} = 15$$

$$n = 16$$

Therefore 16th term of the given AP is 16.

5) Find the number of terms in each of the following AP's.

(i) 7, 13, 19,205

Solution: Given AP is 7, 13, 19, 205

Here $a = 7, d = a_2 - a_1 = 13 - 7 = 6$

Let $a_n = 205$

$$a + (n - 1)d = 205$$

$$7 + (n - 1)(6) = 205$$

$$(n - 1)6 = 205 - 7$$

$$(n - 1)6 = 198$$

$$n - 1 = \frac{198}{6} = 33$$

$$n = 34$$

Therefore the given AP has 34 terms.

(ii) $18, 15\frac{1}{2}, 13, \dots -47$

Solution:

Given AP is $18, 15\frac{1}{2}, 13, \dots -47$

Here $a = 18, d = a_2 - a_1$

$$= \frac{31}{2} - 18 = \frac{31-36}{2} = \frac{-5}{2}$$

Let $a_n = -47$

$$a + (n - 1)d = -47$$

$$18 + (n - 1)\left(\frac{-5}{2}\right) = -47$$

$$(n - 1)\left(\frac{-5}{2}\right) = -47 - 18 = -65$$

$$n - 1 = -65 \times \left(-\frac{2}{5}\right)$$

$$= 13 \times 2 = 26$$

Therefore $n = 26 + 1 = 27$

Hence the given AP has 27 terms.

6) Check whether -150 is a term of the AP: 11, 8, 5, 2,

Solution:

Given AP is 11, 8, 5, 2,

Here $a = 11, d = a_2 - a_1 = 8 - 11 = -3$

Let $a_n = -150$

$$a_n = a + (n - 1)d = -150$$

$$11 + (n - 1)(-3) = -150$$

$$(n - 1)(-3) = -150 - 11$$

$$(n - 1)(-3) = -161$$

$$(n - 1) = \frac{-161}{-3} = 53.66$$

$$n - 1 = 53.66$$

$$n = 54.66$$

But n should be positive integer.

Therefore -150 is not a term of the AP.

7) Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Solution:

Given that 11th term is 38

$$a_{11} = 38$$

$$\Rightarrow a + 10d = 38 \text{ ————— (01)}$$

Also, 16th term is 73

$$a_{16} = 73$$

$$\Rightarrow a + 15d = 73$$

$$a + 15d = 73 \text{ ————— (02)}$$

$$a + 10d = 38 \text{ ————— (01)}$$

$$2 - 1 \Rightarrow 5d = 35$$

$$d = \frac{35}{5} = 7$$

substitute $d = 7$ in 1

$$a + 10(7) = 38$$

$$a + 70 = 38$$

$$a = 38 - 70 = -32$$

Now $a_{31} = a + 30d$

$$= -32 + 30(7) = -32 + 210 = 178$$

Therefore 31st term of an AP is 178.

8) An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Solution:

Number of terms in the AP is 50.

Also given that 3rd term is 12.

$$a_3 = 12$$

$$\Rightarrow a + 2d = 12 \text{ ————— (01)}$$

Last term is 106.

$$a_{50} = 106$$

$$a + (n - 1)d = 106$$

$$a + 49d = 106 \text{ ————— (02)}$$

$$a + 49d = 106 \text{ ————— (02)}$$

$$a + 2d = 12 \text{ ————— (01)}$$

$$2 - 1 \Rightarrow 47d = 94$$

$$d = \frac{94}{47} = 2$$

substitute $d = 2$ in 1

$$a + 2(2) = 12$$

$$a + 4 = 12 \Rightarrow a = 12 - 4 = 8$$

$$\text{Now } a_{29} = a + 28d = 8 + 28(2) = 8 + 56$$

$$a_{29} = 64$$

Therefore 29th term of the given AP is 64.

9) If the 3rd and 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero.

Solution:

Given that 3rd term of an AP is 4.

$$a_3 = 4$$

$$a + 2d = 4 \quad \text{--- (01)}$$

Also, $a_9 = -8$

$$a + 8d = -8 \quad \text{--- (02)}$$

$$a + 8d = -8 \quad \text{--- (02)}$$

$$a + 2d = 4 \quad \text{--- (01)}$$

$$2 - 1 \Rightarrow \frac{6d = -12}{6d = -12}$$

$$d = -\frac{12}{6} = -2$$

Therefore $d = -2$

substitute $d = -2$ in 1

$$a + 2(-2) = 4$$

$$a - 4 = 4 \Rightarrow a = 8$$

Let $a_n = 0$

$$a + (n - 1)d = 0$$

$$8 + (n - 1)(-2) = 0$$

$$(n - 1)(-2) = -8$$

$$n - 1 = \frac{-8}{-2} = 4$$

$$n = 5$$

Therefore 5th term of the given AP is zero.

10) The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution:

Given that $a_{17} - a_{10} = 7$

$$(a + 16d) - (a + 9d) = 7$$

$$a + 16d - a - 9d = 7$$

$$7d = 7 \Rightarrow d = 1$$

Therefore common difference $d = 1$.

11) Which term of the AP 3, 15, 27, 39, will be 132 more than its 54th term?

Solution:

Given AP is 3, 15, 27, 39,

Here $a = 3, d = a_2 - a_1$

$$= 15 - 3 = 12$$

$$\text{Now, } a_{54} = a + 53d = 3 + 53(12)$$

$$= 3 + 636 = 639$$

Required term = 132 more than 54th term

$$= (639 + 132) = 771$$

Let $a_n = 771$

$$\Rightarrow a + (n - 1)d = 771$$

$$3 + (n - 1)(12) = 771$$

$$(n - 1)(12) = 771 - 3$$

$$(n - 1)(12) = 768$$

$$n - 1 = \frac{768}{12} = 64$$

$$n - 1 = 64 \Rightarrow n = 65$$

Hence 65th term is required term.

12) Two AP's have common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms.

Solution:

Let the first AP be $a_1, a_2, a_3, \dots, a_n$ and the 2nd AP be $b_1, b_2, b_3, \dots, b_n$.

Let d be the common difference of two Aps.

Therefore n^{th} term of 1st AP a_n and n^{th} term

2nd AP b_n are $a_n = a_1 + (n - 1)d$ and

$$b_n = b_1 + (n - 1)d$$

Now,

$$a_n - b_n = [a_1 + (n - 1)d] - [b_1 + (n - 1)d]$$

$$= a_1 + (n - 1)d - b_1 - (n - 1)d$$

$$= a_1 - b_1$$

Given that $a_{100} - b_{100} = a_1 - b_1 = 100$

So, $a_{1000} - b_{1000} = 100$.

Hence difference between 1000th terms is also 100.

13) How many 3 digit numbers are divisible by 7?

Solution:

The first 3 digit number divisible by 7 is

$$100 - 2 + 7 = 105.$$

$$\begin{array}{r} 7 \overline{)100} \quad (14 \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

The last 3 digit number divisible by

7 is $999 - 5 = 994$.

Therefore 3 digit number which are divisible by 7 are

105, 112, 119, ... 994.

Here $a = 105, d = 7, l = 994$.

$$a_n = 994$$

$$\Rightarrow a + (n - 1)d = 994$$

$$105 + (n - 1)7 = 994$$

$$(n - 1)7 = 994 - 105$$

$$(n - 1)7 = 889$$

$$n - 1 = \frac{889}{7} = 127$$

$$n = 128$$

Hence number of 3 digit numbers which are divisible by 7 is 128.

$$\begin{array}{r} 7 \overline{)999(142} \\ \underline{29} \\ 28 \\ \underline{19} \\ 14 \\ \underline{5} \end{array}$$

14) How many multiples of 4 lie between 10 and 250?

Solution:

The First multiples of 4 after 10 is 12.

The last multiples of 4 before 250

is $250 - 2 = 248$.

Therefore Number of terms in an AP

whose 1st term $a = 12$ and the last term

$$a_n = 248$$

$$a + (n - 1)d = 248$$

$$12 + (n - 1)4 = 248$$

$$(n - 1)4 = 248 - 12$$

$$(n - 1)4 = 236$$

$$n - 1 = \frac{236}{4} = 59$$

$$n = 60$$

Therefore Number of multiples of 4 lie between 10 and 250 is 60.

$$\begin{array}{r} \cancel{52} \\ 4 \overline{)249} \\ \underline{24} \\ 9 \\ \underline{8} \\ 1 \end{array}$$

15) For what value of n, the nth terms of the AP's 63, 65, 67, ... and 3, 10, 17, ... are equal.

Solution:

Given AP's are 63, 65, 67, ... and 3, 10, 17, ...

In the 1st AP $a = 63, d = 65 - 63 = 2$, and

$$a_n = a + (n - 1)d = 63 + (n - 1)2$$

In the 2nd AP, $b = 3, d = 10 - 3 = 7$, and

$$b_n = b + (n - 1)d = 3 + (n - 1)7$$

Given that $a_n = b_n$

$$63 + (n - 1)2 = 3 + (n - 1)7$$

$$63 + 2n - 2 = 3 + 7n - 7$$

$$61 + 2n = -4 + 7n$$

$$61 + 4 = 7n - 2n$$

$$65 = 5n$$

$$n = \frac{65}{5} = 13$$

Hence, the 13th term of two given AP's are equal.

16) Determine the AP whose third term is 16 and 7th term exceeds the 5th term by 12.

Solution:

Let the AP be $a, a + d, a + 2d, \dots$

Given that $a_3 = 16$

$$a + 2d = 16 \text{ ————— } \textcircled{01}$$

Also it is given that 7th term exceeds the 5th term by 12.

$$a_7 - a_5 = 12$$

$$\Rightarrow a + 6d - (a + 4d) = 12$$

$$a + 6d - a - 4d = 12$$

$$2d = 12 \Rightarrow d = \frac{12}{2} = 6$$

Therefore $d = 6$

substitute $d = 6$ in 1

$$a + 2d = 16 \Rightarrow a + 2(6) = 16$$

$$a + 12 = 16$$

$$a = 16 - 12 = 4$$

Therefore $a = 4$

Then the required AP is $a, a + d, a + 2d, \dots$

4, 4 + 6, 4 + 2(6), ...

4, 10, 16,

17) Find the 20th term from the last term of the AP: 3, 8, 13, ... 253.

Solution:

The given AP from the last is 253, ... 13, 8, 3.

Here $a = 253, d = 8 - 13 = -5, n = 51$.

Therefore $a_{20} = a + 19d = 253 + 19(-5)$

$$= 253 - 95 = 158$$

Hence 20th from the last is 158.

18) The sum of 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.

Solution:

Given sum of 4th and 8th terms of an AP is 24.

$$a_4 + a_8 = 24$$

$$a + 3d + a + 7d = 24$$

$$2a + 10d = 24 \text{ ————— (01)}$$

Also it is given that

Sum of 6th and 10th terms is 44

$$a_6 + a_{10} = 44$$

$$a + 5d + a + 9d = 44$$

$$2a + 14d = 44 \text{ ————— (02)}$$

$$2a + 14d = 44 \text{ ————— (02)}$$

$$2a + 10d = 24 \text{ ————— (01)}$$

$$2 - 1 \Rightarrow 4d = 20$$

$$d = \frac{20}{4} = 5$$

Therefore $d = 5$

substitute $d = 5$ in 1

$$2a + 10(5) = 24$$

$$2a + 50 = 24$$

$$2a = 24 - 50$$

$$2a = -26$$

$$a = -\frac{26}{2} = -13$$

Therefore $a = -13$

Hence, first three term of the AP are

$$a, a + d, a + 2d$$

$$a = -13$$

$$a + d = -13 + 5 = -8 \text{ and}$$

$$a + 2d = -13 + 2(5) = -13 + 10 = -3$$

$$-13, -13 + 5, -13 + 2(5)$$

$$-13, -8, -3.$$

19) Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?

Solution:

The annual salary received by Subba Rao in the year 1995, 1996, 1997 are ₹5000, ₹5200, ₹5400, respectively.

The list of numbers from an AP.

$$\text{Here } a = ₹5000, d = a_2 - a_1$$

$$= ₹5200 - 5000 = ₹200$$

$$\text{Let } a_n = ₹7000$$

$$a + (n - 1)d = ₹7000$$

$$5000 + (n - 1)200 = ₹7000$$

$$(n - 1)200 = ₹7000 - ₹5000 = ₹2000$$

$$n - 1 = ₹\frac{2000}{200} = 10$$

Therefore $n = 11$ years.

Hence in the 11th year, he received ₹7000 as salary.

20) Ramakali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the nth week, her weekly savings become ₹20.75. Find n.

Solution:

Ramkali's savings are in the subsequent weeks are ₹5, ₹5+₹1.75, ₹5+2(1.75)

$$5, 6.75, 8.50, 10.25, \dots$$

$$\text{Here } a = 5, d = ₹1.75.$$

$$\text{let } a_n = 20.75$$

$$a + (n - 1)d = 20.75$$

$$5 + (n - 1)(1.75) = 20.75$$

$$(n - 1)(1.75) = 20.75 - 5$$

$$= 15.75$$

$$n - 1 = \frac{15.75}{1.75} = 1575 = 9$$

$$n = 9 + 1 = 10$$

Hence the 10th week her saving becomes Rs.20.75

Sum of First n Terms of an AP

Note 1 : If first term of an AP is 'a' and its common difference is 'd', then the sum of its first n terms S_n is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

This can also be written as

$$S_n = \frac{n}{2}[a + a_n] \text{ where } a_n \text{ is } n^{\text{th}} \text{ term}$$

If l is the last term then

$$S_n = \frac{n}{2}[a + l]$$

Note 2: The n^{th} term of AP is obtained by the difference of sum of first n terms and the sum to first $(n-1)$ terms. $a_n = S_n - S_{n-1}$

Example 11: Find the sum of the first 22 terms of the AP: 8, 3, - 2,

Solution:

The given AP is 8, 3, - 2,

$$\begin{aligned} \text{Here } a &= 8, d = a_2 - a_1 \\ &= 3 - 8 = -5 \\ n &= 22 \end{aligned}$$

$$\text{We know that } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} \text{Therefore } S_{22} &= \frac{22}{2}[2(8) + 21(-5)] \\ &= 11[16 - 105] = 11[-89] = -979 \end{aligned}$$

Hence, the sum of first 22 terms of AP is - 979

Example 12: If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Solution:

Sum of first 14 terms of an AP is 1050.

$$S_{14} = 1050$$

$$\frac{n}{2}[2a + (n - 1)d] = 1050$$

$$\frac{14}{2}[2a + 13d] = 1050$$

$$2a + 13d = \frac{1050}{7} = 150$$

$$2a + 13d = 150 \text{ ————— } \textcircled{01}$$

Here the first term $a = 10$

$$\text{Therefore } 1 \Rightarrow 2(10) + 13d = 150$$

$$20 + 13d = 150$$

$$13d = 150 - 20$$

$$13d = 130$$

$$d = \frac{130}{13} = 10$$

$$d = 10$$

$$\text{Therefore } a_{20} = a + 19d$$

$$= 10 + 19(10)$$

$$= 10 + 190 = 200$$

Hence 20th term is 200.

Example 13: How many terms of the AP 24, 12, 18, must be taken so that their sum is 78?

Solution:

Given AP is 24, 21, 18, ...

$$\text{Here } a = 24, d = 21 - 24 = -3, S_n = 78$$

$$S_n = 78$$

$$\frac{n}{2}[2a + (n - 1)d] = 78$$

$$\frac{n}{2}[2(24) + (n - 1)(-3)] = 78$$

$$n[48 - 3n + 3] = 78 \times 2 = 156$$

$$n[51 - 3n] = 156$$

$$51n - 3n^2 - 156 = 0$$

$$3n^2 - 51n + 156 = 0$$

$$\div 3, \quad n^2 - 17n + 52 = 0$$

$$(n - 13)(n - 4) = 0$$

$$n - 13 = 0 \quad n - 4 = 0$$

$$n = 13 \quad n = 4$$

Therefore number of terms is either 4 or 13.

Example 14: Find the sum of

(i) The first 1000 positive integers.

Solution:

$$\text{Let } S_n = 1 + 2 + 3 + \dots + 1000$$

$$\text{Here } a = 1, d = 1, l = 1000$$

$$\text{Therefore } S_n = \frac{n}{2}[a + l]$$

$$= \frac{1000}{2}[1 + 1000]$$

$$= 500(1001)$$

$$= 500500$$

(ii) The first n positive integers.

Solution:

$$\text{Let } S_n = 1 + 2 + \dots + n$$

$$\text{Here } a = 1, d = 1, l = n$$

$$\text{Therefore } S_n = \frac{n}{2}[a + l]$$

$$= \frac{n}{2}[1 + n]$$

$$= \frac{n(n + 1)}{2}$$

Example 15: Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3 + 2n$.

Solution:

Given $a_n = 3 + 2n$

Therefore $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2(2) = 3 + 4 = 7$

$a_3 = 3 + 2(3) = 3 + 6 = 9$

$a_4 = 3 + 2(4) = 3 + 8 = 11$

.....

Therefore the list of numbers are 5, 7, 9, 11,

Here $a = 5, d = 7 - 5 = 2, n = 24$

Therefore $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{24} = \frac{24}{2}[2(5) + 23(2)]$$

$$= 12[10 + 46]$$

$$= 12[56] = 672$$

Hence the sum of first 24 terms is 672.

Example 16: A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find:

- a) the production in the first year.
- b) the production in the 10th year.
- c) the total production in first 7 years.

Solution:

Given that the production increases uniformly by a fixed number every year.

Hence the TV manufactured in the 1st, 2nd, 3rd, year will form an AP.

Here $a_3 = 600$ and $a_7 = 700$

$$a + 6d = 700 \text{ ————— (01)}$$

$$a + 2d = 600 \text{ ————— (02)}$$

$$1-2 \Rightarrow 4d = 100$$

$$d = \frac{100}{4} = 25$$

$$d = 25$$

substitute $d = 25$ in 1

$$a + 6(25) = 700$$

$$a + 150 = 700$$

$$a = 700 - 150$$

$$a = 550$$

a) Therefore 1st year production is $a = 550$

$$\begin{aligned} \text{b) } a_{10} &= a + dd = 550 + 9(25) \\ &= 550 + 225 = 775 \end{aligned}$$

Therefore production in the 10th year = 775

$$S_7 = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} S_7 &= \frac{7}{2}[2(550) + 6(25)] = \frac{7}{2}[1100 + 150] \\ &= \frac{7}{2}[1250] = 7 \times 625 = 4375 \end{aligned}$$

c) Therefore total production in the first 7 years is 4375.

EXERCISE 5.3

1) Find the sum of the following Aps:-

a) 2, 7, 12, 10 terms.

Solution:

Given AP is 2, 7, 12, 10 terms.

Here $a = 2, d = 7 - 2 = 5, n = 10$.

Sum of first n terms is

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} \text{Therefore } S_{10} &= \frac{10}{2}[2(2) + 9(5)] \\ &= 5[4 + 45] = 5[49] = 245 \end{aligned}$$

b) - 37, - 33, - 29, to 12 terms.

Solution:

Given AP is - 37, - 33, - 29, to 12 terms.

Here $a = -37, d = -33 - (-37)$

$$= -33 + 37 = 4$$

$$a = -37, d = 4, n = 12$$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} \text{Therefore } S_{12} &= \frac{12}{2}[2(-37) + 11d] \\ &= 6[-74 + 11(4)] \\ &= 6[-74 + 44] \\ &= 6[-30] = -180 \end{aligned}$$

c) 0.6, 1.7, 2.8, to 100 terms.

Solution:

Given AP is 0.6, 1.7, 2.8, to 100 terms.

Here $a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} \text{Therefore } S_{100} &= \frac{100}{2}[2(0.6) + 99(1.1)] \\ &= 50[1.2 + 108.9] \\ &= 50[110.1] \\ &= 5505.0 \end{aligned}$$

d) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms.

Solution:

Given AP is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms.

Here $a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}, n = 11$

$$a = \frac{1}{15}, d = \frac{1}{60}, n = 11$$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} \text{Therefore } S_{11} &= \frac{11}{2}\left[2\left(\frac{1}{15}\right) + 10\left(\frac{1}{60}\right)\right] \\ &= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] \\ &= \frac{11}{2}\left[\frac{4+5}{30}\right] = \frac{11 \times 9}{2 \times 30} \\ &= \frac{33}{20} \end{aligned}$$

2) Find the sums given below.

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

Solution:

Given AP is $7 + 10\frac{1}{2} + 14 + \dots + 84$

Here $a = 7, d = 10\frac{1}{2} - 7 = 3\frac{1}{2}, l = 84$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[a + l]$$

$$\begin{aligned} \text{Therefore } S_{23} &= \frac{23}{2}[7 + 84] \\ &= \frac{23}{2}[91] \\ &= \frac{2093}{2} \\ &= 1046\frac{1}{2} \end{aligned}$$

$$a + (n - 1)d = 84$$

$$7 + (n - 1)\frac{7}{2} = 84$$

$$(n - 1)\frac{7}{2} = 84 - 7$$

$$(n - 1)\frac{7}{2} = 77$$

$$(n - 1) = \frac{77 \times 2}{7}$$

$$(n - 1) = 11 \times 2$$

$$n = 22 + 1 = 23$$

(ii) $34 + 32 + 30 + \dots + 10$

Solution:

Given AP is $34 + 32 + 30 + \dots + 10$

Here $a = 34, d = 32 - 34 = -2, l = 10$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[a + l]$$

$$\text{Therefore } S_n = \frac{13}{2}[34 + 10]$$

$$= \frac{13}{2}[44]$$

$$= 13 \times 22 = 286$$

$$a_n = 10$$

$$a + (n - 1)d = 10$$

$$34 + (n - 1)(-2) = 10$$

$$(n - 1)(-2) = 10 - 34$$

$$n - 1 = \frac{-24}{-2}$$

$$n - 1 = 12$$

$$n = 13$$

(iii) $-5 + (-8) + (-11) + \dots + (-230)$

Solution:

Given AP is $-5 + (-8) + (-11) + \dots + (-230)$

Here $a = -5, d = -8 - 5$

$$= -8 + 5 = -3,$$

$$l = -230$$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[a + l]$$

$$\text{Therefore } S_{76} = \frac{76}{2}[-5 + (-230)]$$

$$= 38[-235] = -8930$$

$$\begin{aligned}
 a + (n - 1)d &= -230 \\
 -5 + (n - 1)(-3) &= -230 \\
 (n - 1)(-3) &= -230 + 5 \\
 n - 1 &= \frac{-225}{-3} \\
 n - 1 &= 75 \\
 n &= 76
 \end{aligned}$$

3) In an AP

(i) Given $a = 5, d = 3, a_n = 50$, find n and S_n

Solution:

Here $a = 5, d = 3, a_n = 50$

$$\begin{aligned}
 a + (n - 1)d &= 50 \\
 5 + (n - 1)(3) &= 50 \\
 (n - 1)(3) &= 50 - 5 \\
 (n - 1)3 &= 45 \\
 (n - 1) &= \frac{45}{3} = 15 \\
 n &= 16
 \end{aligned}$$

We know that sum of first n terms is

$$\begin{aligned}
 S_n &= \frac{n}{2}[a + l] \\
 S_{16} &= \frac{16}{2}[5 + 50] \\
 &= 8 \times 55 = 440
 \end{aligned}$$

Hence $S_{16} = 440$

(ii) Given $a = 7, a_{13} = 35$, find n and S_{13}

Solution:

Here $a = 7, a_{13} = 35$

$$\begin{aligned}
 a + 12d &= 35 \\
 7 + 12d &= 35 \\
 12d &= 35 - 7 \\
 12d &= 28 \\
 d &= \frac{28}{12} = \frac{7}{3} \\
 d &= \frac{7}{3}
 \end{aligned}$$

We know that sum of first n terms is

$$\begin{aligned}
 S_n &= \frac{n}{2}[a + l] \\
 S_{13} &= \frac{13}{2}[7 + 35] \\
 &= \frac{13}{2}[42] = 13 \times 21 = 273
 \end{aligned}$$

Hence $S_{13} = 273$

(iii) Given $a_{12} = 37, d = 3$ find a and S_{12}
Solution:

Here $a_{12} = 37, d = 3$

$$\begin{aligned}
 a + 11d &= 37 \\
 a + 11(3) &= 37 \\
 a + 33 &= 37
 \end{aligned}$$

$$a = 37 - 33$$

$$a = 4$$

We know that sum of first n terms is

$$\begin{aligned}
 S_n &= \frac{n}{2}[a + l] \\
 S_{12} &= \frac{12}{2}[4 + 37] \\
 &= 6[41] = 246
 \end{aligned}$$

Hence $S_{12} = 246$

(iv) Given $a_3 = 15, S_{10} = 125$ find d and a_{10}

Solution:

Here $a_3 = 15, S_{10} = 125$

$$a + 2d = 15 \text{ ————— } \textcircled{01}$$

$$2a + 9d = 25 \text{ ————— } \textcircled{02}$$

$$1 \times 2 \Rightarrow 2a + 4d = 30 \text{ ————— } \textcircled{03}$$

$$2 - 3 \Rightarrow 5d = -5$$

$$d = -1$$

Now, $a_{10} = a + 9d$

$$= 17 + 9(1) = 17 - 9 = 8$$

Therefore $a_{10} = 8$

$$S_{10} = 125$$

We know that sum of first n terms is

$$\frac{10}{2}[2a + 9d] = 125$$

$$2a + 9d = \frac{125}{5}$$

$$2a + 9d = 25 \text{ ————— } \textcircled{02}$$

$d = -1$ in 1

$$a + 2(-1) = 15$$

$$a - 2 = 15$$

$$a = 17$$

Hence $a = 17, d = -1, a_{10} = 8$ and

$S_{10} = 125$

(v) Given $d = 5, S_9 = 75$ find a and a_9

Solution:

Here $d = 5, S_9 = 75$

$$\frac{n}{2}[2a + (n - 1)d] = 75$$

$$\frac{9}{2}[2a + (n - 1)5] = 75$$

$$2a + 8(5) = \frac{75 \times 2}{9}$$

$$2a + 40 = \frac{150}{9}$$

$$2a = \frac{150}{9} - 40$$

$$2a = \frac{150 - 360}{9}$$

$$a = \frac{-210}{9 \times 2}$$

$$a = \frac{-70}{3 \times 2} = \frac{-70}{6} \quad \text{or} \quad = \frac{-35}{3}$$

We know that sum of first n terms is

Now, $a_9 = a + 8d$

$$= \frac{-35}{3} + 8(5) = \frac{-35}{3} + 40$$

$$= \frac{120 - 35}{3} = \frac{85}{3}$$

$$a_9 = \frac{85}{3}$$

Hence $a = \frac{-35}{3}, a_9 = \frac{85}{3}$

(vi) Given $a = 2, d = 8, S_n = 90$ find n and a_n

Solution:

Given $a = 2, d = 8, S_n = 90$

$$\frac{n}{2}[2a + (n - 1)d] = 90$$

$$n[2(2) + (n - 1)8] =$$

90×2

$$n[4 + 8n - 8] = 180$$

$$n[8n - 4] = 180$$

$$8n^2 - 4n - 180 = 0$$

$$(\div 2) \quad 4n^2 - 2n - 90 = 0$$

$$(n - 5)(2n + 9) = 0$$

$$n - 5 = 0 \quad 2n + 9 = 0$$

$$n = 5 \quad 2n = -9$$

$$n = -\frac{9}{2}$$

Therefore $n = 5$ $n = -\frac{9}{2}$ is not possible.

Now, $a_n = a_5 = a + 4d$

$$= 2 + 4(8) = 2 + 32$$

$$a_5 = 34$$

Hence $n = 5, a_5 = 34$

(vii) Given $a = 8, a_n = 62, S_n = 120$ find n and d .

Solution:

Given $a = 8, a_n = 62, S_n = 120$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[a + l] = 120$$

$$\frac{n}{2}[8 + 62] = 120$$

$$\frac{n}{2}[70] = 120$$

$$35n = 120$$

$$n = \frac{120}{35} = 6$$

Therefore $n = 6$

Now, $a_n = 62$, Here $a = 8, n = 6$

$$a + (n - 1)d = 62$$

$$8 + 5d = 62$$

$$5d = 62 - 8 = 54$$

$$d = \frac{54}{5}$$

Hence $n = 6, d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$ find n and 'a'.

Solution:

Given $a_n = 4, d = 2, S_n = -14$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[a + l] = -14$$

$$\frac{n}{2}[a + 4] = -14$$

$$n[a + 4] = -28 \quad \text{--- (01)}$$

Now $a_n = 4$

$$a + (n - 1)d = 4$$

$$a + (n - 1)2 = 4$$

$$a = 4 - (n - 1)2 \quad \text{--- (02)}$$

substitute 2 in 1

$$n[4 - (n - 1)2 + 4] = -28$$

$$n[8 - 2n + 2] = -28$$

$$n[10 - 2n] = -28$$

$$10n - 2n^2 + 28 = 0$$

$$2n^2 - 10n - 28 = 0$$

$$(\div 2) \quad n^2 - 5n - 14 = 0$$

$$(n + 2)(n - 7) = 0$$

$$\Rightarrow n + 2 = 0 \quad \text{or} \quad n - 7 = 0$$

$$n = -2 \quad \text{or} \quad n = 7$$

But $n = -2$ is not possible

Therefore $n = 7$

Now $a_7 = 4$

$$a + 6d = 4$$

$$a + 6(2) = 4$$

$$a + 12 = 4$$

$$a = -8$$

Hence $a = -8$ and $n = 7$

(ix) Given $a = 3, n = 8, S = 192$ find 'd'.

Solution:

Given $a = 3, n = 8, S = 192$

We know that sum of first n terms is

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\text{So,} \quad \frac{8}{2}[2(3) + 7d] = 192$$

$$4[6 + 7d] = 192$$

$$6 + 7d = \frac{192}{4} = 48$$

Therefore $7d = 48 - 6 = 42$

$$d = \frac{42}{7} = 6$$

Therefore $d = 6$

(x) Given $l = 28, S = 144$ and there are total 9 terms. Find 'a'.

Solution:

Here $l = 28, s = 144, n = 9$

$$S_9 = 144$$

$$\frac{9}{2}[a + l] = 144$$

$$9[a + 28] = 144 \times 2$$

$$a + 28 = \frac{144 \times 2}{9}$$

$$a + 28 = 32$$

$$a = 32 - 28$$

$$a = 4$$

4) How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

Solution:

Given AP is 9, 17, 25,

Here $a = 9, d = 17 - 9 = 8$

Let $S_n = 636$

$$\frac{n}{2}[2a + (n - 1)d] = 636$$

$$\frac{n}{2}[2(9) + (n - 1)8] = 636$$

$$n[18 + 8n - 8] = 636 \times 2$$

$$n[10 + 8n] = 636 \times 2$$

$$10n + 8n^2 - 1272 = 0$$

$$(\div 2) \quad 4n^2 + 5n - 636 = 0$$

Here

$$a = 4, b = 5, c = -636$$

Now,

$$\begin{aligned} n &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4(4)(-636)}}{2(4)} \\ &= \frac{-5 \pm \sqrt{25 + 10176}}{8} = \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8} \\ &= \frac{96}{8} = 12 \end{aligned}$$

12 terms must be taken so that their sum is 636.

5) The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

Given $a = 5$, last term $a_n = 45$

$$a + (n - 1)d = 45$$

$$5 + (n - 1)d = 45$$

$$(n - 1)d = 45 - 5$$

$$(n - 1)d = 40 \quad \text{--- (01)}$$

Also, given $S_n = 400$

$$\frac{n}{2}[2a + (n - 1)d] = 400 \quad \text{--- (02)}$$

substitute 1 in 2

$$\frac{n}{2}[2(5) + 40] = 400$$

$$n[10 + 40] = 400 \times 2$$

$$50n = 400 \times 2$$

$$n = \frac{400 \times 2}{50} = 16$$

substitute $n = 16$ in 1

$$(16 - 1)d = 40 \Rightarrow 15d = 40$$

$$d = \frac{40}{15} = \frac{8}{3}$$

Hence number of terms $n = 16$ and common difference $d = \frac{8}{3}$

6) The first and last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Given that $a = 17$ and $a_n = 350, d = 9$

Now $a_n = 350$

$$a + (n - 1)d = 350$$

$$17 + (n - 1)9 = 350$$

$$(n - 1)9 = 350 - 17$$

$$(n - 1)9 = 333$$

$$n - 1 = \frac{333}{9} = 37$$

$$n = 38$$

Now

$$S_{38} = \frac{n}{2} [a + a_n]$$

$$= \frac{38}{2} [17 + 350]$$

$$= 19[367] = 6973$$

Hence number of terms $n = 38$ and

$$S_{38} = 6973$$

7) Find the sum of first 22 terms of an AP whose $d = 7$ and 22nd term is 149.

Solution:

Given $d = 7$ and $a_{22} = 149$

$$a_{22} = 149$$

$$a + 21d = 149$$

$$a + 21(7) = 149$$

$$a + 147 = 149$$

$$a = 149 - 147$$

$$a = 2$$

$$\text{Now } S_{22} = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{22}{2} [2(2) + 21(7)]$$

$$= 11[4 + 147] = 11[151] = 1661$$

Hence sum of first 22nd terms is 1661.

8) Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution:

Given $a = 14$ and $a_3 = 18$

$$a + d = 14 \quad \text{--- (01)}$$

$$a + 2d = 18 \quad \text{--- (02)}$$

$$2 - 1 \Rightarrow$$

$$d = 4$$

substitute $d = 4$ in 1

$$a + 4 = 14$$

Therefore $a = 14 - 4 = 10$

$$\begin{aligned} \text{Now } S_{51} &= \frac{51}{2} [2a + 50d] \\ &= \frac{51}{2} [2(10) + 50(4)] \\ &= \frac{51}{2} [20 + 200] \\ &= \frac{51}{2} \times 220 \\ &= 110 \times 51 = 5610 \end{aligned}$$

Hence $S_{51} = 5610$

9) If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of n terms.

Solution:

$$S_7 = 49$$

$$\frac{n}{2} [2a + 6d] = 49$$

$$\frac{7}{2} [2a + 6d] = 49$$

$$2a + 6d = \frac{49 \times 2}{7}$$

$$2a + 6d = 14$$

$$(\div 2) \quad a + 3d = 7$$

Also given that $S_n = 289$

$$\frac{17}{2} [2a + 16d] = 289$$

$$2a + 16d = \frac{289 \times 2}{17}$$

$$2a + 16d = 34$$

$$(\div 2) \quad a + 8d = 17$$

$$a + 8d = 17 \quad \text{--- (01)}$$

$$a + 3d = 7 \quad \text{--- (02)}$$

$$2 - 1 \Rightarrow 5d = 10$$

$$d = \frac{10}{5} = 2$$

Therefore substitute $d = 2$ in 1

$$a + 3(2) = 7$$

$$a + 6 = 7$$

$$a = 7 - 6 = 1$$

$$a = 1$$

$$\text{Now } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 + (n - 1)2]$$

$$= \frac{n}{2}[2 + 2n - 2]$$

$$= \frac{n}{2} \times 2n$$

$$= n^2$$

10) Show that a_1, a_2, \dots, a_n form an AP where a_n is defined as below.

(i) $a_n = 3 + 4n$, find sum of 15 terms.

Solution:

$$\text{Given } a_n = 3 + 4n$$

$$\text{Therefore } a = a_1 = 3 + 4(1) = 3 + 4 = 7$$

$$a_2 = 3 + 4(2) = 3 + 8 = 11$$

$$\text{Therefore } d = a_2 - a_1 = 11 - 7 = 4$$

$$S_{15} = \frac{15}{2}[2a + 14d]$$

$$S_{15} = \frac{15}{2}[2(7) + 14(4)] = \frac{15}{2}[14 + 56]$$

$$S_{15} = \frac{15}{2}[70] = 15 \times 35 = 525$$

$$\text{Hence } S_{15} = 525$$

(ii) Given $a_n = 9 - 5n$

Solution:

$$a = a_1 = 9 - 5(1) = 9 - 5 = 4$$

$$a_2 = 9 - 5(2) = 9 - 10 = -1$$

$$\text{Therefore } d = a_2 - a_1 = -1 - 4 = -5$$

$$S_{15} = \frac{15}{2}[2a + 14d] = \frac{15}{2}[2(4) + 14(-5)]$$

$$S_{15} = \frac{15}{2}[8 - 70] = \frac{15}{2}[-62] = -31 \times 15$$

$$\text{Hence } S_{15} = -465$$

11) If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (S_1)? What is sum of first two terms? What is the second term? Similarly find 3rd, the 10th and n^{th} terms.

Solution:

$$\text{Given } S_n = 4n - n^2$$

$$\text{First term } a = S_1 = 4(1) - 1^2$$

$$= 4 - 1 = 3$$

$$\text{Sum of first two terms } S_2 = 4(2) - 2^2$$

$$S_2 = 8 - 4 = 4$$

$$\text{Therefore 2nd term} = S_2 - S_1$$

$$a_2 = 4 - 3 = 1$$

$$\text{Therefore common difference } d = a_2 - a_1$$

$$d = 1 - 3 = -2$$

$$\text{Now } a_3 = a + 2d = 3 + 2(-2)$$

$$= 3 - 4 = -1$$

$$a_{10} = a + 9d$$

$$= 3 + 9(-2) = 3 - 18 = -15$$

$$n^{\text{th}} \text{ term } a_n = a + (n - 1)d = 3 + (n - 1)(-2)$$

$$= 3 - 2n + 2 = 5 - 2n$$

Hence, first term $a = 3$

sum of first 2 terms = 4

Second term = 1

3rd term = -1

10th term = -15

n^{th} term = $5 - 2n$

12) Find the sum of the first 40 positive integers divisible by 6.

Solution:

First 40 positive integers divisible by 6 are 6, 12, 18, 24, 240.

$$\text{Here } a = 6, d = 6, l = 240$$

$$\text{Sum of 40 integers} = S_n = \frac{40}{2}[a + l]$$

$$\text{Now } S_{40} = 20[6 + 240]$$

$$= 20[246]$$

$$= 4920$$

$$\text{Hence } S_{40} = 4920$$

13) Find the sum of first 15 multiples of 8.

Solution:

First 15 multiple of 8 are 8, 16, 24, 120

Here $a = 8, d = 8, l = 120$

$$\text{Therefore } S_n = \frac{15}{2} [a + l]$$

$$\begin{aligned} \text{Now } S_{15} &= \frac{15}{2} [8 + 120] = \frac{15}{2} [128] \\ &= 15 \times 64 = 960 \end{aligned}$$

Hence $S_{15} = 960$

14) Find the sum of odd numbers between 0 and 50.

Solution:

The odd numbers between 0 and 50 are 1, 3, 5, 7.... 49, which are in AP.

Here $a = 1, d = 3 - 1 = 2, l = 49$

$$l = 49$$

$$a + (n - 1)d = 49$$

$$1 + (n - 1)2 = 49$$

$$(n - 1)2 = 48$$

$$n - 1 = \frac{48}{2}$$

$$n - 1 = 24 \Rightarrow n = 25$$

Therefore sum of 25 terms = S_{25}

$$= 25[a + l] = \frac{25}{2} [1 + 49]$$

$$= \frac{25}{2} \times 50 = 25 \times 25$$

$$S_{25} = 625$$

15) A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for first day, ₹250 for 2nd day ₹300 for 3rd day etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Solution:

The penalty for each succeeding day is ₹50 for the first day, 2nd day, 3rd day etc will form an AP.

Here $a = 200, d = 50, n = 30$

penalty for 30 days = S_{30}

$$\text{Therefore } S_{30} = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{30}{2} [2(200) + 29(50)] = 15[400 + 1450]$$

$$= 15[1850] = ₹27750$$

Hence, the contractor has to pay ₹27750 as penalty.

16) A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize as ₹20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let the value of the first prize be ₹ x

Then, the value of the 2nd prize ₹ $x - 20$

the value of 2nd prize ₹ $(x - 20) - 20$

$$₹x - 40$$

and so on.

the list of the number form an AP whose first term $a = ₹x$ and the common difference

$$d = a_3 - a_2 = a - 40 - (a - 20)$$

$$= a - 40 - a + 20 = -20$$

Here ₹700 used for 7 cash prize.

$$S_7 = 700$$

$$\frac{n}{2} [2a + (n - 1)d] = 700$$

$$\frac{7}{2} [2x + 6(-20)] = 700$$

$$7[2x + (-120)] = 700 \times 2$$

$$2x - 120 = \frac{700 \times 2}{7} = 200$$

$$2x = 200 + 120 = 320$$

$$x = \frac{320}{2} = 160$$

Hence the 1st prize value = ₹160

2nd prize value = ₹160 - 20 = ₹140

3rd prize value = ₹140 - 20 = ₹120

4th prize value = ₹120 - 20 = ₹100

5th prize value = ₹100 - 20 = ₹80

6th prize value = ₹80 - 20 = ₹60

7th prize value = ₹60 - 20 = ₹40

17) In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and on till class XII. There are three sections of each class. How many trees will be planted by the student? (2014)

Solution:

Number of sections in each class = 3.

Number of trees that are planted }
by the students of class I } = $1 \times 3 = 3$

Number of trees that are planted }
by the students of class II } = $2 \times 3 = 6$

6

and so on.

Number of trees that are planted }
by the students of class XII } = $12 \times 3 = 36$

36

Thus the list of Numbers 3, 6, 9, 36 form an AP.

Here $a = 3, d = 6 - 3 = 3, l = 36$

$$a + (n - 1)d = 36$$

$$3 + (n - 1)3 = 36$$

$$(n - 1)3 = 36 - 3 = 33$$

$$n - 1 = \frac{33}{3} = 11$$

$$n = 12$$

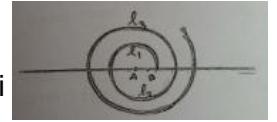
Hence the number of trees planted

$$\text{by the students} = S_{12} = \frac{n}{2}[a + l]$$

$$= \frac{12}{2}[3 + 36] = 6[39] = 234$$

18) A spiral is made up of successive semi-circles with centres alternately at A and B, starting with centre at A of radii 0.5 cm, 1.0 cm, 1.5 cm, 2 cm as shown in below figure. What is the total length of such spiral made up of thirteen consecutive semi circles? (take $\pi = \frac{22}{7}$)

Solution:



Length of successive semi circles are l_1, l_2, \dots, l_{13} with centres A, B

Therefore length of 1st semi circle = $\pi r = \pi(0.5)$

Length of 2nd semi circle = $\pi(1)$

Length of 3rd semi circle = $\pi(1.5)$ etc.

Therefore length of 13th semi circle = $\pi(6.5)$.

The list of numbers $0.5\pi, \pi, 1.5\pi, 2\pi, \dots, 6.5\pi$ form an AP.

Sum of 13 semi circles = $0.5\pi + \pi + 1.5\pi + \dots + 6.5\pi = 0.5\pi[1 + 2 + 3 + \dots + 13]$

$$= 0.5 \times \frac{22}{7} \times \frac{13}{2} [2a + 12d]$$

$$= 0.5 \times \frac{22}{7} \times \frac{13}{2} [2(1) + 12(1)]$$

$$= \frac{5}{10} \times \frac{22}{7} \times \frac{13}{2} \times 14 = 11 \times 3 = 143 \text{ cm.}$$

19) 200 logs are stacked in the following manner 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on.

(i) How many rows, are the 200 logs placed and how many logs are in the top row?

(ii) Which value is depicted in the pattern of log?

Solution:

Number of logs stacked in each row is 20, 19, 18, 17,

The list of numbers from an AP with $a = 20, d = 19 - 20 = -1$ and $S_n = 200$

$$S_n = 200$$

$$\frac{n}{2}[2a + (n - 1)d] = 200$$

$$n[2(20) + (n - 1)(-1)] = 200 \times 2$$

$$n[40 - n + 1] = 400$$

$$n[41 - n] = 400$$

$$41n - n^2 - 400 = 0$$

$$n^2 - 41n + 400 = 0$$

$$(n - 25)(n - 16) = 0$$

$$n - 25 = 0 \text{ or } n - 16 = 0$$

$$\text{When } n = 25, \quad a_n = a + (n - 1)d$$

$$= 20 + 24(-1)$$

$$= 20 - 24 = -4$$

The number of logs in the last row $a_n = -4$ is not possible.

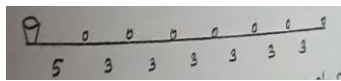
$$\text{Therefore } n = 16, a_n = a + (n - 1)d$$

$$= 20 + 15(-1)$$

$$= 20 - 15 = 5$$

Hence the number of rows is 16 and the number of log in the top row is 5.

- 20) In a potato race, a bucket is placed at the starting point, which is 5m from the first potato and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in a line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run.



Solution:

The distance run by the competitor to pick up the first potato = $2 \times 5 = 10$

The distance run by the competitor to pick up the 2nd potato = $2 \times (5 + 3) = 16$

For 3rd potato = $2 \times (5 + 3 + 3)$
 $= 2 \times 11 = 22$ etc

Here the list of number 10, 16, 22, 28,..... form an AP.

Here $a = 10, d = 16 - 10 = 6, n = 10$

Therefore $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{10} = \frac{10}{2}[2(10) + 9(6)]$$

$$= 5[20 + 54]$$

$$= 5[74] = 370$$

Hence the total distance the competitor has to run is 370 m.

06 TRIANGLES

Similar Figures:

Two geometrical figures are said to be similar, if they have same shape but not necessarily the same size.

Note:

All the congruent figures are similar but the similar figures need not be congruent.

Similar Polygons:

Two polygons of same number of sides are similar, if

- a) all the corresponding angles are equal.
- b) all the corresponding sides are in the same ratio (or proportion).

Similarity of Triangles:

Two triangles are said to be similar, if

- a) their corresponding angles are equal.
- b) their corresponding sides are proportional.

EXERCISE 6.1

1) Fill in the blanks using the correct word given in brackets.

(i) All circles are ----- (Congruent, similar)

Ans: similar

[Since all circles have same shape but size can vary]

(ii) All squares ----- (similar, congruent)

Ans: similar.

[since all square have same shape but size Can vary].

(iii) All ----- triangles are similar.

[isocetes, Equilateral]

Ans: Equilateral

[since all equilateral triangles have same shape but size can vary].

(iv) Two polygons of the same number of sides are similar, it

a) their corresponding angles are ----- and

b) their corresponding sides are ----- --- (equal, proportional)

Ans: a) their corresponding angles are equal.

b) their corresponding sides are proportional.

2) Give two different examples of pair of (i) similar figures (ii) non-similar figures.

(i) Similar figures:

- a) pair of equilateral triangles are similar figures.
- b) pair of squares are similar figures.

(ii) Non-Similar figures:

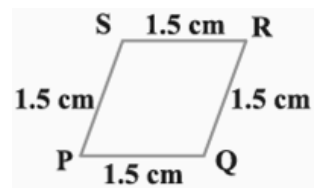
- a) A triangles and a quadrilateral form a pair of non-similar figures.
- b) A square and a circle form a pair of non similar figures.

State whether similar or not:

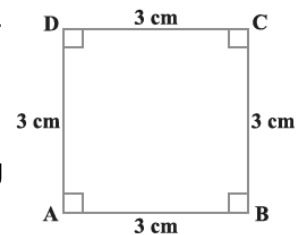
Solution:

$$\frac{PQ}{AB} = \frac{1.5}{3} = \frac{15}{30} = \frac{1}{2}$$

$$\frac{QR}{BC} = \frac{1.5}{3} = \frac{15}{30} = \frac{1}{2}$$



Clearly the corresponding sides of PQRS and ABCD are proportional.



But their corresponding angles are not equal.

Hence PQRS and ABCD are not similar.

Theorem 6.1: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio. (Thales Theorem).

Given:

In triangle ABC, a line DE is parallel to BC, intersect AB at D and AC at E.

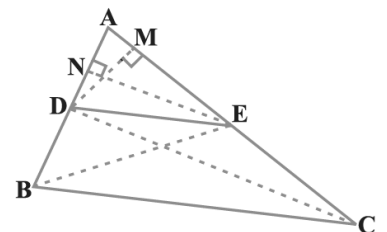
To prove:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction:

Join BE and CD.

Draw $EF \perp r$ AD and $DG \perp r$ AE.



Proof:

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} \times \text{Base} \times \text{Height}}{\frac{1}{2} \times \text{Base} \times \text{Height}}$$

$$= \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DE \times EF} = \frac{AD}{DB} \quad \text{--- (01)}$$

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DEC} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \quad \text{--- (02)}$$

But $\triangle BDE$ and $\triangle DEC$ have the same base DE and between the parallel lines DE and BC.

Therefore Area of BDE = Area of $\triangle DEC$ --- (03)

From 1, 2, 3 $\frac{AD}{DB} = \frac{AE}{EC}$

Theorem 6.2:

Converse of Basic proportionality

Theorem:- If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Given:

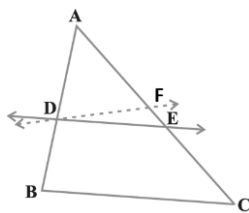
In $\triangle ABC$, a line DE intersect AB at D and AC at E

Such that $\frac{AD}{DB} = \frac{AE}{EC}$ --- (01)

To prove $DE \parallel BC$.

Proof:

Let us assume that DE is not parallel to BC.



Now, draw a line DF parallel to BC.

Therefore by BPT $\frac{AD}{DB} = \frac{AF}{FC}$

From 1 and 2 $\frac{AE}{EC} = \frac{AF}{FC}$

Add 1 to both side of equation 3

$$\frac{AE}{EC} = 1 = \frac{AF}{FC} + 1$$

$$\Rightarrow \frac{AE + EC}{EC} = \frac{AF + FC}{FC} \Rightarrow \frac{AC}{EC} = \frac{AC}{FC}$$

$$\Rightarrow EC = FC$$

$\Rightarrow E$ and F are coincides.

Hence DE is parallel to BC .

$DE \parallel BC$.

Solution:

Given:

In $\triangle ABC$, DE is parallel to BC.

Therefore by BPT $\frac{AD}{AB} = \frac{AE}{AC}$ --- (01)

To prove:

$$\frac{AD}{AB} = \frac{AE}{AC}$$

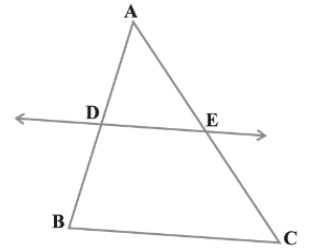
From 1, $\frac{DB}{AD} = \frac{EC}{AE}$ --- (02)

add 1 to both sides of equation 2.

$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

$$\Rightarrow \frac{DB + AD}{AD} = \frac{EC + AE}{AE}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$$



Example 2: ABCD is a trapezium with $AB \parallel DC$. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB.

Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

Given:

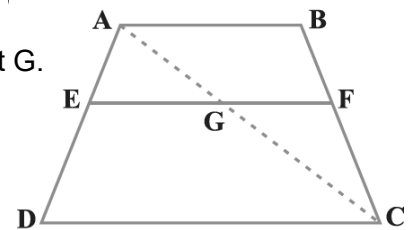
ABCD is a trapezium with $AB \parallel DC$. E and F are points on non-parallel sides such that $EF \parallel AB$.

Construction:

Joint AC meets EF at G.

To prove:

$$\frac{AE}{ED} = \frac{BF}{FC}$$



Solution:

Now $AB \parallel DC$ and $EF \parallel AB$

Therefore $DC \parallel EF$.

In $\triangle ADC$, $EG \parallel DC$

Therefore by BPT, $\frac{AE}{ED} = \frac{AG}{GC}$ --- (01)

In $\triangle ABC$, $FG \parallel AB$

Therefore by BPT, $\frac{CF}{FB} = \frac{CG}{GA} \Rightarrow \frac{AG}{GC} = \frac{BF}{FC}$ --- (02)

From 1 and 2

$\frac{AE}{ED} = \frac{BF}{FC}$ Hence the proof.

Example 1: If a line intersect sides AB and AC of $\triangle ABC$ at D and E respectively, and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$.

to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$.

Example 3: In the Figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$

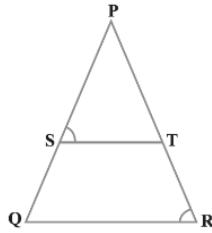
. Prove that $\triangle PQR$ is an isosceles triangle.

Solution:

In $\triangle PQR$, $\frac{PS}{SQ} = \frac{PT}{TR}$ and

$$\angle PST = \angle PRQ \quad \text{--- (01)}$$

Now, $\frac{PS}{SQ} = \frac{PT}{TR}$



Therefore by converse of BPT, $ST \parallel QR$

$$\Rightarrow \angle PST = \angle PQR \quad \text{--- (02)}$$

(corresponding angle)

From 1 and 2

$$\angle PQR = \angle PQR$$

$\Rightarrow PQ = PR$ (sides opposite to equal angles are equal)

$\Rightarrow \triangle PQR$ is an isosceles triangle.

EXERCISE 6.2

1) (i) In the figure $DE \parallel BC$, Find EC .

Solution:

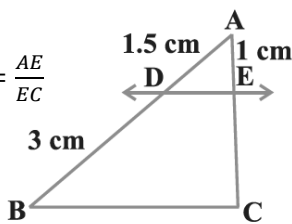
In $\triangle ABC$, $DE \parallel BC$

Therefore By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{1.5}{3} = \frac{1}{EC}$$

$$\frac{15}{30} = \frac{1}{EC} \Rightarrow \frac{1}{EC} = \frac{1}{2}$$

$$\Rightarrow EC = 2 \text{ cm}$$



(ii) In the figure $DE \parallel BC$, find AD .

Solution:

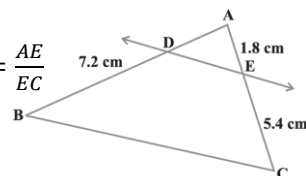
In $\triangle ABC$, $DE \parallel BC$

Therefore By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{1.8 \times 7.2}{5.4} \Rightarrow \frac{18}{54} \times 7.2$$

$$= 2.4 \text{ cm}$$



2) E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state $EF \parallel QR$.

(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

Solution:

In $\triangle PQR$, E and F are points on PQ and PR.

$$\text{Now, } \frac{PE}{EQ} = \frac{3.9}{3} = \frac{39}{30} = \frac{13}{10} = 1.3 \quad \text{--- (01)}$$

$$\text{and } \frac{PF}{FR} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2} \quad \text{--- (02)}$$

$$\text{From 1 and 2 } \frac{PE}{EQ} \neq \frac{PF}{FR}$$

$\Rightarrow EF$ is not parallel to QR

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $FR = 9$ cm

Solution:

In $\triangle PQR$, E and F are points on PQ and PR.

$$\text{Now, } \frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9} \quad \text{--- (01)}$$

$$\text{and } \frac{PF}{FR} = \frac{8}{9} \quad \text{--- (02)}$$

$$\text{From 1 and 2 } \frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore By converse of BPT, $EF \parallel QR$

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm

Solution:

In $\triangle PQR$, E and F are points on PQ and PR.

$$PQ = 1.28, PE = 0.18$$

$$\text{Therefore, } EQ = PQ - PE = 1.28 - 0.18 = 1.1$$

$$\text{Now, } \frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{18}{110} \quad \text{--- (01)}$$

$$PR = 2.56$$

$$PF = 0.36$$

$$\text{Therefore } FR = PR - PF = 2.56 - 0.36$$

$$= 2.20$$

$$\text{Therefore } \frac{PF}{FR} = \frac{0.36}{2.2} = \frac{36}{220} = \frac{18}{110} \quad \text{--- (02)}$$

$$\text{From 1 and 2 } \frac{PE}{EQ} = \frac{PF}{FR}$$

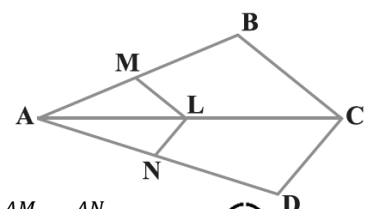
Therefore by CBPT, $EF \parallel QR$

3) In the figure $LM \parallel CB$ and $LN \parallel CD$. Prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$

Solution:

In $\triangle ABC$, $LM \parallel BC$



$$\text{Therefore BY BPT, } \frac{AM}{AB} = \frac{AN}{AD} \quad \text{--- (01)}$$

In $\triangle ACD$, $LN \parallel CD$

Therefore BY BPT, $\frac{AL}{LC} = \frac{AN}{ND}$ ——— (02)

From 1 and 2 $\frac{AM}{MB} = \frac{AN}{ND} \Rightarrow \frac{MB}{AM} = \frac{ND}{AN}$ ——— (03)

Add, 1 to both sides of equation 3

$$\frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$$

$$\Rightarrow \frac{MB+AM}{AM} = \frac{ND+AN}{AN}$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

Hence the proof.

4) In the figure $DE \parallel AC$ and $DF \parallel AE$. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

Solution:

In $\triangle ABC$, $DE \parallel AC$

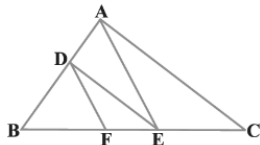
Therefore BY BPT, $\frac{BE}{EC} = \frac{BD}{DA}$ ——— (01)

Also, In $\triangle ABE$, $DF \parallel AE$

Therefore BY BPT, $\frac{BF}{FE} = \frac{BD}{DA}$ ——— (02)

From 1 and 2 $\frac{BE}{EC} = \frac{BF}{FE}$

Hence the proof.



5) In the figure $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.

Solution:

In $\triangle POQ$, $ED \parallel QO$

Therefore BY BPT, $\frac{PE}{EQ} = \frac{PD}{DO}$ ——— (01)

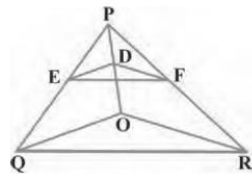
Also, In $\triangle POR$, $DF \parallel OR$

Therefore by BPT, $\frac{PD}{DO} = \frac{PF}{FR}$ ——— (02)

From 1 and 2 $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore by Converse of Basic Proportionality theorem (CBPT), $EF \parallel QR$

Hence the proof.

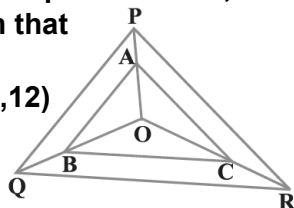


6) In the figure A, B and C are points on OP, OQ an OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$. (2010,12)

Solution:

In $\triangle OPQ$, $AB \parallel PQ$

Therefore by BPT, $\frac{OA}{AP} = \frac{OB}{BQ}$ ——— (01)



In $\triangle OPR$, $AC \parallel PR$

Therefore by BPT, $\frac{OC}{CR} = \frac{OA}{AP}$ ——— (02)

From 1 and 2 $\frac{OB}{BQ} = \frac{OC}{CR}$

Therefore by converse of BPT, $BC \parallel QR$

Hence the proof.

7) Using Thales theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Given: In $\triangle ABC$, D is mid point of AB. $DE \parallel BC$.

To prove: $AE = EC$.

Proof:

D is mid point of AB.

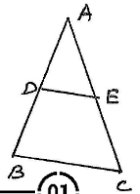
Therefore $AD = DB \Rightarrow \frac{AD}{DB} = 1$ ——— (01)

Also, $DE \parallel BC$,

Therefore By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$ ——— (02)

From 1 and 2 $\frac{AE}{EC} = 1 \Rightarrow AE = EC$

Hence the proof.



8) Using converse of Basic proportionality Theorem, prove that the line joining the mid points of any two sides of a triangle is parallel to the third side.

Solution:

Consider the triangle ABC,

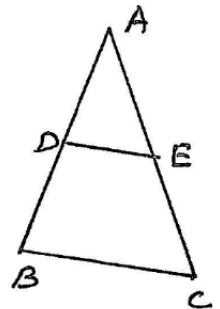
in which D and E are the mid point of AB and AC respectively.

Therefore $\frac{AD}{DB} = 1$ and $\frac{AE}{EC} = 1$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Therefore by converse of BPT, $DE \parallel BC$.

Hence the proof.



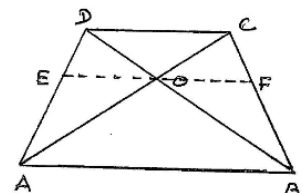
9) ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Solution:

Given:

ABCD is a trapezium in which $AB \parallel DC$.

Diagonals intersect at O.



To prove:

$$\frac{AO}{BO} = \frac{CO}{DO}$$

Construction:

Through O, Draw EF parallel to AB also $EF \perp$ to CD.

Proof:

In $\triangle ACD$, $EO \parallel CD$.

Therefore $\frac{AE}{ED} = \frac{AO}{OC}$ ————— (01)

Also, in $\triangle ABD$, $EO \parallel AB$.

Therefore By BPT, $\frac{AE}{ED} = \frac{BO}{OD}$ ————— (02)

From 1 and 2 $\frac{AO}{OC} = \frac{BO}{OD} \Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$

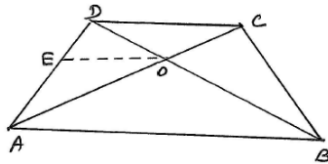
Hence proved.

10) The diagonals of a quadrilateral ABCD intersect each other at the point O, such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Solution:

Given:

ABCD is a Quadrilateral in which the diagonals intersect at O, such that



$$\frac{AO}{BO} = \frac{CO}{DO} \Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$$
 ————— (01)

To prove:

ABCD is a trapezium, to prove $AB \parallel CD$.

Construction:

Through O, draw $OE \parallel AB$.

Proof:

In $\triangle ABD$, $EO \parallel AB$

Therefore By BPT, $\frac{DE}{EA} = \frac{DO}{OB}$ or $\frac{AE}{ED} = \frac{BO}{OD}$ ————— (02)

From 1 and 2 $\frac{AE}{ED} = \frac{AO}{CO}$

Therefore By converse of BPT, $EO \parallel DC$.

Now $EO \parallel AB$ (by construction) and $EO \parallel DC$ (by proof)

Therefore $AB \parallel DC$

\Rightarrow ABCD is a trapezium.

AAA Similarity Criterion

Theorem:

If in two triangles, corresponding angles are equal, then their corresponding sides are in the

same ratio, they are proportional and hence the two triangles are similar.

SSS Similarity Criterion

Theorem:

If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

SAS Similarity Criterion

Theorem:

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Example 4: If the figure, if $PQ \parallel RS$, prove that $\triangle POQ \sim \triangle SOR$.

Solution:

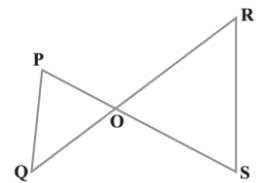
Given $PQ \parallel RS$

Therefore $\angle P = \angle S$ and

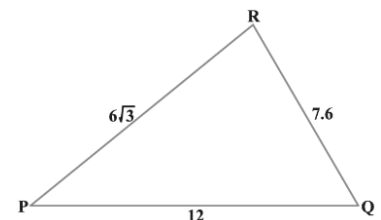
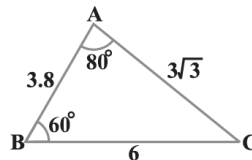
$\angle Q = \angle R$ (alternative angles)

Also, $\angle POQ = \angle SOR$ (vertically opposite angles)

Therefore By AAA criteria, $\triangle POQ \sim \triangle SOR$



Example 5: From the figure find $\angle P$.



Solution:

In $\triangle ABC$ and $\triangle PQR$

$$\frac{AB}{PQ} = \frac{3.8}{6\sqrt{3}} = \frac{1}{2}, \frac{BC}{QR} = \frac{6}{7.6} = \frac{1}{2}, \frac{AC}{PR} = \frac{3\sqrt{3}}{12} = \frac{1}{2}$$

Therefore $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \Rightarrow \triangle ABC \sim \triangle PQR$

\Rightarrow their corresponding angles are equal.

$\angle C = \angle P$

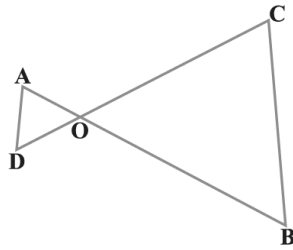
$$\begin{aligned} \angle C &= 180 - (\angle A + \angle B) \\ &= 180 - (80 + 60) \\ &= 180 - 140 = 40 \end{aligned}$$

$\angle P = 40$

Example 6: In the figure $OA \cdot OB = OC \cdot OD$.

Show that $\angle A = \angle C$ and $\angle B = \angle D$.

Solution:



$$OA \cdot OB = OC \cdot OD$$

$$\text{So } \frac{OA}{OC} = \frac{OD}{OB} \text{ ————— (01)}$$

$$\angle AOD = \angle BOC \text{ ————— (02)}$$

From 1 and 2

two sides are proportional and one angle is equal.

Therefore By SAS similarity

$$\triangle OAB \sim \triangle OCD$$

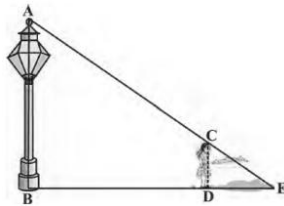
$$\Rightarrow \angle A = \angle C \text{ and } \angle B = \angle D$$

(corresponding angles of similar triangle)

Example 7: A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2m/s. If the lamp is 3.6m above the ground, find the length of her shadow after 4 seconds.

Solution:

Let AB be the lamp post, CD be the girl and D be the position of the girl after



4 sec.

Let $DE = x$ be the length of the shadow of the girl.

Given $CD = 90 \text{ cm} = 0.9 \text{ m}$, $AB = 3.6 \text{ m}$.

Speed of the girl = 1.2 m/sec.

$BD =$ Distance of the girl from the lamp-post after 4 sec

$$= \text{speed} \times \text{time}$$

$$= 1.2 \text{ m/sec} \times 4 \text{ sec} = 4.8 \text{ m}$$

In $\triangle ABE$ and $\triangle CDE$

$\angle B = \angle D$ and $\angle E$ is common.

Therefore $\triangle ABE \sim \triangle CDE$

Therefore their corresponding sides are proportional.

$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\frac{3.6}{0.9} = \frac{BD + DE}{DE}$$

$$\frac{36}{9} = \frac{4.8 + x}{x} \Rightarrow 4 = \frac{4.8 + x}{x}$$

$$\Rightarrow 4x = 4.8 + x$$

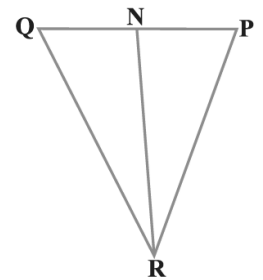
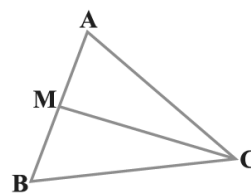
$$4x - x = 4.8$$

$$3x = 4.8$$

$$x = \frac{4.8}{3} = 1.6 \text{ m}$$

Hence length of her shadow after 4 sec 1.6 m.

Example 8: In the figure CM and RN are respectively the median of $\triangle ABC$ and $\triangle PQR$. If $\triangle ABC \sim \triangle PQR$. Prove that



(i) $\triangle AMC \sim \triangle PNR$

Solution:

Given $\triangle ABC \sim \triangle PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \text{ and ————— (01)}$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \text{ ————— (02)}$$

Also, medians bisect the opposite sides.

$$AM = BM \Rightarrow AB = 2AM$$

$$QN = NP \Rightarrow PQ = 2PN$$

$$\text{Therefore from 1 } \frac{AB}{PQ} = \frac{AC}{PR}$$

$$\text{(i.e.) } \frac{2AM}{2PN} = \frac{AC}{PR} \Rightarrow \frac{AM}{PN} = \frac{AC}{PR} \text{ ————— (03)}$$

In $\triangle AMC$ and $\triangle PNR$

$$\frac{AM}{PN} = \frac{AC}{PR} \text{ and } \angle A = \angle P$$

Therefore By SAS property, $\triangle AMC \sim \triangle PNR$

(ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

In $\triangle AMC$ and $\triangle PNR$, $\triangle AMC \sim \triangle PNR$

$$\frac{AM}{PN} = \frac{AC}{PR} = \frac{CM}{RN}$$

consider, $\frac{AC}{PR} = \frac{CM}{RN} \Rightarrow \frac{AB}{PQ} = \frac{CM}{RN}$ from 1

(iii) $\Delta CMB \sim \Delta RNQ$

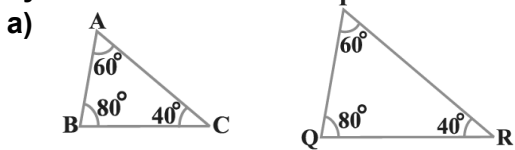
From 1 $\frac{AB}{PQ} = \frac{BC}{QR}$

But from (Result ii) $\frac{AB}{PQ} = \frac{CM}{RN}$

$\Rightarrow \frac{CM}{RN} = \frac{2BM}{2QN} = \frac{BM}{QN}$

Therefore $\frac{BC}{QR} = \frac{CM}{RN} = \frac{BM}{QN} \Rightarrow \Delta CMB \sim \Delta RNQ$

1) State which pairs of triangles in the figure are similar. Write the similarity criterion used by you for answering the question and also write the pair of similar triangles in the symbolic form:-



Solution:

In ΔABC and ΔPQR

$\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

By AAA Similarity criteria, $\Delta ABC \sim \Delta PQR$

b)

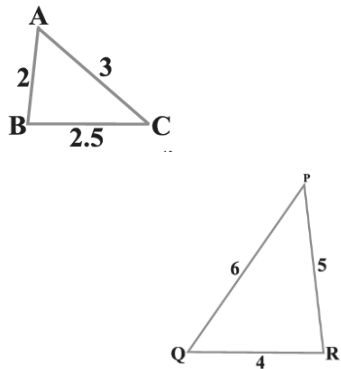
Solution:

In ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{2}{4} = \frac{1}{2}$

$\frac{AC}{PR} = \frac{3}{6} = \frac{1}{2}$

$\frac{BC}{QR} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$



Therefore By SSS Similarity criteria, $\Delta ABC \sim \Delta PQR$

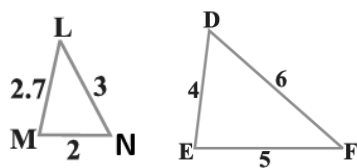
c)

Solution:

$\frac{MN}{DE} = \frac{2}{4} = \frac{1}{2}$

$\frac{LN}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{EF} = \frac{2.7}{5} = \frac{27}{50}$



Here $\frac{MN}{DE} = \frac{LN}{DF} = \frac{1}{2}$ But not equal to $\frac{LM}{EF}$

Therefore ΔLMN is not similar to ΔDEF

d)

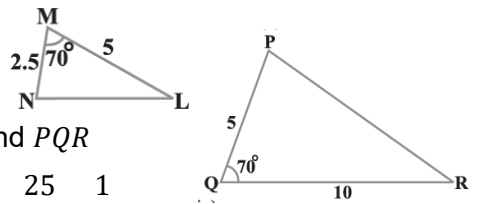
Solution:

In ΔLMN and ΔPQR

$\frac{MN}{PQ} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$

$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$ and $\angle M = \angle Q = 70^\circ$

Therefore By SAS criteria, $\Delta LMN \sim \Delta PQR$



e)

Solution:

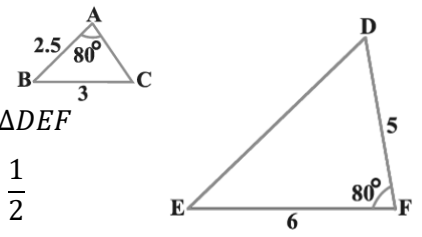
In ΔABC and ΔDEF

$\frac{AB}{DF} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$

$\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$ and $\angle A = \angle F = 80^\circ$

ΔABC is not similar to ΔDEF .

Since the sides included angle is not given.



f)

Solution:

In ΔDEF

$\angle F = 180 - (70 + 80)$
 $= 180 - 150 = 30^\circ$

In ΔPQR ,

$\angle P = 180 - (80 + 30)$
 $= 180 - 110 = 70^\circ$

In ΔDEF and ΔPQR

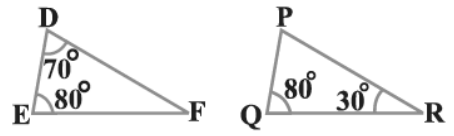
$\angle D = \angle P = 70^\circ$

$\angle E = \angle Q = 80^\circ$

$\angle F = \angle R = 30^\circ$

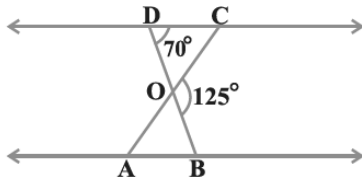
Therefore by AAA Similarity criteria,

$\Delta DEF \sim \Delta PQR$



2) In the figure $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle COD = 70^\circ$ Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.

Solution:



BD is a line

Therefore $\angle DOC + \angle BOC = 180^\circ$

$$\angle DOC + 125 = 180 \Rightarrow \angle DOC = 180 - 125$$

$$\angle DOC = 55^\circ$$

Therefore $\angle AOB = 55^\circ$ (vertically opposite angles are equal)

Given $\angle COD = 70^\circ$

Therefore $\angle ABO = 70^\circ$ (alternate angles are equal)

In $\triangle OCD$,

$$\angle ODC + \angle DCO + \angle COD = 180^\circ$$

$$70^\circ + \angle DCO + 55^\circ = 180^\circ$$

$$\angle DCO + 125 = 180$$

$$\text{Therefore } \angle DCO = 180 - 125 = 55^\circ$$

$$\text{Therefore } \angle DCO = 55^\circ$$

$$\text{Therefore } \angle OAB = 55^\circ$$

(alternate angles are equal)

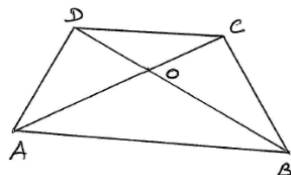
Hence $\angle DOC = 55^\circ$

$$\angle DCO = 55^\circ$$

$$\angle OAB = 55^\circ$$

3) Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$

Solution:



ABCD is a trapezium such that $AB \parallel DC$.

Diagonals AC and BD intersect at O.

consider the $\triangle OAB$ and $\triangle OCD$

$$\angle DOC = \angle AOB \text{ (vertically opposite angles)}$$

$$\angle CDO = \angle OBA \text{ (alternate angles)}$$

$$\angle DCO = \angle OAB \text{ (alternate angles)}$$

Therefore $\triangle OCD \sim \triangle OAB$

\Rightarrow their corresponding sides are proportional.

$$(i.e) \frac{OD}{OB} = \frac{OC}{OA} = \frac{CD}{AB}$$

Therefore $\frac{OB}{OD} = \frac{OA}{OC}$ (Reciprocal of 1st two terms)

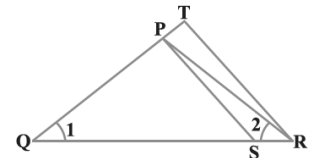
Hence proved.

4) In the figure $\frac{QR}{QS} = \frac{QT}{PR}$ (2010) and $\angle 1 = \angle 2$ show that $\triangle PQS \sim \triangle TQR$.

Solution:

$$\angle 1 = \angle 2 \text{ (given)}$$

$$\Rightarrow PQ = PR$$



(sides opposite to equal angles are equal)

$$\text{Given that } \frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ} \text{ (PR = PQ)}$$

$$\Rightarrow \frac{QS}{QR} = \frac{PQ}{QT} \text{ (reciprocal)}$$

In $\triangle PQS$ and $\triangle TQR$

$$\angle Q \text{ is common and } \frac{QS}{QR} = \frac{PQ}{QT}$$

Therefore $\triangle PQS \sim \triangle TQR$ (By SAS property)

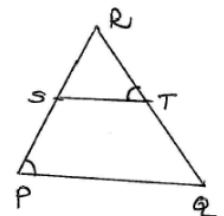
5) S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$ (2010)

Solution:

In $\triangle RPQ$ and $\triangle RTS$

$$\angle P = \angle T \text{ (Given)}$$

$\angle R$ is common



Therefore $\triangle RPQ \sim \triangle RTS$ (by AA similarity)

6) In the given figure, if $\triangle ABE \cong \triangle ACD$, Show that $\triangle ADE \sim \triangle ABC$ (2015).

Note:

[Two triangles are congruent. If their corresponding angles and sides are equal.

Two triangles are similar if their corresponding angles are equal and their corresponding sides are proportional]

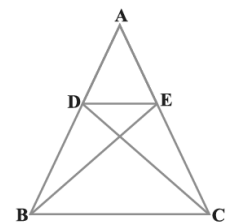
Solution:

$$\triangle ABE \cong \triangle ACD$$

$$\Rightarrow AB = AC \text{ and } AE = AD$$

$$\Rightarrow \frac{AB}{AC} = 1 \text{ and } \frac{AD}{AE} = 1$$

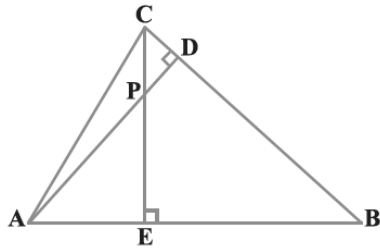
$$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE} \text{ and } \angle A \text{ is common.}$$



Therefore $\triangle ABC \sim \triangle ADE$ (by SAS similarity)

- 7) In the figure altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that
 (i) $\triangle AEP \sim \triangle CDP$

Solution:



Given AD, CE are altitudes of $\triangle ABC$ intersect at P.

In $\triangle AEP$ and $\triangle CDP$

$$\angle AEP = \angle CDP = 90^\circ$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

By AA similarity $\triangle AEP \sim \triangle CDP$

- (ii) $\triangle ABD \sim \triangle CBE$

Solution:

In $\triangle ABD$ and $\triangle CBE$

$$\angle ADB = \angle CEB = 90^\circ$$

$\angle B$ is common

Therefore By AA similarity $\triangle ABD \sim \triangle CBE$

- (iii) $\triangle AEP \sim \triangle ADB$

Solution:

In $\triangle AEP$ and $\triangle ADB$

$$\angle AEP = \angle ADB = 90^\circ$$

$\angle A$ is common

Therefore By AA similarity $\triangle AEP \sim \triangle ADB$

- (iv) $\triangle PDC \sim \triangle BEC$

Solution:

In $\triangle PDC$ and $\triangle BEC$

$$\angle PDC = \angle BEC = 90^\circ$$

$\angle C$ is common

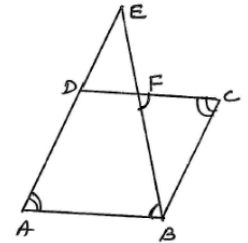
Therefore

$\triangle PDC \sim \triangle BEC$ (By AA similarity)

- 8) E is a point on the side AD produced of a parallelogram ABCD and BE intersect CD of F. Show that $\triangle ABE \sim \triangle CFB$.

Solution:

ABCD is a parallelogram,
 produce AD to E such that
 BE intersect DC at F.



In a parallelogram ABCD

$$\angle A = \angle C \text{ (opposite angles of a } ll^{gm} \text{ are equal).}$$

In $\triangle ABE$ and $\triangle CBF$

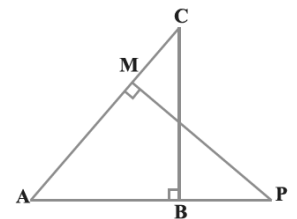
$$\angle CFB = \angle EBA \text{ (alternate angles)}$$

Therefore By AA similarity $\triangle ABE \sim \triangle CBF$

- 9) In the figure, ABC and AMP are two right angled triangles, right angled at B and M. Prove that

- (i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Solution:

- (i) In $\triangle ABC$ and $\triangle AMP$

$$\angle ABC = \angle AMP = 90^\circ$$

$\angle A$ is common

Therefore By AA similarity $\triangle ABC \sim \triangle AMP$

- (ii) $\triangle ABC \sim \triangle AMP$ (proved)

\Rightarrow Their corresponding sides proportional.

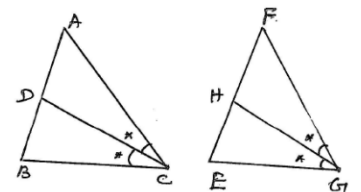
$$\frac{AC}{AP} = \frac{BC}{PM}$$

Hence proved

- 10) CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC \sim \triangle FEG$ respectively. If $\triangle ABC \sim \triangle FEG$ show that

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

Solution:



Given $\triangle ABC \sim \triangle FEG$.

Therefore all corresponding angles are equal.

$$\angle CAB = \angle GFE$$

$$\angle ACB = \angle EGF$$

$$\angle CAD = \angle GFH \text{ ————— (01)}$$

$$\frac{1}{2} \angle ACB = \frac{1}{2} \angle EGF \Rightarrow \angle ACD = \angle FGH \text{ ————— (02)}$$

From 1 and 2

$\Delta ACD \sim \Delta EGH$ (By AA similarity)

$$\Rightarrow \frac{AD}{FH} = \frac{AC}{FG} = \frac{CD}{GH} \Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

(ii) $\Delta DCB \sim \Delta HGE$

Solution:

In ΔABC and ΔFEG are similar.

\Rightarrow their corresponding angles are equal.

so, $\angle ABC = \angle FEG$ ——— (01)

$\angle ACB = \angle FGE$

$$\frac{1}{2} \angle ACB = \frac{1}{2} \angle FGE$$

$\Rightarrow \angle DCB = \angle HGE$ ——— (02)

From 1 and 2 $\Delta DCB \sim \Delta HGE$ (by AA similarity)

Hence proved.

(iii) $\Delta DCA \sim \Delta HGF$

Solution:

In ΔABC and $\Delta FEGG$ are similar (Given)

so, $\angle BAC = \angle EFG$ ——— (01)

and $\angle ACB = \angle FGE$

$$\frac{1}{2} \angle ACB = \frac{1}{2} \angle FGE$$

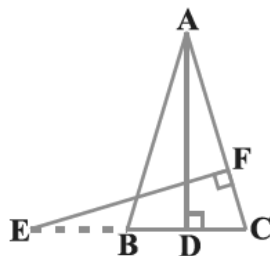
$\Rightarrow \angle ACD = \angle FGH$ ——— (02)

From 1 and 2

$\Delta DCA \sim \Delta HGF$ (by AA similarity)

11) In the figure, E is a point on side CB produced of an isosceles triangle ABC with AB=AC.

If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$ (2019)



Solution:

Given:

ΔABC is an isosceles triangle with $AB=AC$.

$\Rightarrow \angle B = \angle C$ ——— (01)

(angle opposite to equal sides are equal)

$AD \perp BC$ and $EF \perp AC$.

Therefore $\angle ADB = \angle EFC = 90^\circ$ ——— (02)

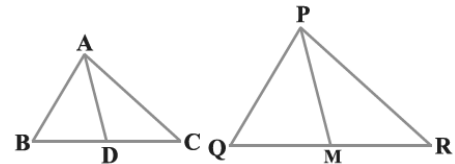
In ΔABD and ΔECF

$\angle B = \angle C$ and $\angle ADB = \angle EFC = 90^\circ$

Therefore By AA similarity, $\Delta ABD \sim \Delta ECF$

12) Sides AB and BC and median AD of a triangle ABC are respectively proportional to side PQ and QR and median Pm of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Solution:



Given:

ΔABC and ΔPQR in which AD and PM are their medians respectively, and also given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

Now $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (Given)

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \Rightarrow \Delta ADB \sim \Delta PQM$$

$\Rightarrow \angle B = \angle Q$ ——— (01)

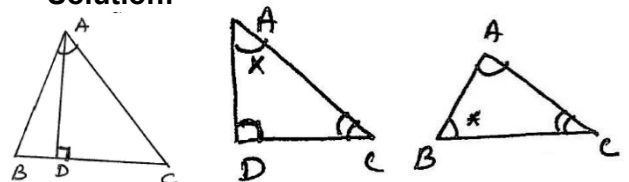
In $\Delta s ABC$ and PQR , $\frac{AB}{PQ} = \frac{BC}{QR}$ (Given) ——— (02)

From 1 and 2 $\Delta ABC \sim \Delta PQR$ (by SAS similarity)

Hence proved.

13) D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$ (2015)

Solution:



In ΔABC , D is a point on BC and Join AD.

In ΔABC and ΔADC

$\angle ADC = \angle BAC$

and $\angle C$ is common

By AA similarity $\Delta ABC \sim \Delta ADC$

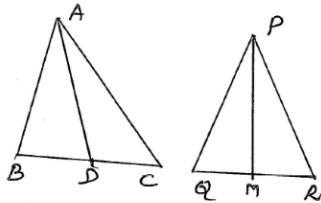
$$\Rightarrow \frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{DC}$$

consider $\frac{BC}{AC} = \frac{AC}{DC} \Rightarrow AC^2 = BC \cdot DC$

Hence proved.

14) Sides AB and AC and Median AD of a triangle ABC are respectively proportional to side PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Solution:



Given:

ΔABC and ΔPQR in which AD and PM are the medians, such that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \text{ ————— (01)}$$

To prove:

$$\Delta ABC \sim \Delta PQR$$

In ΔABC and ΔPQR $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ ————— (02)}$$

From 1 and 2

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \Rightarrow \Delta ABD \sim \Delta PQM$$

$$\Rightarrow \angle B = \angle Q \text{ ————— (03)}$$

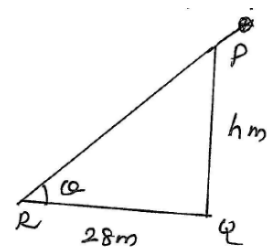
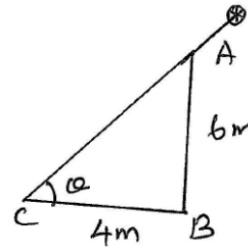
From 1 $\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle B = \angle Q$

Therefore By SAS similarity $\Delta ABC \sim \Delta PQR$

15) A verticle pole of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.

Solution:

Let AB=6 m be a pole cast a shadow BC = 4 m.



Now, let PQ = h m be cast a shadow

QR = 28 m.

In ΔABC and ΔPQR

$$\angle ABC = \angle PQR = 90^\circ$$

$\angle ACB = \angle PRQ = \theta$ [sun rays have same inclination]

Therefore $\Delta ABC \sim \Delta PQR$ (By AA similarity)

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

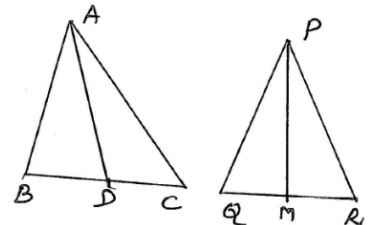
$$\frac{6}{h} = \frac{4}{28} \Rightarrow 4h = 28 \times 6$$

$$h = 7 \times 6 = 42m$$

Hence the height of the tower $h = 42m$.

16) If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Solution:



Given:

AD and PM are the medians of ΔABC and ΔPQR . Also, $\Delta ABC \sim \Delta PQR$

To prove:

$$\frac{AB}{PQ} = \frac{AD}{PM}$$

Proof:

In ΔABC and ΔPQR ,

$\Delta ABC \sim \Delta PQR$ (Given)

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} \text{ and } \angle B = \angle Q \text{ ————— (01)}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \text{ ————— (02)}$$

From 1 and 2

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ and } \angle B = \angle Q$$

$\Rightarrow \Delta ABD \sim \Delta PQM$ (By SAS similarity)

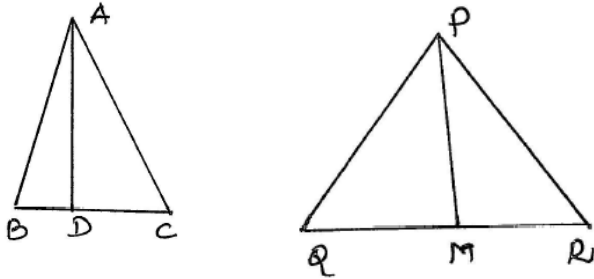
$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Therefore $\frac{AB}{PQ} = \frac{AD}{PM}$

Hence proved.

Area of Similar Triangle

Theorem: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. (2019, 2020).



Given: $\Delta ABC \sim \Delta PQR$

To prove:

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

Construction:

Draw altitudes AD and PM of ΔABC and ΔPQR

Proof: $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \angle A = \angle P$$

$$\angle B = \angle Q \quad \text{and}$$

$$\angle C = \angle R \quad \text{--- (01)}$$

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \quad \text{--- (02)}$$

In ΔABD and ΔPQM

$$\angle B = \angle Q \quad \text{and} \quad \angle D = \angle M = 90$$

Therefore By AA similarity $\Delta ABD \sim \Delta PQM$.

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM} \quad \text{--- (03)}$$

$$\text{Now, } \frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM}$$

$$= \frac{BC}{QR} \times \frac{AD}{PM} = \frac{AB}{PQ} \times \frac{AB}{PQ} \quad \text{[From Equation 2 \& 3]}$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Similarly we can prove that $\left(\frac{AC}{PR}\right)^2$ and $\left(\frac{BC}{QR}\right)^2$

$$\text{Therefore } \frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

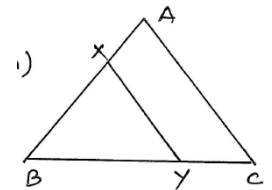
Hence proved.

Example 9: In the figure, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal area. Find the ratio $\frac{AX}{AB}$.

Solution:

XY is parallel to AC of

ΔABC ($XY \parallel AC$) (Given)



Also, XY divides the triangle into two parts of equal area.

$$\text{Area of } \Delta ABC = 2(\text{area of } \Delta XBY) \quad \text{--- (01)}$$

Now, In ΔABC and ΔXBY , $XY \parallel AC$

Therefore $\angle BAC = \angle BXY$ and

$\angle BYX = \angle BCA$, $\angle B$ is common.

$\Rightarrow \Delta ABC \sim \Delta XBY$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta XBY} = \left(\frac{AB}{XB}\right)^2$$

$$\text{From 1: } \frac{2(\text{area of } \Delta XBY)}{\text{area of } \Delta XBY} = \frac{(AB)^2}{(XB)^2}$$

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1} \Rightarrow \frac{AB}{XB} = \frac{\sqrt{2}}{1}$$

$$\text{Therefore } \frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(\sqrt{2} - 1)}{(\sqrt{2})^2} = \frac{2 - \sqrt{2}}{2}$$

$$\text{Therefore } \frac{AX}{AB} = \frac{2 - \sqrt{2}}{2}$$

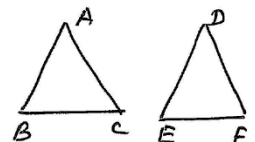
EXERCISE 6.4

1) Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64cm^2 and 121cm^2 .

If $EF = 15.4\text{ cm}$. Find BC .

Solution:

Given, $\Delta ABC \sim \Delta DEF$



$$\text{Therefore } \frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{64}{121} = \left(\frac{BC}{EF}\right)^2 \Rightarrow \left(\frac{BC}{EF}\right)^2 = \frac{8^2}{11^2}$$

$$\Rightarrow \frac{BC}{EF} = \frac{8}{11}$$

$$\frac{BC}{15.4} = \frac{8}{11}$$

$$\Rightarrow BC = \frac{8}{11} \times 15.4 = 11.2 \text{ cm}$$

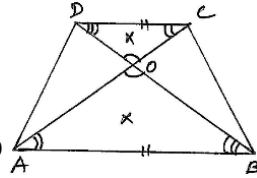
Therefore BC = 11.2 cm.

2) Diagonals of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O.

If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

Solution:

Given ABCD is a trapezium in which $AB \parallel DC$ and $AB = 2CD$



$$\Rightarrow \frac{AB}{CD} = \frac{2}{1} \text{ ————— (01)}$$

In $\triangle OAB$ and $\triangle OCD$

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle DCO = \angle OAB$ (alternate angles)

$\angle CDB = \angle DBA$ (alternate angles)

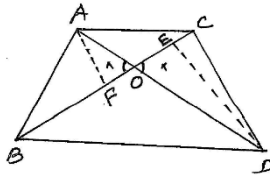
Therefore $\triangle OAB \sim \triangle OCD$

$$\text{Now, } \frac{\text{area of } \triangle AOB}{\text{area of } \triangle COD} = \left(\frac{AB}{CD}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

Hence ratio of area of $\triangle AOB$ and $\triangle COD$ is 4:1.

3) In the figure ABC and DBC are two triangles on the same base BC. If AD intersect BC at O, Show that $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DBC} = \frac{AO}{DO}$.

$$\text{that } \frac{\text{area of } \triangle ABC}{\text{area of } \triangle DBC} = \frac{AO}{DO}$$



Solution:

From the figure triangles ABC and DBC stands on the same base BC.

Construction:

Draw $AF \perp BC$ and $DE \perp BC$.

Proof: In $\triangle AOF$ and $\triangle EOD$

$\angle AOF = \angle EOD$ (vertically opposite)

$\angle AFO = \angle DEO = 90^\circ$

Therefore By AA similarity $\triangle AOF \sim \triangle DOE$.

$$\Rightarrow \frac{AF}{DF} = \frac{AO}{DO} \text{ ————— } 1$$

$$\text{Now, } \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DBC} = \frac{\frac{1}{2} \times BC \times AF}{\frac{1}{2} \times BC \times DE}$$

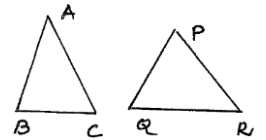
$$= \frac{AF}{DE} = \frac{AO}{DO} \text{ (From 1)}$$

Hence proved.

4) If the areas of two similar triangles are equal, prove that they are congruent, (2010)

Solution:

Let two triangles be $\triangle ABC$ and $\triangle PQR$.



Also, given that $\triangle ABC \sim \triangle PQR$ and area of $\triangle ABC = \text{area of } \triangle PQR$

$$\Rightarrow \frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = 1$$

$$\left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2 = 1$$

$$\Rightarrow \left(\frac{AB}{PQ}\right)^2 = 1$$

$$\Rightarrow \frac{AB}{PQ} = 1$$

$$\Rightarrow AB = PQ$$

Similarly $AC = PQ$ and $BC = QR$.

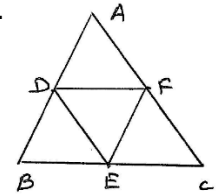
By SSS congruence rule $\triangle ABC \cong \triangle PQR$.

5) D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.

Solution:

In $\triangle ABC$, D, E, F are mid points of AB, BC, AC respectively.

Join D, E, F.



By mid point theorem, $DF = \frac{1}{2}BC$ and

$$DE = \frac{1}{2}AC, \quad EF = \frac{1}{2}AB.$$

$$\Rightarrow \frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB} = \frac{1}{2}$$

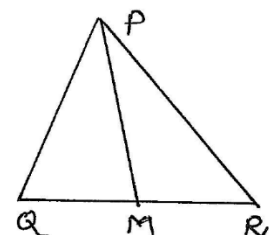
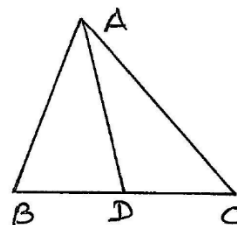
$\Rightarrow \triangle DEF \sim \triangle ABC$ (By SSS similarity)

$$\text{Now } \frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC} = \left(\frac{DE}{AC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Hence the required ratio is 1:4.

6) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:



Let AD be the median of $\triangle ABC$.

D is mid point of BC.

$$\text{Therefore } BC = 2BD \Rightarrow BD = \frac{1}{2}BC.$$

Also, PM is median of $\triangle PQR$.

M is mid point of QR $\Rightarrow QR = 2QM$.

Therefore $QM = \frac{1}{2}QR$.

$\Delta ABC \sim \Delta PQR$ (Given)

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}, \angle B = \angle Q. \text{---(01)}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \text{---(02)}$$

In ΔABD and ΔPQM , $\angle ABD = \angle PQM$

$$\text{and } \frac{AB}{PQ} = \frac{BD}{QM} \text{ (from 1 and 2)}$$

Therefore By SAS similarity $\Delta ABD \sim \Delta PQM$.

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \text{---(03)}$$

$$\begin{aligned} \text{Now, } \frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} &= \left(\frac{AB}{PQ}\right)^2 \\ &= \left(\frac{BD}{QM}\right)^2 \text{ (by 3)} \end{aligned}$$

Hence proved.

7) Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals. (2010, 2018).

Solution:

Let ABCD be a square of length 'a'.

Then the diagonal $BD = a\sqrt{2}$
(By Pythagoras Theorem)

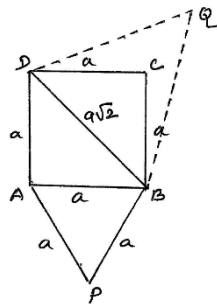
Now construct two equilateral triangles ABP and BDQ .

Therefore $\Delta ABP \sim \Delta BDQ$
(equilateral triangles are similar)

$$\begin{aligned} \Rightarrow \frac{\text{area of } \Delta APB}{\text{area of } \Delta BDQ} &= \frac{AB^2}{BD^2} = \frac{a^2}{(a\sqrt{2})^2} \\ &= \frac{a^2}{2a^2} = \frac{1}{2} \end{aligned}$$

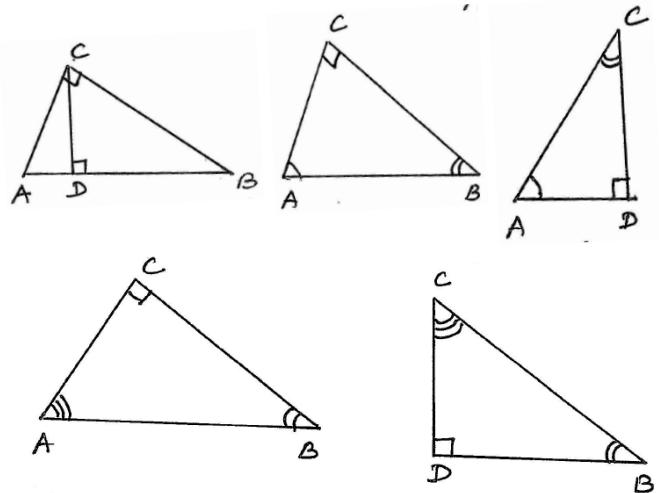
Therefore area of $APB = \frac{1}{2}$ area of BDQ

Hence proved.



Theorem 6.8

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (Pythagoras Theorem)



Given:

ΔABC is a right angled triangle at C and Draw $CD \perp r$ to AB (hypotenuse)

To prove: $AB^2 = AC^2 + BC^2$

Proof: ABC is a right angled triangle and $CD \perp r AB$.

In ΔABC and ADC ,

$\angle A$ is common and $\angle C = \angle D = 90^\circ$

Therefore By AA similarity

$\Delta ABC \sim \Delta ADC$

$$\Rightarrow \frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

$$\text{Now } \frac{AB}{AC} = \frac{AC}{AD} \Rightarrow AC^2 = AB \cdot AD \text{---(01)}$$

In ΔABC and ΔBCD

$\angle C = \angle D = 90^\circ$, $\angle B$ is common.

Therefore By AA similarity

$\Delta ABC \sim \Delta BCD$

$$\Rightarrow \frac{AB}{BC} = \frac{AC}{CD} = \frac{BC}{BD}$$

$$\text{Now } \frac{AB}{BC} = \frac{BC}{BD} \Rightarrow BC^2 = AB \cdot BD \text{---(02)}$$

Adding 1 and 2

$$\begin{aligned} AC^2 + BC^2 &= AB \cdot AD + AB \cdot BD \\ &= AB(AD + BD) \\ &= AB \cdot AB \\ &= AB^2 \end{aligned}$$

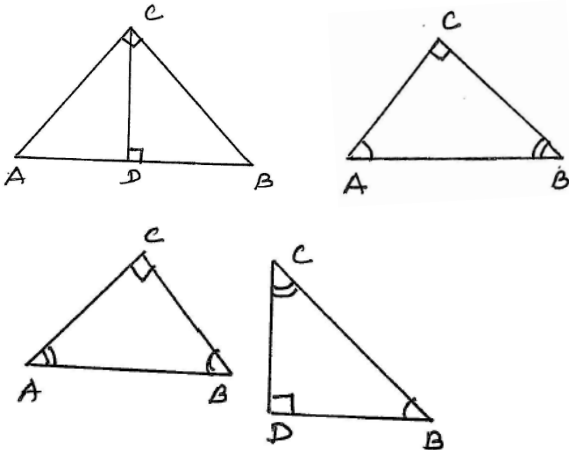
Therefore $AB^2 = AC^2 + BC^2$

Hence proved.

Theorem 6.7

If a perpendicular is drawn from the vertex of the right angled of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Example 10: In the figure $\angle ACB = 90^\circ$ and $CD \perp r AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.



Solution:

By Theorem, $\triangle ABC \sim \triangle ADC$

$$\Rightarrow \frac{AC}{AD} = \frac{AB}{AC}$$

$$\Rightarrow AC^2 = AB \cdot AD \text{ ————— (01)}$$

Similarly $\triangle ABC \sim \triangle BCD$

$$\Rightarrow \frac{BC}{BD} = \frac{AB}{BC}$$

$$\Rightarrow BC^2 = AB \cdot BD$$

From 1 and 2

$$\frac{BC^2}{AC^2} = \frac{AB \cdot BD}{AB \cdot AD} = \frac{BD}{AD}$$

Hence proved.

Example 11: A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.

Solution:

Let AB be the ladder and AC be the wall with window at A.

Also, BC = 2.5 m, AC = 6 m.

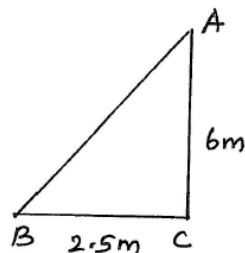
By Pythagoras Theorem,

$$AB^2 = AC^2 + BC^2$$

$$= 6^2 + (2.5)^2 = 36 + 6.25 = 42.25$$

$$\text{Therefore } AB = \sqrt{42.25} = 6.5$$

Therefore Length of the ladder AB = 6.5 m.



Example 12: In the figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$

Solution: In the figure $AD \perp r BC$.

Therefore ABD is a right angled triangle.

By Pythagoras Theorem,

$$AB^2 = AD^2 + BD^2 \text{ ————— (01)}$$

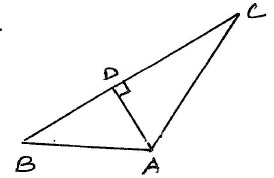
In right angled triangle ADC,

$$AC^2 = AD^2 + CD^2 \text{ ————— (02)}$$

$$\text{From 1 and 2 } AD^2 = AB^2 - BD^2 = AC^2 - CD^2$$

$$\Rightarrow AB^2 + CD^2 = AC^2 + BD^2$$

Hence proved.



Example 13:

BL and CM are medians of a triangle AE at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.

Solution:

BL and CM are medians of

the triangle ABC, at $\angle A = 90^\circ$

In $\triangle ABC$,

$$BC^2 = AB^2 + AC^2 \text{ ————— (01)}$$

(by Pythagoras Theorem)

In $\triangle ABL$, $BL^2 = AL^2 + AB^2$

$$= \left(\frac{1}{2}AC\right)^2 + AB^2$$

$$= \frac{AC^2}{4} + AB^2 \Rightarrow \frac{AC^2 + 4AB^2}{4}$$

$$\Rightarrow 4BL^2 = AC^2 + 4AB^2 \text{ ————— (02)}$$

In $\triangle AMC$, $CM^2 = AM^2 + AC^2$

$$= \left(\frac{1}{2}(AB)\right)^2 + AC^2$$

$$= \frac{AB^2}{4} + AC^2 \Rightarrow \frac{AB^2 + 4AC^2}{4}$$

$$\text{Therefore } \Rightarrow 4CM^2 = AB^2 + 4AC^2 \text{ ————— (03)}$$

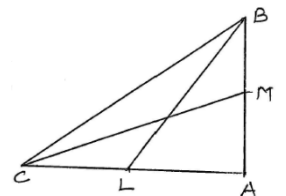
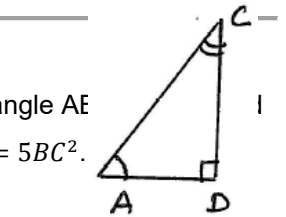
2 + 3 \Rightarrow

$$4BL^2 + 4CM^2 = AC^2 + 4AB^2 + AB^2 + 4AC^2$$

$$= 5AB^2 + 5AC^2$$

$$= 5(AB^2 + AC^2)$$

$$= 5BC^2 \text{ (from 1)}$$

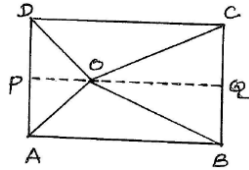


Example 14: O is any point inside a rectangle

ABCD. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.

Given: ABCD is rectangle, O is any point inside ABCD.

To prove: $OB^2 + OD^2 = OA^2 + OC^2$.



Construction: Through O Draw $PQ \parallel AB$.

So that P lies on AD and Q lies on BC.

Proof: $PQ \parallel AB$

$\Rightarrow PQ \perp r$ to AD and $PQ \perp r$ BC [Since ABCD is a rectangle].

Therefore ABQP and PQCD are rectangle.

Now, In $\triangle OPA$, $OA^2 = OP^2 + PA^2$ ——— (01)

In $\triangle OPD$, $OD^2 = OP^2 + PD^2$ ——— (02)

In $\triangle OBQ$, $OB^2 = OQ^2 + BQ^2$ ——— (03)

In $\triangle OCQ$, $OC^2 = OQ^2 + QC^2$ ——— (04)

2 + 3 \Rightarrow

$$\begin{aligned} OB^2 + OD^2 &= OQ^2 + BQ^2 + OP^2 + PD^2 \\ &= OQ^2 + PA^2 + OP^2 + QC^2 \\ &= OA^2 + OC^2 \quad [BQ = PA, PD = QC] \end{aligned}$$

Therefore $OB^2 + OD^2 = OA^2 + OC^2$

Hence proved.

1) Sides of triangle are given below. Determine which of them are right triangles. In case of right – triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

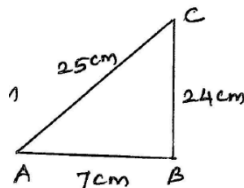
Let AB = 7 cm, BC = 24 cm, AC = 25 cm.

Now, $AB^2 = 49$

$BC^2 = 24^2 = 576$

$AC^2 = 25^2 = 625$

Now, $AB^2 + BC^2 = 49 + 576$
 $= 625$
 $= AC^2$



Therefore ABC is right triangle and its hypotenuse is 25 cm.

(ii) 3 cm, 8 cm, 6 cm

Let AB = 3 cm, BC = 8 cm, AC = 6 cm.

$AB^2 = 3^2 = 9$

$BC^2 = 8^2 = 64$

$AC^2 = 6^2 = 36$

Now, $AB^2 + AC^2 = 9 + 36 = 45$
 $\neq BC^2$

Therefore ABC is not a right triangle.

(iii) 50 cm, 80 cm, 100 cm

Let AB = 50 cm, BC = 80 cm, AC = 100 cm.

$AB^2 = 50^2 = 2500$

$BC^2 = 80^2 = 6400$

$AC^2 = 100^2 = 10000$

Now, $AB^2 + BC^2 = 2500 + 6400$
 $= 8900 \neq AC^2$

Therefore ABC is not right triangle.

(iv) 13 cm, 12 cm, 5 cm

Let AB = 13 cm, BC = 12 cm, AC = 5 cm.

Now, $AB^2 = 13^2 = 169$

$BC^2 = 12^2 = 144$

$AC^2 = 5^2 = 25$

$BC^2 + AC^2 = 144 + 25 = 169 = AB^2$

Therefore ABC is right triangle, its hypotenuse is 13 cm.

2) PQR is a right triangle at P and M is a point on QR such that $PM \perp r$ QR.

Show that $PM^2 = QM \cdot MR$. (2019)

Solution:

PQR is a triangle right angle at P.

M is a point on QR such that $PM \perp r$ QR.

In $\triangle PQR$ and $\triangle PMR$.

$\angle P = \angle M = 90^\circ$ and $\angle Q$ is common

Therefore $\triangle PQR \sim \triangle PMR$ ——— (01)

(By AA similarity)

In $\triangle PQR$ and $\triangle PMR$.

$\angle P = \angle PMR = 90^\circ$ and $\angle R$ is common

Therefore By AA similarity

$\triangle PQR \sim \triangle PMR$ ——— (02)

From 1 and 2 $\triangle PMQ \sim \triangle PMR$

From 1

$\frac{PQ}{QM} = \frac{PR}{PM} = \frac{QR}{PQ}$

$\Rightarrow \frac{PQ}{PR} = \frac{QM}{PM}$ ——— (03)

From 2

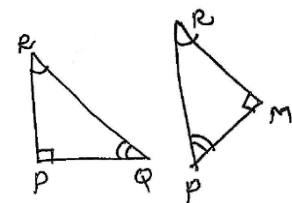
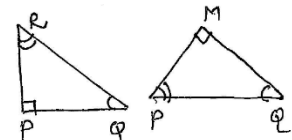
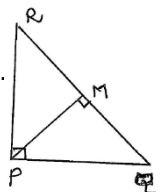
$\frac{PQ}{PM} = \frac{PR}{MR} = \frac{QR}{PR}$

$\Rightarrow \frac{PQ}{PR} = \frac{PM}{MR}$ ——— (04)

From 3 and 4 $\frac{QM}{PM} = \frac{PM}{MR}$

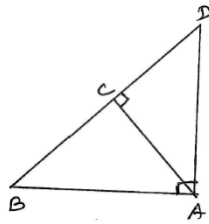
$\Rightarrow PM^2 = QM \cdot MR$

Hence proved.



3) In the figure ABD is a triangle right angled at A and $AC \perp r BD$. Show that

- (i) $AB^2 = BC \cdot BD$
- (ii) $AC^2 = BC \cdot DC$
- (iii) $AD^2 = BD \cdot CD$



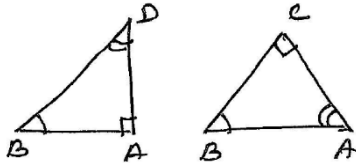
Solution:

(i) ABD is a triangle right angled at A and $AC \perp r BD$.

In $\triangle ABD$ and $\triangle ABC$

$\angle BAD = \angle BCA = 90^\circ$

and $\angle B$ is common



Therefore By AA similarity $\triangle ABD \sim \triangle ABC$

$\Rightarrow \frac{AB}{BC} = \frac{AD}{AC} = \frac{BD}{AB}$ ————— (01)

Consider $\frac{AB}{BC} = \frac{BD}{AB} \Rightarrow AB^2 = BC \cdot BD$

Hence proved.

(ii) From 1 and 2 $\triangle ABC$ and $\triangle ACD$ are similar.

Consider From 1 $\frac{AB}{AD} = \frac{BC}{AC}$ and

From 2 $\frac{AB}{AD} = \frac{AC}{CD}$

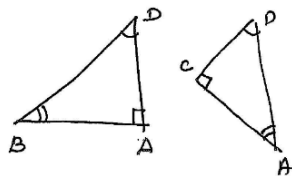
Therefore $\frac{BC}{AC} = \frac{AC}{CD} \Rightarrow AC^2 = BC \cdot CD$

Hence proved.

(iii) In $\triangle ABD$ and $\triangle ACD$

$\angle BAD = \angle ACD = 90^\circ$

$\angle D$ is common



Therefore $\triangle ABD \sim \triangle ACD$.

$\Rightarrow \frac{AB}{AC} = \frac{AD}{CD} = \frac{BD}{AD}$ ————— (02)

Consider $\frac{AD}{CD} = \frac{BD}{AD} \Rightarrow AD^2 = CD \cdot BD$

Hence proved.

4) ABC is an isosceles triangle right angled at C.

Prove that $AB^2 = 2AC^2$. (2014, 15)

Solution: ABC is an isosceles triangle right angled at C.

Therefore $AC = BC$.

Now ABC is right angled triangle.

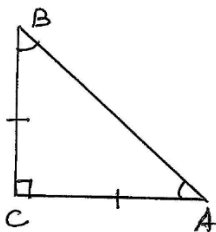
Therefore $AB^2 = AC^2 + BC^2$

But $BC = AC$ ($\triangle ABC$ is an Isosceles)

Therefore $AB^2 = AC^2 + AC^2$

$= 2AC^2$

Hence $AB^2 = 2AC^2$



5) ABC is an isosceles triangle with $AC=BC$. If

$AB^2 = 2AC^2$, Prove that ABC is a right triangle.

Solution: ABC is an isosceles triangle with $AC = BC$.

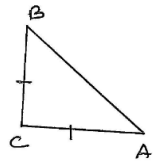
Given $AC = BC$ and $AB^2 = 2AC^2$

Now $AC^2 + BC^2 = AC^2 + AC^2$ [$BC = AC$]

$= 2AC^2$

$= AB^2$

\Rightarrow ABC is a right angled triangle (By converse of Pythagoras Theorem)



6) ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Solution: ABC is an equilateral triangle of side 2a.

$AD \perp r$ to BC of $\triangle ABC$.

Now In $\triangle s$ ABD and ACD

$AD = AD$ (common side)

$\angle ADB = \angle ADC = 90^\circ$ and $AB = AC$ (sides of an equilateral $\triangle ABC$).

Therefore $\triangle ABD \cong \triangle ACD$.

Then $BD = DC = \frac{1}{2}BC = a$.

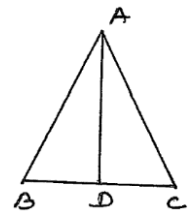
In $\triangle ADB$, $AB^2 = AD^2 + BD^2$

$(2a)^2 = AD^2 + a^2$

$4a^2 - a^2 = AD^2 \Rightarrow AD^2 = 3a^2$

Therefore altitude $AD = \sqrt{3a^2}$

$= a\sqrt{3}$



7) Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of the diagonals. (2015) (OR)

ABCD is a rhombus. Prove that $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$. (CBSE 2005)

Solution:

Given:

ABCD is a rhombus.

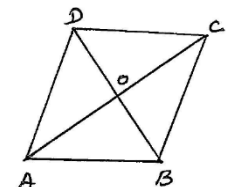
Therefore $AB = BC = CD = AD$

let the diagonals AC and BD intersect at O.

In a rhombus the diagonals bisect each other at right angles.

Therefore

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$ and



$$OA = OC, OB = OD.$$

In right angled triangle AOB .

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4} = \frac{AC^2 + BD^2}{4}$$

$$\Rightarrow AC^2 + BD^2 = 4AB^2$$

$$= AB^2 + AB^2 + AB^2 + AB^2$$

$$= AB^2 + BC^2 + CD^2 + AD^2$$

[Because $AB = BC = CD = AD$]

Hence proved.

8) In the figure, O is a point in the interior of a triangle ABC , $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

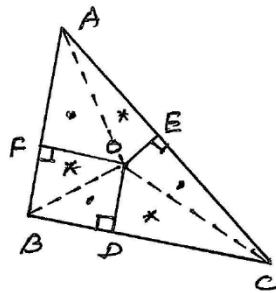
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Given:

O is a point in the interior of a triangle ABC and $OD \perp BC$,

$OE \perp AC$, $OF \perp AB$.



(i) In right angled $\triangle OAF, OBD, OCE$. (dot triangles)

$$OA^2 = OF^2 + AF^2$$

$$OB^2 = OD^2 + BD^2$$

$$OC^2 = OE^2 + CE^2$$

Adding all the results.

$$OA^2 + OB^2 + OC^2 = OF^2 + OD^2 + OE^2 + AF^2 + BD^2 + CE^2$$

Therefore,

$$OA^2 + OB^2 + OC^2 - OF^2 - OD^2 - OE^2 = AF^2 + BD^2 + CE^2 \quad \text{--- 1}$$

Hence the proof (i).

(ii) In right angled triangle OEA, OFB, ODC

(* triangles)

$$OA^2 = OE^2 + AE^2$$

$$OB^2 = OF^2 + BF^2$$

$$OC^2 = OD^2 + CD^2$$

Adding all the results.

$$OA^2 + OB^2 + OC^2 = OE^2 + OF^2 + OD^2 + AE^2 + BF^2 + CD^2$$

$$\Rightarrow OA^2 + OB^2 + OC^2 - OE^2 - OF^2 - OD^2 =$$

$$AE^2 + BF^2 + CD^2 \quad \text{--- (02)}$$

From 1 and 2

$$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$$

9) A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Solution: Let AC be the ladder

and BC be the wall.

Also $AC = 10$ m, $BC = 8$ m.

In right angled triangle ABC ,

$AC^2 = AB^2 + BC^2$ (By Pythagoras Theorem)

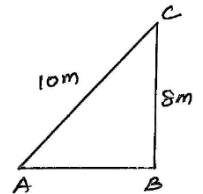
$$10^2 = AB^2 + 8^2$$

$$100 = AB^2 + 64 \Rightarrow AB^2 = 100 - 64$$

$$AB^2 = 36$$

$$AB = \sqrt{36} = 6 \text{ m.}$$

Hence, the distance between the foot of the ladder and base of wall $AB = 6$ m.



10) A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution: Let $AC = 25$ m be guy

wire attached to the vertical pole

BC of height 18 m. To keep the wire

taut, let it be fixed to stake at A .

Now ABC is a right angled triangle at C .

By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

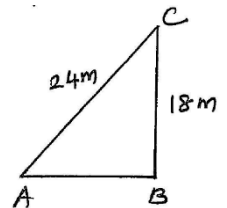
$$24^2 = AB^2 + 18^2$$

$$576 = AB^2 + 324$$

$$\text{Therefore } AB^2 = 576 - 324 = 252$$

$$\text{Therefore } AB = \sqrt{252} = 6\sqrt{7} \text{ m.}$$

The stake may be placed at a distance of $6\sqrt{7}$ m from the base of the pole.



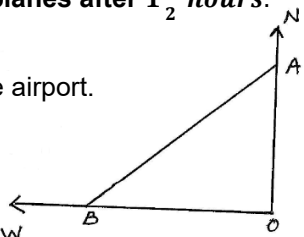
- 11) An aeroplane leaves an airport and flies due north at a speed of 1000 km/hr. At the same time, another plane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours.

Solution:

Let O be the position of the airport.

Let the first flight starts

from O and goes upto A

at a speed of 1000 km/hr. 

Time taken to reach the point A is $1\frac{1}{2}$ hours.

Therefore Distance $OA = \text{speed} \times \text{time}$

$$= 1000 \times \frac{3}{2} = 1500 \text{ km.}$$

Let the second flight starts from O goes upto B at a speed of 1200 km/hr.

Time taken to reach the point B is $1\frac{1}{2}$ hours.

Therefore Distance $OB = \text{speed} \times \text{time}$

$$= 1200 \times \frac{3}{2} = 1800 \text{ km.}$$

Distance between two planes after $1\frac{1}{2}$ hours is AB.

$$\begin{aligned} \text{Now } AB &= \sqrt{OA^2 + OB^2} = \sqrt{1500^2 + 1800^2} \\ &= \sqrt{2250000 + 3240000} = \sqrt{5490000} \\ &= \sqrt{9 \times 61 \times 100 \times 100} = 300\sqrt{16} \text{ km.} \end{aligned}$$

- 12) Two poles of height 6 m and 11 m stand on a plane ground. If the distance between the feet of the pole is 12 m, find the distance between their tops.

Solution:

Let AD and BC be the poles of height 6 m and 11 m respectively.

Let CD be the distance Between their tops.

Given $AD = 6 \text{ m.}$

$BC = 11 \text{ m, } AB = DE = 12 \text{ m}$

Now $BE = 6 \text{ m}$

[Because $AD = BE$]

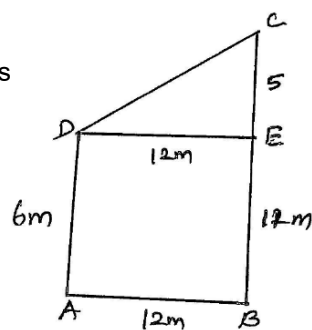
Therefore $EC = BC - BE = 11 - 6 = 5 \text{ m.}$

In right angled triangle DEC,

$$DC^2 = DE^2 + EC^2 = 12^2 + 5^2 = 144 + 25 = 169$$

Therefore $DC = \sqrt{169} = 13 \text{ m.}$

Hence the distance between their tops = 13m.



- 13) D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$ (2019).

Solution:

Consider a right angled triangle ACB, at $\angle C = 90^\circ$.

D and E are point on sides AC and BC respectively.

In right angled triangle AEC,

$$AE^2 = EC^2 + AC^2 \text{ ————— (01)}$$

[By Pythagoras Theorem]

In right angled triangle BCD,

$$BD^2 = BC^2 + CD^2 \text{ ————— (02)}$$

In right angled triangle ECD,

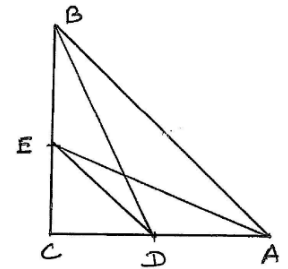
$$ED^2 = EC^2 + CD^2 \text{ ————— (03)}$$

$$\text{In } \triangle ACB, AB^2 = AC^2 + BC^2 \text{ ————— (04)}$$

Adding 1 + 2

$$\begin{aligned} \Rightarrow AE^2 + BD^2 &= EC^2 + AC^2 + BC^2 + CD^2 \\ &= (EC^2 + CD^2) + (AC^2 + BC^2) \\ &= ED^2 + AB^2 \end{aligned}$$

Hence the proof.



- 14) The perpendicular from A on side BC of a $\triangle ABC$ intersect BC at D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$. (2005, 2009, 2012, 2019)

Given:

In $\triangle ABC$, through A Draw $AD \perp r$ to BC.

Also, $DB = 3CD$

Now $BC = BD + DC$

$$= 3CD + DC$$

$$= 4CD$$

Therefore $CD = \frac{1}{4}BC$

From Equation 1, $DB = 3 \left[\frac{1}{4}BC \right] = \frac{3}{4}BC$

In right angled triangle ABD,

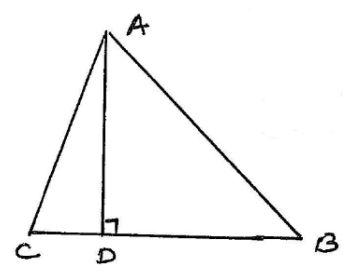
$$AB^2 = AD^2 + DB^2 \text{ ————— (01)}$$

In right angled triangle ACD,

$$AC^2 = AD^2 + DC^2 \text{ ————— (02)}$$

Equation 1 – 2,

$$\begin{aligned} \Rightarrow AB^2 - AC^2 &= AD^2 + DB^2 - (AD^2 + DC^2) \\ &= AD^2 + DB^2 - AD^2 - DC^2 \\ &= DB^2 - DC^2 \text{ (Since } DB = \frac{3}{4}BC \text{ and } CD = \frac{1}{4}BC) \\ &= \left(\frac{3}{4}BC \right)^2 - \left(\frac{1}{4}BC \right)^2 \end{aligned}$$



$$\begin{aligned}
 &= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 \\
 &= \left(\frac{9}{16} - \frac{1}{16}\right)BC^2 = \frac{8}{16}BC^2 \\
 &= \frac{1}{2}BC^2
 \end{aligned}$$

Therefore $2[AB^2 - AC^2] = BC^2$

$$2AB^2 - 2AC^2 = BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Hence proved.

15) In an equilateral triangle ABC , D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. (2014, 2018)

Given:

In an equilateral triangle ABC , D is a point on BC such that $BD = \frac{1}{3}BC$.

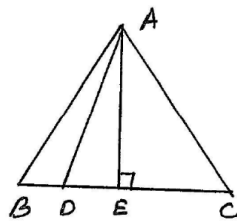
Draw $AE \perp r$ to BC .

Therefore $BE = EC = \frac{1}{2}BC$

Now $DE = BE - BD$

$$= \frac{1}{2}BC - \frac{1}{3}BC$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right)BC \Rightarrow \left(\frac{3-2}{6}\right)BC = \frac{1}{6}BC.$$



In right angled triangle ABE ,

$$AB^2 = AE^2 + BE^2 \quad \text{--- (01)}$$

In right angled triangle ADE ,

$$AD^2 = AE^2 + DE^2 \quad \text{--- (02)}$$

$1 - 2 \Rightarrow$

$$AB^2 - AD^2 = (AE^2 + BE^2) - (AE^2 + DE^2)$$

$$= AE^2 + BE^2 - AE^2 - DE^2$$

$$= BE^2 - DE^2$$

$$= \left(\frac{1}{2}BC\right)^2 - \left(\frac{1}{6}BC\right)^2$$

$$= \frac{BC^2}{4} - \frac{BC^2}{36} = \frac{9BC^2 - BC^2}{36}$$

$$= \frac{8BC^2}{36} = \frac{2BC^2}{9}$$

Therefore $9(AB^2 - AD^2) = 2BC^2$

$$9AB^2 - 9AD^2 = 2AB^2$$

(Because $AB = BC = CA$)

$$9AB^2 - 2AB^2 = 9AD^2$$

$$7AB^2 = 9AD^2$$

Hence proved.

16) In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Solution:

Let ABC be an equilateral triangle.

Draw $AD \perp r$ BC and AD

is altitude of ΔABC .

Therefore $BD = DC = \frac{1}{2}BC$

In right angled triangle ABD ,

$$AB^2 = AD^2 + BD^2$$

$$= AD^2 + \left(\frac{1}{2}BC\right)^2$$

$$= AD^2 + \frac{BC^2}{4}$$

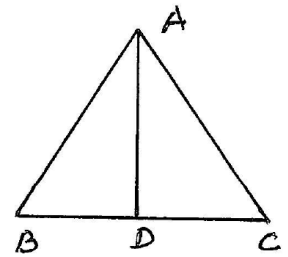
$$= AD^2 + \frac{AB^2}{4} \quad [\text{because } AB = BC = AC]$$

$$AB^2 - \frac{AB^2}{4} = AD^2$$

$$\frac{4AB^2 - AB^2}{4} = AD^2 \Rightarrow \frac{3AB^2}{4} = AD^2$$

Therefore $3AB^2 = 4AD^2$

Hence proved.



17) Tick the correct answer and justify:- In

ΔABC , $AB = 6\sqrt{3}$ cm, $AC = 12$ cm, $BC = 6$ cm. The angle B is

Solution:

$$AB^2 + BC^2 = (6\sqrt{3})^2 + 6^2$$

$$= 36 \times 3 + 36$$

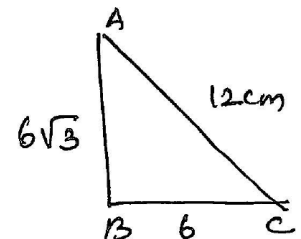
$$= 108 + 36$$

$$= 144$$

$$= 12^2$$

$$= AC^2 \Rightarrow \angle B = 90^\circ$$

(By converse of Pythagoras Theorem)



07 COORDINATE GEOMETRY

Axes of co-ordinates:-

In the figure Ox and OY are called as x - axis and y - axis respectively and both together are known as axes of coordinates.

Origin:

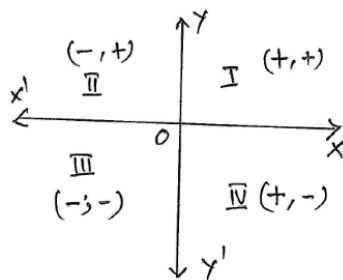
Origin is the point of intersection of the axes of co-ordinates.

Abscissa:

The distance of the point P from y - axis is called its abscissa. That is x - coordinate is called abscissa.

Ordinate:

The distance of the point P from x - axis is called its ordinate. y - coordinate is called ordinate.



Distance between any two

Points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{(OR)}$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example 1: Do the points (3,2), (-2,-3), (2,3) form a triangle? If so, name the type of triangle.

Let the points are A(3,2), B(-2,-3) and C(2,3)

$$AB = \sqrt{(-2 - 3)^2 + (-3 - 2)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2} = \sqrt{25 + 25}$$

$$= \sqrt{50} = 7.07 \text{ (app)}$$

$$BC = \sqrt{(2 - (-2))^2 + (3 + 3)^2}$$

$$= \sqrt{4^2 + 6^2} = \sqrt{16 + 36}$$

$$= \sqrt{52} = 7.21 \text{ (app)}$$

$$AC = \sqrt{(2 - 3)^2 + (3 - 2)^2}$$

$$= \sqrt{1 + 1} = \sqrt{2} = 1.414 \text{ (app)}$$

Given points A, B and C will form a triangle. Since sum of any two sides is greater than the third side.

$$\text{Now } AB^2 + AC^2 = (\sqrt{50})^2 + (\sqrt{2})^2$$

$$= 50 + 2 = 52$$

$$= BC^2$$

By prthagores Theorem, $\angle A = 90^\circ$

Hence $\triangle ABC$ is a right angled triangle.

Example 2: Show that the points (1,7), (4,2) (-1,-1) and (-4,4) are the vertices of a square.

Solution:

Let the points be A(1,7), B(4,2), C(-1,-1) and D(-4,4)
To prove given points form a square.

That is to prove all the sides are equal and diagonals also be equal.

$$\text{Now } AB = \sqrt{(4 - 1)^2 + (2 - 7)^2} = \sqrt{3^2 + (-5)^2}$$

$$= \sqrt{9 + 25} = \sqrt{34}$$

$$BC = \sqrt{(-1 - 4)^2 + (-1 - 2)^2} = \sqrt{5^2 + 3^2}$$

$$= \sqrt{25 + 9} = \sqrt{34}$$

$$CD = \sqrt{(-4 + 1)^2 + (4 + 1)^2} = \sqrt{(-3)^2 + 5^2}$$

$$= \sqrt{9 + 25} = \sqrt{34}$$

$$AD = \sqrt{(-4 - 1)^2 + (4 - 7)^2} = \sqrt{(-5)^2 + (-3)^2}$$

$$= \sqrt{25 + 9} = \sqrt{34}$$

$$\text{Also, } AC = \sqrt{(-1 - 1)^2 + (-1 - 7)^2} = \sqrt{(-2)^2 + (-8)^2}$$

$$= \sqrt{4 + 64} = \sqrt{68}$$

$$BD = \sqrt{(-4 - 4)^2 + (4 - 2)^2} = \sqrt{(-8)^2 + (2)^2}$$

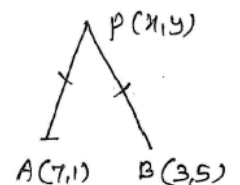
$$= \sqrt{64 + 4} = \sqrt{68}$$

Here $AB = BC = CD = AD$ and diagonals $AC = BD$.
Therefore ABCD is a square.

Example 4: Find a relation between x and y such that the point (x, y) is equidistance from the points (7,1) and (3,5).

Solution:

Let P(x, y) be equidistant from the points A(7,1) and B(3,5).



$$PA = PB \Rightarrow PA^2 = PB^2$$

$$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$$

$$x^2 + 49 - 14x + y^2 + 1 - 2y$$

$$= x^2 + 9 - 6x + y^2 + 25 - 10y$$

$$14x - 6x + 2y - 10y = 49 + 1 - 9 - 25$$

$$8x - 8y = 16$$

$$(\div 8) \quad x - y = 2$$

Example 5: Find a point on the y - axis which is equidistant from the points A(6,5) and B(-4,3).

Solution:

Any point on the y - axis be of the form (0, y).

Let P(0, y) be the point on the y - axis which is equidistant from A(6,5) and B(-4,3).

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$(0 - 6)^2 + (y - 5)^2 = (0 + 4)^2 + (y - 3)^2$$

$$36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$$

$$36 = 10y - 6y$$

$$4y = 36$$

$$y = 9$$

Hence the required point is (0,9).

EXERCISE 7.1

1) Find the distance between the following pairs of points.

a) (2,3), (4,1)

Let the points be A(2,3) and B(4,1)

Distance between A and B =

$$\begin{aligned} &\sqrt{(4-2)^2 + (1-3)^2} \\ &= \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{4 \times 2} \\ &= 2\sqrt{2} \end{aligned}$$

b) (-5,7), (-1,3)

Let the points be A(-5,7) and B(-1,3)

Distance between A and B

$$\begin{aligned} &= \sqrt{(-1+5)^2 + (3-7)^2} \\ &= \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{2 \times 16} \\ &= 4\sqrt{2} \end{aligned}$$

c) (a,b), (-a,-b)

Let the points be A(a,b) and B(-a,-b)

Distance between A and B =

$$\begin{aligned} &\sqrt{(-a-a)^2 + (-b-b)^2} \\ &= \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} \\ &= \sqrt{4(a^2 + b^2)} = 2\sqrt{a^2 + b^2} \end{aligned}$$

2) Find the distance between the points (0,0) and (36,15).

Solution:

Let the points be A(0,0) and B(36,15).

$$\begin{aligned} \text{Distance AB} &= \sqrt{(36-0)^2 + (15-0)^2} \\ &= \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39 \end{aligned}$$

3) Determine if the points (1,5), (2,3), (-2,-11) are collinear.

Solution:

Let the points be A(1,5), B(2,3), and C(-2,-11).

We know that Distance between A(x_1, y_1) and B(x_2, y_2)

$$d = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} \text{Now, } AB &= \sqrt{(2-1)^2 + (3-5)^2} \\ &= \sqrt{1 + (-2)^2} = \sqrt{1+4} = \sqrt{5} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-2-2)^2 + (-11-3)^2} \\ &= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} \\ &= \sqrt{212} = \sqrt{2^2 \times 53} = 2\sqrt{53} \\ AC &= \sqrt{(-2-1)^2 + (-11-5)^2} \\ &= \sqrt{(-3)^2 + (-16)^2} \\ &= \sqrt{9+256} = \sqrt{265} \end{aligned}$$

Hence $AC \neq AB + BC$

Therefore A, B, C are non-collinear points.

4) Check whether (5, - 2), (6, 4) and (7,- 2) are vertices of an isosceles triangle.

Solution:

Let the points be A(5,- 2), B(6, 4) and C(7, - 2).

Distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\text{Now } AB = \sqrt{(6-5)^2 + (4-(-2))^2}$$

$$= \sqrt{1^2 + 36} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2}$$

$$= \sqrt{1+36} = \sqrt{37}$$

$$AC = \sqrt{(7-5)^2 + (-2-(-2))^2}$$

$$= \sqrt{2^2 + 0} = \sqrt{4} = 2$$

Here $AB = BC = \sqrt{37}$

Therefore A, B, C will form the vertices of an isosceles triangle.

5) Name the type of quadrilateral formed if any, by the following points and give reasons for your answer.

a) (- 1, - 2), (1,0), (- 1, 2), (- 3, 0)

Solution:

Let the points be A(- 1, - 2), B(1, 0), C(- 1, 2) and D(- 3, 0)

$$\text{Now } AB = \sqrt{(1-(-1))^2 + (0+2)^2} = \sqrt{2^2 + 2^2}$$

$$= \sqrt{4+4} = \sqrt{8} = \sqrt{2 \times 4} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{(-2)^2 + 2^2}$$

$$= \sqrt{4+4} = \sqrt{8} = \sqrt{2 \times 4} = 2\sqrt{2}$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2}$$

$$= \sqrt{(-2)^2 + (-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = \sqrt{2 \times 4} = 2\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2 + (0-(-2))^2}$$

$$= \sqrt{(-3+1)^2 + (2)^2} = \sqrt{(-2)^2 + 2^2}$$

$$= \sqrt{4+4} = \sqrt{8} = \sqrt{2 \times 4} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2 + (2-(-2))^2}$$

$$= \sqrt{0^2 + 4^2} = 4$$

$$\begin{aligned} \text{Diagonal BD} &= \sqrt{(-3-1)^2 + (0-0)^2} \\ &= \sqrt{(-4)^2} = \sqrt{16} = 4 \end{aligned}$$

Here four sides $AB = BC = CD = AD$ and
Diagonals $AC = BD$.

Therefore given points form a square.

b) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

Solution:

Let the points be $A(-3, 5), B(3, 1), C(0, 3)$ and
 $D(-1, -4)$

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} \text{Now AB} &= \sqrt{(3 - (-3))^2 + (1 - 5)^2} = \sqrt{36 + 16} \\ &= \sqrt{52} = \sqrt{4 \times 13} = 2\sqrt{13} \end{aligned}$$

$$\begin{aligned} \text{BC} &= \sqrt{(0 - 3)^2 + (3 - 1)^2} = \sqrt{9 + 4} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} \text{CD} &= \sqrt{(-1 - 0)^2 + (-4 - 3)^2} \\ &= \sqrt{(-1)^2 + (-7)^2} \\ &= \sqrt{1 + 49} = \sqrt{50} = \sqrt{2 \times 25} = 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{AD} &= \sqrt{(-1 - (-3))^2 + (-4 - 5)^2} \\ &= \sqrt{2^2 + (-9)^2} = \sqrt{4 + 81} \\ &= \sqrt{85} \end{aligned}$$

$$\begin{aligned} \text{AC} &= \sqrt{(0 + 3)^2 + (3 - 5)^2} \\ &= \sqrt{9 + 4} = \sqrt{13} \end{aligned}$$

$$\begin{aligned} \text{Diagonal BD} &= \sqrt{(-1 - 3)^2 + (-4 - 1)^2} \\ &= \sqrt{(-4)^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41} \end{aligned}$$

$$\text{Now AC} + \text{BC} = \sqrt{13} + \sqrt{13} = 2\sqrt{13} = \text{AB}$$

Therefore A, B, C are collinear.

Hence the given points does not form a
Quadrilateral.

c) $(4, 5), (7, 6), (4, 3), (1, 2)$

Solution:

Let the points be $A(4, 5), B(7, 6), C(4, 3)$ and
 $D(1, 2)$

$$\begin{aligned} \text{Now AB} &= \sqrt{(7 - 4)^2 + (6 - 5)^2} = \sqrt{3^2 + 1^2} \\ &= \sqrt{9 + 1} = \sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{BC} &= \sqrt{(4 - 7)^2 + (3 - 6)^2} = \sqrt{(-3)^2 + (-3)^2} \\ &= \sqrt{9 + 9} = \sqrt{2 \times 9} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{CD} &= \sqrt{(1 - 4)^2 + (2 - 3)^2} = \sqrt{(-3)^2 + (-1)^2} \\ &= \sqrt{9 + 1} = \sqrt{10} \end{aligned}$$

$$\text{AD} = \sqrt{(1 - 4)^2 + (2 - 5)^2}$$

$$\begin{aligned} &= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{2 \times 9} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Diagonal AC} &= \sqrt{(4 - 4)^2 + (3 - 5)^2} \\ &= \sqrt{0 + (-2)^2} = \sqrt{4} = 2 \end{aligned}$$

$$\begin{aligned} \text{Diagonal BD} &= \sqrt{(1 - 7)^2 + (2 - 6)^2} \\ &= \sqrt{(-6)^2 + (-4)^2} \\ &= \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13} \end{aligned}$$

Here $AB = CD$ and $BC = AD$.

Diagonals $AC \neq BD$.

Therefore the given points form a
Parallelogram.

6) Find the point on the x - axis which is
equidistant from $(2, -5)$ and $(-2, 9)$.

Solution:

Let the point on the x - axis be $A(x, 0)$ which
equidistant from $B(2, -5)$ and $C(-2, 9)$.

Therefore $AB = AC$

$$\Rightarrow AB^2 = AC^2$$

$$\Rightarrow (2 - x)^2 + (-5 - 0)^2 = (-2 - x)^2 + (9 - 0)^2$$

$$4 + x^2 - 4x + 25 = 4 + x^2 + 4x + 81$$

$$-4x + 29 = 4x + 85$$

$$-4x - 4x = 85 - 29$$

$$-8x = 56$$

$$x = \frac{56}{-8} = -7$$

Hence the required point is $A(-7, 0)$.

7) Find the values of y for which the distance
between the points $P(2, -3)$ and $Q(10, y)$ is 10
unity.

Solution:

Given points are $P(2, -3)$ and $Q(10, y)$.

By data, $PQ = 10$.

$$\sqrt{(10 - 2)^2 + (y + 3)^2} = 10$$

$$\sqrt{8^2 + y^2 + 9 + 6y} = 10$$

squaring on both sides,

$$64 + y^2 + 9 + 6y = 100$$

$$y^2 + 6y + 73 - 100 = 0$$

$$y^2 + 6y - 27 = 0$$

$$(y + 9)(y - 3) = 0$$

$$\Rightarrow y + 9 = 0 \quad | \quad y - 3 = 0$$

$$y = -9 \quad | \quad y = 3$$

Hence the value of P are -9 and 3 .

8) If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$.

Find the value of x . Also find the distances QR and PR .

Solution:

Given that $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$.

$$QP = QR$$

$$\Rightarrow QP^2 = QR^2$$

$$\Rightarrow (5 - 0)^2 + (-3 - 1)^2 = (x - 0)^2 + (6 - 1)^2$$

$$\Rightarrow 25 + 16 = x^2 + 25$$

$$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$$

Therefore the coordinates of R are $(4, 6)$ and $R(-4, 6)$.

Case (i)

Distance between the points $Q(0, 1)$ and $R(4, 0)$ is

$$\sqrt{(4 - 0)^2 + (6 - 1)^2} = \sqrt{16 + 25} = \sqrt{41}$$

Distance between the points $P(5, -3)$ and $R(4, 6)$ is

$$PR = \sqrt{(4 - 5)^2 + (6 + 3)^2} = \sqrt{(-1)^2 + 9^2} \\ = \sqrt{1 + 81} = \sqrt{82}$$

Case (ii)

Distance between the points $Q(0, 1)$ and $R(-4, 6)$ is

$$QR = \sqrt{(-4 - 0)^2 + (6 - 1)^2} = \sqrt{16 + 25} = \sqrt{41}$$

Distance between the points $P(5, -3)$ and $R(-4, 6)$ is

$$PR = \sqrt{(-4 - 5)^2 + (6 + 3)^2} \\ = \sqrt{(-9)^2 + 9^2} = \sqrt{81 + 81} = \sqrt{2 \times 81} = 9\sqrt{2}$$

9) Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Solution:

Given that the point $A(x, y)$ is equidistant from the point $B(3, 6)$ and $C(-3, 4)$.

$$AB = AC$$

$$\Rightarrow AB^2 = AC^2$$

$$(3 - x)^2 + (6 - y)^2 = (-3 - x)^2 + (4 - y)^2$$

$$9 + x^2 - 6x + 36 + y^2 - 12y$$

$$= 9 + x^2 + 6x + 16 + y^2 - 8y$$

$$-6x - 12y + 45 = 6x - 8y + 25$$

$$-6x - 6x - 12y + 8y = 25 - 45$$

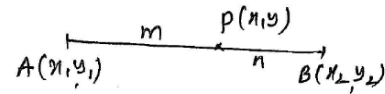
$$-12x - 4y = -20$$

$$(\div -4) \quad 3x + y = 5 \text{ or } 3x + y - 5 = 0$$

Section formula:

Internal division of a line segment:

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points and $P(x, y)$ is a point on the line segment joining A and B such that $AP : PB = m_1 : m_2$ then the point P is said to divide line segment AB internally in the ratio $m_1 : m_2$.



Coordinates of P are given by $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

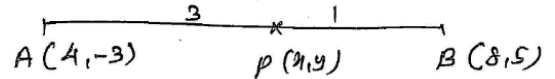
known as section formula.

Example 6: Find the coordinates of the point which divides the line segment joining the points $(4, -3)$ and $(8, 5)$ in the ratio $3 : 1$ internally.

Solution:

Let $P(x, y)$ be the required points.

Given P divides AB internally in the ratio $3:1$



Here $m_1 : m_2 = 3 : 1$

$$(x_1, y_1) = (4, -3)$$

$$(x_2, y_2) = (8, 5)$$

$$\text{By section formula } P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \\ = \left(\frac{3(8) + 1(4)}{3 + 1}, \frac{3(5) + 1(-3)}{3 + 1}\right) = \left(\frac{24 + 4}{4}, \frac{15 - 3}{4}\right) = \left(\frac{28}{4}, \frac{12}{4}\right) = (7, 3)$$

Therefore the required point is $(7, 3)$.

Example 7: In what ratio does the point $(-4, 6)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ [CBSE 2019].

Solution:

Let the point $P(-4, 6)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$.

By section formula,

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Here $P(x, y) = (-4, 6)$

$$(x_1, y_1) = (-6, 10)$$

$$(x_2, y_2) = (3, -8)$$

$$\text{Therefore } (-4, 6) = \left(\frac{m_1(3) + m_2(-6)}{m_1 + m_2}, \frac{m_1(-8) + m_2(10)}{m_1 + m_2}\right)$$

$$\Rightarrow -4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$$

$$\Rightarrow (m_1 + m_2)(-4) = 3m_1 - 6m_2$$

$$-4m_1 - 4m_2 = 3m_1 - 6m_2$$

$$-4m_1 - 3m_1 = -6m_2 + 4m_2$$

$$-7m_1 = -2m_2$$

$$\frac{m_1}{m_2} = \frac{-2}{-7} = \frac{2}{7}$$

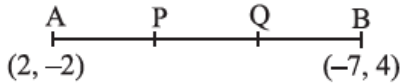
Therefore $m_1 : m_2 = 2 : 7$

Example 8: Find the coordinates of the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4).

Solution:

Let P and Q be the points of trisection of AB.

AP = PQ = QB



Therefore P divides AB internally in the ratio 1 : 2.

Therefore By section formula, $(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2})$

$$= (\frac{1(-7) + 2(2)}{1+2}, \frac{1(4) + 2(-2)}{1+2})$$

$$= (\frac{-7+4}{3}, \frac{4-4}{3}) \Rightarrow (\frac{-3}{3}, 0) = (-1, 0)$$

Therefore the point p is (-1, 0).

Now, Q divides AB in the ratio 2 : 1.

$$\text{Therefore the point Q} = (\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1})$$

$$= (\frac{-14+2}{3}, \frac{8-2}{3}) = (\frac{-12}{3}, \frac{6}{3}) = (-4, 2)$$

Therefore the point Q is (-4, 2).

Example 9: Find the ratio in which the y - axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the point of intersection.

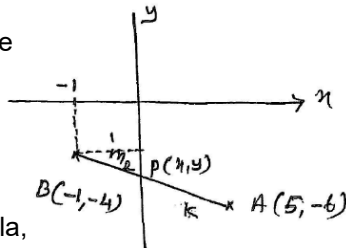
Solution:

Let A(5, -6) and B(-1, -4) be

the given line segment.

Let y - axis divides AB in

the ratio K : 1.



Therefore By section formula,

$$P = (\frac{K(-1)+1(5)}{1+K}, \frac{K(-4)+1(-6)}{1+K})$$

The point P is on the y - axis.

Any point on the y – axis, x coordinate is zero.

$$\text{Therefore } \frac{-K+5}{1+K} = 0 \Rightarrow -K + 5 = 0$$

$$\Rightarrow K = 5$$

Hence the required ratio is 5 : 1.

Therefore the coordinates of P = $(\frac{5(-1)+1(5)}{1+5}, \frac{5(-4)+1(-6)}{1+5})$

$$= (\frac{-5+5}{6}, \frac{-20-6}{6}) = (0, \frac{-26}{6}) = (0, \frac{-13}{3})$$

Hence the required point of intersection is $(0, \frac{-13}{3})$.

Example 10: If the A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram taken in order, find the value of P.

Solution:

Given points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram.

We know that, In a parallelogram, diagonals bisect each other.

mid point of AC = mid point of BD.

$$(\frac{6+9}{2}, \frac{4+1}{2}) = (\frac{8+P}{2}, \frac{3+2}{2})$$

$$\Rightarrow (\frac{15}{2}, \frac{5}{2}) = (\frac{8+P}{2}, \frac{5}{2})$$

$$\Rightarrow \frac{15}{2} = \frac{8+P}{2} \Rightarrow 8+P = 15$$

$$P = 15 - 8$$

$$P = 7$$

EXERCISE 7.2

1) Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3.

Solution:

Let P(x, y) divides the line joining A(-1, 7) and B(4, -3) in the ratio 2 : 3.

Here $m_1 : m_2 = 2 : 3$

$$(x_1, y_1) = A(-1, 7)$$

$$(x_2, y_2) = B(4, -3)$$

$$\text{Therefore the point P} = (\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2})$$

$$= (\frac{3(-1)+2(4)}{2+3}, \frac{3(7)+2(-3)}{2+3})$$

$$= (\frac{-3+8}{5}, \frac{21-6}{5}) = (\frac{5}{5}, \frac{15}{5})$$

$$= (1, 3)$$

2) Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Solution:

Let P and Q be the points of trisection of AB.

AP = PQ = QB.

Therefore P divides AB $A(4, -1), B(-2, -3)$ in the ratio 1 : 2.

Therefore the coordinates of

$$P = (\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}) = (\frac{1(-2)+2(4)}{1+2}, \frac{1(-3)+2(-1)}{1+2})$$

$$= (\frac{-2+8}{3}, \frac{-3-2}{3}) \Rightarrow (\frac{6}{3}, \frac{-5}{3}) = (2, \frac{-5}{3})$$

Therefore the coordinates of P is $(2, \frac{-5}{3})$.

Let Q divides AB in the ratio 2 : 1.

$$\begin{aligned} \text{Therefore the coordinates of Q} &= \left(\frac{2(-2)+1(4)}{1+2}, \frac{2(-3)+1(-1)}{1+2} \right) \\ &= \left(\frac{-4+4}{3}, \frac{-6-1}{3} \right) \\ &= \left(0, \frac{-7}{3} \right) \end{aligned}$$

3) Find the ratio in which the line segment joining the points (- 3, 10) and (6, - 8) is divided by (- 1, 6).

Solution:

Let P(-1, 6) divides the line segment joining A(- 3, 10) and B(6, -8) in the ratio $m_1 : m_2$.

By section formula, $P(-1, 6) =$

$$\left(\frac{m_1(6)+m_2(-3)}{m_1+m_2}, \frac{m_1(-8)+m_2(10)}{m_1+m_2} \right)$$

$$\Rightarrow -1 = \frac{6m_1 - 3m_2}{m_1 + m_2}$$

$$\Rightarrow -(m_1 + m_2) = 6m_1 - 3m_2$$

$$\Rightarrow -m_1 - m_2 = 6m_1 - 3m_2$$

$$-m_1 - 6m_1 = -3m_2 + m_2$$

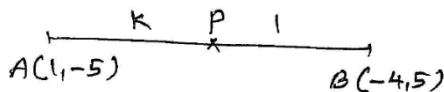
$$-7m_1 = -2m_2$$

$$\frac{m_1}{m_2} = \frac{-2}{-7} = \frac{2}{7}$$

Therefore $m_1 : m_2 = 2 : 7$

4) Find the ratio in which the line segment joining A(1, - 5) and B(- 4, 5) is divided by the x – axis. Also find the coordinates of the point of division.

Solution:



Let x - axis divides the line joining A(1, - 5) and B(- 4, 5) in the ratio K : 1 at P.

Therefore By section formula, the point P is

$$= \left(\frac{K(-4) + 1(1)}{1 + K}, \frac{K(5) + 1(-5)}{1 + K} \right)$$

Any point on the x - axis, y coordinate is zero.

Equating y – coordinate of P to zero.

$$\frac{5K - 5}{1 + K} = 0 \Rightarrow 5K - 5 = 0$$

$$\Rightarrow 5K = 5$$

$$\Rightarrow K = \frac{5}{5} = 1$$

Therefore $K = 1$

Therefore the required ratio is 1 : 1.

Therefore the coordinates of point P is

$$\left(\frac{1(-4)+1(1)}{1+1}, \frac{1(5)+1(-5)}{1+1} \right) = \left(\frac{-4+1}{2}, \frac{5-5}{2} \right)$$

$$= \left(-\frac{3}{2}, 0 \right)$$

5) If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution:

Let A(1, 2), B(4, y), C(x, 6) and D(3, 5) are the vertices of a parallelogram.

We know that, In a parallelogram, diagonals bisect each other.

mid point of AC = mid point of BD.

$$\left(\frac{1+x}{2}, \frac{2+6}{2} \right) = \left(\frac{4+3}{2}, \frac{y+5}{2} \right)$$

$$\Rightarrow \frac{1+x}{2} = \frac{7}{2} \qquad \frac{8}{2} = \frac{y+5}{2}$$

$$\Rightarrow x + 1 = 7 \qquad y + 5 = 8$$

$$x = 7 - 1 \qquad y = 8 - 5$$

Therefore $x = 6 \qquad y = 3$

6) Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, - 3) and B is (1, 4).

Solution:

Given, AB is diameter of a circle and the centre C is (2, - 3). Let the coordinate of A is (a, b).

We know that,

centre of the circle = mid point of diameter

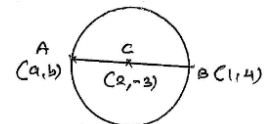
$$(2, -3) = \left(\frac{a+1}{2}, \frac{b+4}{2} \right)$$

$$\Rightarrow \frac{a+1}{2} = 2 \text{ and } \frac{b+4}{2} = -3$$

$$a + 1 = 4 \qquad b + 4 = -6$$

$$a = 4 - 1 \qquad b = -6 - 4$$

$$a = 3 \qquad b = -10$$

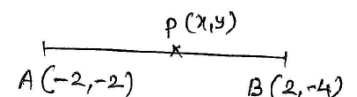


Therefore the coordinates of A is (3, - 10).

7) If A and B are (- 2, - 2) and (2, - 4) respectively, find the coordinates of P such that $AP = \frac{3}{7} AB$ and P lies on the line segment AB.

Solution:

Given, $AP = \frac{3}{7} AB$



$$\frac{AB}{AP} = \frac{7}{3} \Rightarrow \frac{AP + PB}{AP} = \frac{7}{3}$$

$$\frac{AP + PB}{AP} = \frac{7}{3}$$

$$1 + \frac{PB}{AP} = \frac{7}{3}$$

$$\frac{PB}{AP} = \frac{7}{3} - 1 \Rightarrow \frac{PB}{AP} = \frac{7-3}{3}$$

$$\Rightarrow \frac{PB}{AP} = \frac{4}{3}$$

Therefore $\frac{AP}{PB} = \frac{3}{4} \Rightarrow AP : PB = 3 : 4$

Therefore the point P divides AB in the ratio 3 : 4

By section formula,

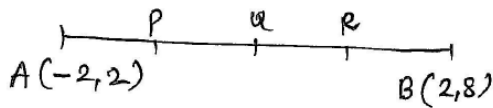
$$P(x, y) = \left(\frac{3(2) + 4(-2)}{3 + 4}, \frac{3(-4) + 4(-2)}{3 + 4} \right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7} \right) = \left(\frac{-2}{7}, \frac{-20}{7} \right)$$

Therefore the coordinates of P is $\left(\frac{-2}{7}, \frac{-20}{7} \right)$.

8) Find the coordinates of the points which divide the line segment joining A(- 2, 2) and B(2, 8) into four equal parts.

Solution:



Let P, Q, R be the points of four section of AB.

Therefore P divides AB in the ratio 1 : 3.

Therefore the coordinates of

$$P \text{ is } \left(\frac{1(2)+3(-2)}{1+3}, \frac{1(8)+3(2)}{1+3} \right)$$

$$= \left(\frac{2-6}{4}, \frac{8+6}{4} \right) = \left(\frac{-4}{4}, \frac{14}{4} \right) = \left(-1, \frac{7}{2} \right)$$

Now, Q divides AB in the ratio 1 : 1.

Therefore coordinates of Q is

$$= \left(\frac{1(2)+1(-2)}{1+1}, \frac{1(8)+1(2)}{1+1} \right) = \left(\frac{2-2}{2}, \frac{8+2}{2} \right) = \left(0, \frac{10}{2} \right) = (0, 5)$$

Now, R divides AB in the ratio 3 : 1.

Therefore coordinates of R is

$$= \left(\frac{3(2)+1(-2)}{3+1}, \frac{3(8)+1(2)}{3+1} \right) = \left(\frac{6-2}{4}, \frac{24+2}{4} \right) = \left(\frac{4}{4}, \frac{26}{4} \right) = \left(1, \frac{13}{2} \right)$$

Hence the required points are

P(-1, $\frac{7}{2}$), Q(0, 5), R(1, $\frac{13}{2}$).

9) Find the area of a rhombus if its vertices are (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) taken in order [Area of rhombus = $\frac{1}{2} \times$ product of diagonals]

Solution:

Let the vertices of rhombus be A(3, 0), B(4, 5), C(-1, 4) and D(-2, -1).

$$\text{Diagonal AC} = \sqrt{(-1 - 3)^2 + (4 - 0)^2}$$

$$= \sqrt{16 + 16} = \sqrt{2 \times 16} = 4\sqrt{2}$$

$$\text{Diagonal BD} = \sqrt{(-2 - 4)^2 + (-1 - 5)^2}$$

$$= \sqrt{36 + 36} = \sqrt{2 \times 36} = 6\sqrt{2}$$

Therefore Area of rhombus = $\frac{1}{2} \times$ product of diagonals

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= \frac{1}{2} \times 4 \times 6 \times 2$$

$$= 24 \text{ sq units.}$$

Area of a Triangle

Area of a triangle when its base and heights are given

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

Area of a triangle when the coordinates of its vertices are given.

Area of a triangle when vertices are

$A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are given by =

$$\frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2} [(x_1y_2 + x_2y_3 + x_3y_1) - (y_1x_2 + y_2x_3 + y_3x_1)]$$

Area of a trapezium:

Area of trapezium = $\frac{1}{2}(\text{sum of parallel sides}) \times$ distance between them.

Example 11: Find the area of a the triangle whose vertices are (1, - 1), (- 4, 6) and (- 3, - 5).

Solution:

Let the vertices of a triangle be A(1, - 1), B(- 4, 6) and C(- 3, - 5).

$$\text{Now Area of the triangle } ABC = \frac{1}{2} \begin{vmatrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{vmatrix}$$

$$= \frac{1}{2} [(1(6) - 4(-5) - 3(-1)) - (-1(-4) + 6(-3) - 5(1))]$$

$$= \frac{1}{2} [(6 + 20 + 3) - (4 - 18 - 5)]$$

$$= \frac{1}{2} [29 - 19]$$

$$= \frac{1}{2} [29 + 19] = \frac{1}{2} [48]$$

$$= 24 \text{ sq units.}$$

Example 12: Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C(7, -4).

Solution:

Given points are A(5, 2), B(4,7) and C(7, -4).

$$\text{Area of the triangle } ABC = \frac{1}{2} \begin{vmatrix} 5 & 4 & 7 & 5 \\ 2 & 7 & -4 & 2 \end{vmatrix}$$

$$= \frac{1}{2} [(5(7) + 4(-4) + 7(2)) - (2(4) + 7(7) - 4(5))]]$$

$$= \frac{1}{2} [(35 - 16 + 4) - (8 + 49 - 20)]$$

$$= \frac{1}{2} [33 - (37)] = \frac{1}{2} [-4] = -2$$

But Area cannot be negative.

Therefore Area of the triangle ABC = 2 sq units.

Example 13: Find the area of the triangle formed by the points P(-1.5, 3), Q(6, -2) and R(-3, 4).

Solution:

Vertices of the triangle are P(-1.5, 3), Q(6, -2), R(-3, 4).

$$\text{Area of the triangle } PQR = \frac{1}{2} \begin{vmatrix} -1.5 & 6 & -3 & -1.5 \\ 3 & -2 & 4 & 3 \end{vmatrix}$$

$$= \frac{1}{2} [(3 + 24 - 9) - (18 + 6 - 6)]$$

$$= \frac{1}{2} [18 - 18] = \frac{1}{2} [0] = 0$$

Therefore Area of the triangle PQR = 0.

Example 14: Find the value of K if the points A(2, 3), B(4, k), C(6, -3) are collinear.

Solution:

Given that the points are collinear.

⇒ Area of the triangle = 0.

$$= \frac{1}{2} \begin{vmatrix} 2 & 4 & 6 & 2 \\ 3 & k & -3 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} [(2k - 12 + 18) - (12 + 6k - 6)] = 0$$

$$\Rightarrow 2k + 6 - (6 + 6k) = 0$$

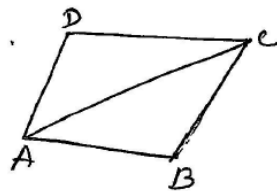
$$\Rightarrow 2k + 6 - 6 - 6k = 0$$

$$\Rightarrow 2k - 6k = 0$$

$$\Rightarrow -4k = 0 \Rightarrow k = 0$$

Example 15: If A(-5, 7), B(-4, -5), C(-1, -6) and D(4,5) are the vertices of a quadrilateral find the area of the Quadrilateral ABCD.

Solution: ABCD is a quadrilateral by joining A and C, we get two triangle ABC and ACD.



Area of the Quadrilateral

$$= \text{Area of } \Delta ABC + \text{Area of } \Delta ACD.$$

$$\text{Now Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} -5 & -4 & -1 & -5 \\ 7 & -5 & -6 & 7 \end{vmatrix}$$

$$= \frac{1}{2} [(25 + 24 - 7) - (-28 + 5 + 30)]$$

$$= \frac{1}{2} [(49 - 7) - (35 - 28)]$$

$$= \frac{1}{2} [42 - (7)] = \frac{1}{2} [42 - 7]$$

$$\text{Area of } \Delta ABC = \frac{35}{2} \text{ sq units.}$$

$$\text{Area of the triangle } ACD = \frac{1}{2} \begin{vmatrix} -5 & -1 & 4 & -5 \\ 7 & -6 & 5 & 7 \end{vmatrix}$$

$$= \frac{1}{2} [(30 - 5 + 28) - (-7 - 24 - 25)]$$

$$= \frac{1}{2} [(58 - 5) - (-56)]$$

$$= \frac{1}{2} [53 + 56] = \frac{1}{2} [109] \text{ sq units.}$$

Therefore Area of the Quadrilateral

$$ABCD = \text{Area of } ABC + \text{Area of } ACD$$

$$= \frac{35}{1} + \frac{109}{2} = \frac{144}{2} = 72 \text{ sq units.}$$

EXERCISE 7.3

1) Find the area of the triangle whose vertices are

(i) (2,3), (-1, 0), (2, -4)

Solution:

Let the vertices of the triangle be A(2, 3), B(-1, 0), C(2, -4).

$$\text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} 2 & -1 & 2 & 2 \\ 3 & 0 & -4 & 3 \end{vmatrix}$$

$$= \frac{1}{2} [(2(0) - 1(-4) + 2(3) - (3(-1) + 0(2) - 4(2)))]$$

$$= \frac{1}{2} [(0 + 4 + 6) - (-3 + 0 - 8)]$$

$$= \frac{1}{2} [10 - (-11)] = \frac{1}{2} [10 + 11] = \frac{21}{2}$$

= 10.5 sq units.

(ii) (-5, -1), (3, -5), (5, 2)

Solution: Let the vertices of the triangle be

A(-5, -1), B(3, -5), C(5, 2).

$$\text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} -5 & 3 & 5 & -5 \\ -1 & -5 & 2 & -1 \end{vmatrix}$$

$$= \frac{1}{2} [(25 + 6 - 5) - (3 - 25 - 10)]$$

$$= \frac{1}{2} [26 - (-38)]$$

$$= \frac{1}{2} [26 + 38] = \frac{1}{2} [64] = 32$$

= 32 sq units.

2) In each of the following find the value of 'K' for which the points are collinear.

(i) (7, -2), (5, 1), (3, k)

Solution:

Let the points be A(7, -2), B(5, 1) and C(3, k).

Given that points are collinear.

⇒ Area of the triangle is zero.

$$\frac{1}{2} \begin{vmatrix} 7 & 5 & 3 & 7 \\ -2 & 1 & k & -2 \end{vmatrix} = 0$$

$$\Rightarrow (7 + 5k - 6) - (-10 + 3 + 7k) = 0$$

$$1 + 5k - (-7 + 7k) = 0$$

$$1 + 5k + 7 - 7k = 0$$

$$8 - 2k = 0 \Rightarrow 2k = 8$$

$$k = \frac{8}{2} = 4$$

$$k = 4$$

(ii) (8, 1), (k, -4), (2, -5)

Solution:

Let the points be A(8, 1), B(k, -4) and C(2, -5).

Given that the points are collinear.

⇒ Area of $\Delta ABC = 0$

$$\frac{1}{2} \begin{vmatrix} 8 & k & 2 & 8 \\ 1 & -4 & -5 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (-32 - 5k + 2) - (k - 8 - 40) = 0$$

$$\Rightarrow -30 - 5k - k + 48 = 0$$

$$\Rightarrow 18 - 6k = 0$$

$$\Rightarrow 6k = 18$$

$$\Rightarrow k = \frac{18}{6} = 3$$

$$k = 3$$

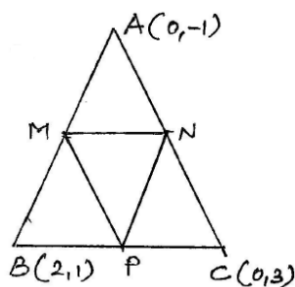
- 3) Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, -1), (2, 1), (0, 3). Find the ratio of this area to the area of the given triangle.

Solution:

Let A(0, -1), B(2, 1), C(0, 3)

be the vertices of ΔABC and

M, N, P be the mid point of AB, AC and BC respectively.



The coordinates of M = mid point of AC

$$= \left(\frac{0+2}{2}, \frac{-1+1}{2} \right)$$

$$= (1, 0)$$

The coordinates of N = mid point of AC

$$= \left(\frac{0+0}{2}, \frac{-1+3}{2} \right)$$

$$= (0, 1)$$

The coordinates of P = mid point of BC

$$= \left(\frac{2+0}{2}, \frac{1+3}{2} \right) = (1, 2)$$

$$\text{Area of } MNP = \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \end{vmatrix}$$

$$= \frac{1}{2} [(1 + 0 + 0) - (0 + 1 + 2)]$$

$$= \frac{1}{2} [1 - (3)] = \frac{1}{2} [-2] = -1$$

Area can't be negative.

Therefore Area of $\Delta MNP = 1$ squnit.

$$\text{Now, Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} 0 & 2 & 0 & 0 \\ -1 & 1 & 3 & -1 \end{vmatrix}$$

$$= \frac{1}{2} [(0 + 6 + 0) - (-2 + 0 + 0)]$$

$$= \frac{1}{2} [6 + 2] = \frac{8}{2} = 4 \text{ squnit.}$$

Therefore Required ratio

= Area of ΔMNP : Area of ΔABC .

= 1 : 4

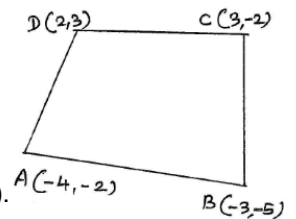
- 4) Find the area of the quadrilateral whose vertices, taken in order are (-4, -2), (-3, -5), (3, -2) and (2, 3).

Solution:

Let the vertices of the

quadrilateral be A(-4, -2),

B(-3, -5), C(3, -2) and D(2, 3).



Now Area of the quadrilateral

$$ABCD = \frac{1}{2} \begin{vmatrix} -4 & -3 & 3 & -4 \\ -2 & -5 & -2 & -2 \end{vmatrix}$$

$$= \frac{1}{2} [(20 + 6 + 9 - 4) - (6 - 15 - 4 - 12)]$$

$$= \frac{1}{2} [31 - (-25)] = \frac{1}{2} [31 + 25] = \frac{1}{2} [56]$$

Area of the quadrilateral ABCD = 28 sq unit.

- 5) The median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A(4, -6), B(3, -2) and C(5, 2).

Solution:

Given the points are

A(4, -6), B(3, -2) and C(5, 2).

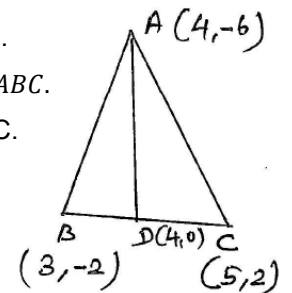
Given that AD is median of ΔABC .

Therefore D is mid point of BC.

Therefore coordinates of

D = mid point of BC.

$$= \left(\frac{3+5}{2}, \frac{-2+2}{2} \right) = (4, 0)$$



$$\text{Now, Area of } \Delta ADC = \frac{1}{2} \begin{vmatrix} 4 & 3 & 4 & 4 \\ -6 & -2 & 0 & -6 \end{vmatrix}$$

$$= \frac{1}{2} [(-8 + 0 - 24) - (-18 - 8 + 0)]$$

$$= \frac{1}{2}[-32 - (-26)] = \frac{1}{2}[-32 + 26]$$

$$= \frac{1}{2}[-6] = -3$$

But Area can't be negative.

Therefore Area of $\triangle ADC = 3$ sq unit.

$$\text{Area of } \triangle ADC = \frac{1}{2} \begin{vmatrix} 4 & 4 & 5 & 4 \\ -6 & 0 & 2 & -6 \end{vmatrix}$$

$$= \frac{1}{2}[(0 + 8 - 30) - (-24 + 0 + 8)]$$

$$= \frac{1}{2}[-22 - (-16)] = \frac{1}{2}[-22 + 16]$$

$$= \frac{-6}{2} = -3$$

Therefore Area of $\triangle ADC = 3$ sq unit.

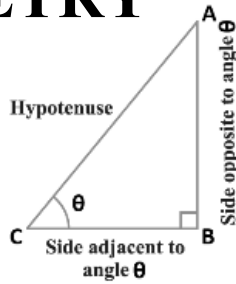
Thus Area of $\triangle ABD = \text{Area of } \triangle ADC$

Hence, the median of $\triangle ABC$, divides it into two triangle of equal areas.

08 INTRODUCTION TO TRIGONOMETRY

Trigonometric Ratios:

- 1) $\sin \theta = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- 2) $\cos \theta = \frac{\text{adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- 3) $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AB}{BC}$
- 4) $\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{adjacent side}} = \frac{AC}{AB} = \frac{1}{\sin \theta}$
- 5) $\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{adjacent side}} = \frac{AC}{BC}$
- 6) $\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{BC}{AB}$
- 7) $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- 8) $\cot \theta = \frac{\cos \theta}{\sin \theta}$



Example 1: Given $\tan A = \frac{4}{3}$, find the other trigonometric ratios of the angle A.

Solution:

$$\text{Given } \tan A = \frac{4}{3} = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$

In right angled triangle

$$AC^2 = AB^2 + BC^2$$

$$= 3^2 + 4^2 = 9 + 16$$

$$AC^2 = 25$$

$$\Rightarrow AC = \sqrt{25} = 5$$

$$\text{Therefore } \sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5}$$

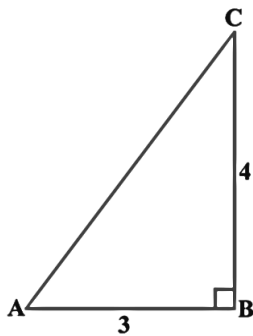
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{3}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4}$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{3}$$

$$\cot A = \frac{1}{\tan A} = \frac{3}{4}$$



Example 2: If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$ then prove that $\angle B = \angle Q$.

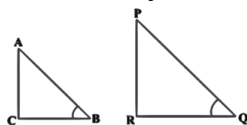
Solution:

In $\triangle ABC$ and $\triangle PQR$

$$\sin B = \sin Q$$

$$\Rightarrow \frac{AC}{AB} = \frac{PR}{PQ} \Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = K(\text{say})$$

By Pythagoras Theorem,



$$BC = \sqrt{AB^2 - AC^2} \text{ and } QR = \sqrt{PQ^2 - PR^2}$$

$$\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{(KPQ)^2 - (KPR)^2}}{\sqrt{PQ^2 - PR^2}}$$

$$= \frac{\sqrt{K^2(PQ^2 - PR^2)}}{\sqrt{PQ^2 - PR^2}} \text{ from 1}$$

$$= \frac{K\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = K \text{ (02)}$$

from 1 and 2

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \triangle ABC \sim \triangle PQR$$

$$\Rightarrow \angle B = \angle Q$$

Hence the proof.

Example 3 : Consider $\triangle ACB$, right angled at C, in which $AB = 29$ units, $BC = 21$ units and $\angle ABC = \theta$. Determine the values of a) $\cos^2 \theta + \sin^2 \theta$

b) $\cos^2 \theta - \sin^2 \theta$

Solution:

In right angled triangle ACB,

$$AC = \sqrt{AB^2 - BC^2}$$

$$= \sqrt{29^2 - 21^2}$$

$$= \sqrt{(29 - 21)(29 + 21)}$$

$$= \sqrt{8 \times 50} = \sqrt{400} = 20$$

$$AC = 20$$

$$\text{a) } \cos^2 \theta + \sin^2 \theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2$$

$$= \frac{441}{841} + \frac{400}{841} = \frac{441+400}{841}$$

$$= \frac{841}{841} = 1$$

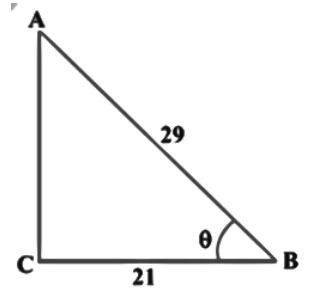
$$\text{Therefore } \cos^2 \theta + \sin^2 \theta = 1$$

$$\text{b) } \cos^2 \theta - \sin^2 \theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2$$

$$= \frac{441}{841} - \frac{400}{841} \Rightarrow \frac{441-400}{841}$$

$$= \frac{41}{841}$$

$$\text{Therefore } \cos^2 \theta - \sin^2 \theta = \frac{41}{841}$$



Example 4: In a right angled triangle ABC, right – angled at B, if $\tan A = 1$ then verify that $2 \sin A \cos A = 1$.

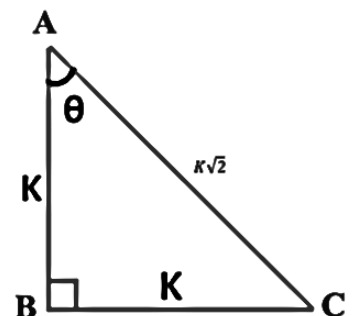
Solution:

In $\triangle ABC$, $\tan A = 1$

$$\frac{BC}{AB} = 1$$

$$\Rightarrow BC = AB.$$

let $AB = BC = K$ (say)



$$\begin{aligned} \text{Now, } AC &= \sqrt{AB^2 + BC^2} = \sqrt{K^2 + K^2} \\ &= \sqrt{2K^2} = K\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Now } 2 \sin A \cos A &= 2 \left(\frac{K}{K\sqrt{2}} \right) \left(\frac{K}{K\sqrt{2}} \right) \\ &= 2 \times \frac{1}{2} = 1 \end{aligned}$$

Therefore $2 \sin A \cos A = 1$
Hence proved.

Example 5: In $\triangle OPQ$, right angled at P $OP = 7$ cm, and $OQ - PQ = 1$ cm. Determine the values of $\sin Q$ and $\cos Q$.

Solution:

In $\triangle OPQ$, we have

$$OQ^2 = OP^2 + PQ^2 \quad \text{--- (1)}$$

$$\text{Given } OQ - PQ = 1 \Rightarrow OQ = 1 + PQ$$

$$\text{Therefore } 1 \Rightarrow (1 + PQ)^2 = OP^2 + PQ^2$$

$$1 + PQ^2 + 2PQ = OP^2 + PQ^2$$

$$1 + 2PQ = OP^2$$

$$1 + 2PQ = 7^2 = 49$$

$$2PQ = 49 - 1 = 48$$

$$PQ = 48/2$$

$$PQ = 24$$

$$\text{Now, } OQ = 1 + PQ$$

$$= 1 + 24 = 25$$

$$\text{Therefore } \sin Q = \frac{OP}{OQ} = \frac{7}{25}, \quad \cos Q = \frac{PQ}{OQ} = \frac{24}{25}$$



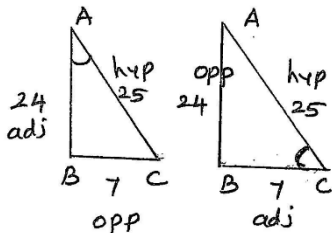
EXERCISE 8.1

1) In $\triangle ABC$, right angled at B, $AB = 24$ cm, $BC = 7$ cm
Determine a) $\sin A$, $\cos A$ b) $\sin C$, $\cos C$.

Solution:

In right angled triangle

$$\begin{aligned} ABC, AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{24^2 + 7^2} \\ &= \sqrt{576 + 49} \\ &= \sqrt{625} = 25 \end{aligned}$$



a) **Sin A, Cos A**

In $\triangle ABC$,

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{7}{25}, \quad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{24}{25}$$

b) **Sin C, Cos C**

In $\triangle ABC$,

$$\sin C = \frac{\text{opp}}{\text{hyp}} = \frac{24}{25}, \quad \cos C = \frac{\text{adj}}{\text{hyp}} = \frac{7}{25}$$

2) In the figure, find $\tan P - \cot R$.

Solution:

In right angled triangle

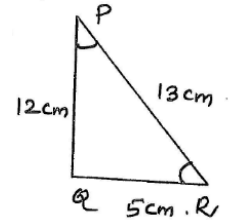
$$QR^2 = PR^2 - PQ^2$$

$$= 13^2 - 12^2 = 169 - 144$$

$$= 25$$

$$\text{Therefore } QR = \sqrt{25} = 5$$

$$\text{Now, } \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$



3) In $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Solution:

$$\text{Given } \sin A = \frac{3}{4}$$

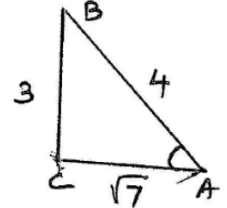
$$\frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{3}{4}$$

$$\Rightarrow BC = 3, AB = 4$$

$$\text{In } \triangle ABC, AC = \sqrt{AB^2 - BC^2}$$

$$= \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$$

$$\text{Therefore } \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{7}}{4}, \quad \tan A = \frac{\text{opp}}{\text{adj}} = \frac{3}{\sqrt{7}}$$



4) Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Solution:

$$\text{Given } 15 \cot A = 8$$

$$\Rightarrow \cot A = \frac{8}{15}$$

$$\frac{\text{adj}}{\text{opp}} = \frac{AB}{BC} = \frac{8}{15}$$

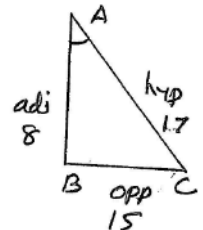
$$\Rightarrow AB = 8, BC = 15.$$

$$\text{In } \triangle ABC, AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289}$$

$$AC = 17$$

$$\text{Therefore } \sin A = \frac{\text{opp}}{\text{hyp}} = \frac{15}{17}, \quad \sec A = \frac{\text{adj}}{\text{hyp}} = \frac{8}{17}$$



5) Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution:

$$\text{Given } \sec \theta = \frac{13}{12}$$

$$\frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$$

$$\frac{AC}{BC} = \frac{13}{12}$$

$$\Rightarrow AC = 13, BC = 12$$

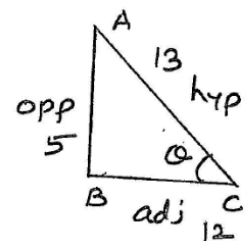
In $\triangle ABC$,

$$AB = \sqrt{AC^2 - BC^2}$$

$$= \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144} = \sqrt{25}$$

$$AB = 5$$



Other ratios are

$$\begin{array}{l} \sin \theta = \frac{5}{13} \\ \cos \theta = \frac{12}{13} \\ \tan \theta = \frac{5}{12} \end{array} \quad \left| \quad \begin{array}{l} \operatorname{cosec} \theta = \frac{13}{5} \\ \sec \theta = \frac{13}{12} \\ \cot \theta = \frac{12}{5} \end{array} \right.$$

6) If $\angle A$ and $\angle B$ are acute angles such that

$\cos A = \cos B$, show that $\angle A = \angle B$.

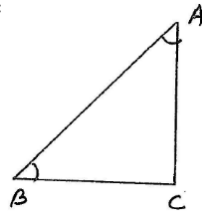
Solution:

In triangle $\triangle ABC$,

$$\cos A = \cos B$$

$$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB} \Rightarrow AC = BC$$

$$\Rightarrow \angle B = \angle A$$



[Therefore in a triangle, angles opposite to equal sides are equal].

Hence proved.

7) If $\cot \theta = \frac{7}{8}$, evaluate a) $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$ b)

$\cot^2 \theta$.

Solution:

$$\text{Given } \cot \theta = \frac{7}{8}$$

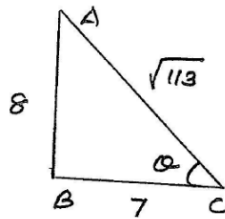
$$\frac{\text{adj}}{\text{opp}} = \frac{7}{8}$$

$$BC = 7, AB = 8$$

$$\text{In } \triangle ABC, AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{8^2 + 7^2}$$

$$= \sqrt{64 + 49} = \sqrt{113}$$



$$\text{a) } \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)} = \frac{1-\sin^2 \theta}{1-\cos^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta$$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$$\text{Therefore } \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)} = \frac{49}{64}$$

$$\text{b) } \cot^2 \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$$\text{Therefore } \cot^2 \theta = \frac{49}{64}$$

8) If $3 \cot A = 4$, check whether

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A \text{ or not}$$

(2014).

Solution:

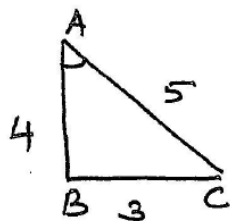
$$\text{Given } 3 \cot A = 4$$

$$\Rightarrow \cot A = \frac{4}{3}$$

$$\frac{\text{adj}}{\text{opp}} = \frac{AB}{BC} = \frac{4}{3}$$

$$AB = 4, BC = 3$$

$$\text{In right angled triangle } ABC, AC = \sqrt{AB^2 + BC^2}$$



$$\begin{aligned} &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} = \sqrt{25} \end{aligned}$$

$$AC = 5$$

$$\text{Now, } \sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AC} = \frac{3}{5}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC} = \frac{4}{5}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AB} = \frac{3}{4}$$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$\text{LHS} = \frac{7}{25} \text{ ————— } \textcircled{01}$$

$$\text{RHS} = \cos^2 A - \sin^2 A$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25}$$

$$\text{RHS} = \frac{7}{25} \text{ ————— } \textcircled{02}$$

from 1 and 2

$$\text{LHS} = \text{RHS}$$

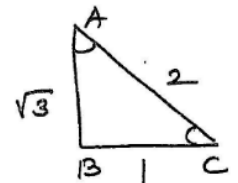
9) In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ then find the value of a) $\sin A \cos C + \cos A \sin C$ b) $\cos A \cos C - \sin A \sin C$.

Solution:

$$\text{Given } \tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BC = 1, AB = \sqrt{3}$$



$$\text{In right angled triangle } ABC, AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(\sqrt{3})^2 + 1^2}$$

$$= \sqrt{3+1} = \sqrt{4} = 2$$

a) $\sin A \cos C + \cos A \sin C$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

b) $\cos A \cos C - \sin A \sin C$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

10) In $\triangle PQR$, right angled at Q, $PR+QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$. (CBSE 2010).

Solution:

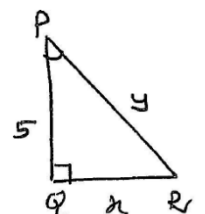
$$\text{Let } QR = x, PR = y, PQ = 5 \text{ cm}$$

$$\text{Given } PR + QR = 25$$

$$y + x = 25$$

$$y = 25 - x$$

In right angled triangle, PQR,



$$PR^2 = PQ^2 + QR^2$$

$$y^2 = 5^2 + x^2$$

$$(25 - x)^2 = 25 + x^2$$

$$625 + x^2 - 50x = 25 + x^2$$

$$50x = 625 - 25$$

$$50x = 600 \Rightarrow x = \frac{600}{50} = 12$$

Now, $y = 25 - x$

$$y = 25 - 12 = 13$$

Therefore $QR = x = 12, PR = y = 13, PQ = 5 \text{ cm}$.

Now $\sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13},$

$$\tan P = \frac{QR}{PQ} = \frac{12}{5}$$

11) State whether the following statements are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

Ans: False, because $\tan A$ lies between $-\infty$ to $+\infty$.

(ii) $\sec A = \frac{12}{5}$ for some value of $\angle A$.

Ans: True, because $\sec A$ is always greater than 1.

(iii) $\cos A$ is the abbreviation used for cosecant of $\angle A$.

Ans: False, because $\cos A$ is abbreviation of cosine

A.

(iv) $\cot A$ is the product of \cot and A .

Ans: $\cot A$ is symbol, not the product of \cot and A . (false).

(v) $\sin \theta = \frac{4}{3}$ for some $\angle Q$.

Ans: False, because $\sin \theta$ is always less than or equal to 1. Therefore $\sin \theta = \frac{4}{3}$ is not possible.

Trigonometric ratios of $0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90°

θ	0°	30°	45°	60°	90°
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	α
Cosec θ	α	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	α
Cot θ	α	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example 7: In ΔPQR , right angled at Q , $PQ = 3 \text{ cm}$ and $PR = 6 \text{ cm}$. Determine $\angle QPR$ and $\angle PRQ$.

Solution:

Given $PQ = 3 \text{ cm}, PR = 6 \text{ cm}$.

$$\sin R = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}$$

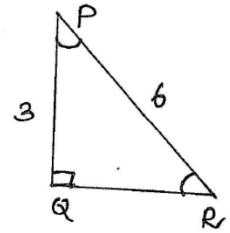
$$\sin R = \frac{1}{2} \Rightarrow \angle R = 30^\circ$$

$$\angle PRQ = 30^\circ$$

$$\cos P = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}$$

$$\cos P = \frac{1}{2} \Rightarrow \angle P = 60^\circ$$

$$\angle QPR = 60^\circ$$



Example 8: If $\sin(A - B) = \frac{1}{2}, \cos(A + B) = \frac{1}{2}, 0 < A + B \leq 90^\circ, A > B$ find A .

Solution:

Given $\sin(A - B) = \frac{1}{2}$

$$\Rightarrow A - B = 30^\circ \text{ ———— } \textcircled{01} \text{ from table}$$

Also, $\cos(A + B) = \frac{1}{2}$

$$\Rightarrow A + B = 60^\circ \text{ ———— } \textcircled{02}$$

$$A + B = 30^\circ \text{ ———— } \textcircled{01}$$

$$A - B = 60^\circ \text{ ———— } \textcircled{02}$$

$$1 + 2 \Rightarrow \frac{2A = 90^\circ}{A = 45^\circ}$$

$$A = 45^\circ$$

substitute $A = 45^\circ$ in 2

$$45^\circ + B = 60^\circ$$

$$B = 60^\circ - 45^\circ$$

$$B = 15^\circ$$

EXERCISE 8.2

1) Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$\begin{aligned} & \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1 \end{aligned}$$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$\begin{aligned} &= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 2(1) + \frac{3}{4} - \frac{3}{4} = 2 \end{aligned}$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})} = \frac{\sqrt{3}}{2(\sqrt{2}+\sqrt{6})}$$

$$= \frac{\sqrt{3}}{2(\sqrt{2}+\sqrt{6})} \times \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}} = \frac{\sqrt{3}(\sqrt{2}-\sqrt{6})}{2[(\sqrt{2})^2 - (\sqrt{6})^2]}$$

$$= \frac{\sqrt{6}-\sqrt{18}}{2[2-6]} = \frac{\sqrt{6}-\sqrt{9 \times 2}}{2(-4)}$$

$$= \frac{\sqrt{6}-3\sqrt{2}}{-8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\frac{\sin 30 + \tan 45 - \operatorname{cosec} 60}{\sec 30 + \cos 60 + \cot 45}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{2(2) + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} = \frac{3\sqrt{3} - 4}{4 + 3\sqrt{3}} \times \frac{4 - 3\sqrt{3}}{4 - 3\sqrt{3}}$$

$$= \frac{12\sqrt{3} - 9\sqrt{3}\sqrt{3} - 16 + 12\sqrt{3}}{16 - (3\sqrt{3})^2}$$

$$= \frac{24\sqrt{3} - 9 \times 3 - 16}{16 - 9 \times 3} = \frac{24\sqrt{3} - 27 - 16}{16 - 27}$$

$$= \frac{24\sqrt{3} - 43}{-11} = \frac{43 - 24\sqrt{3}}{11}$$

(v) $\frac{5 \cos^2 60 + 4 \sec^2 30 - \tan^2 45}{\sin^2 30 + \cos^2 30}$

Note: ($\sin^2 30 + \cos^2 30 = 1$)

$$= \frac{5(\frac{1}{2})^2 + 4(\frac{2}{\sqrt{3}})^2 - 1^2}{1} = 5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1$$

$$= \frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12}$$

$$= \frac{79 - 12}{12} = \frac{67}{12}$$

2) Choose the correct answer and justify your

choice: (i) $\frac{2 \tan 30}{1 + \tan^2 30}$ (A) sin 60 (B) cos 60

(C) tan 60 (D) sin 30

Solution:

(i) $\frac{2 \tan 30}{1 + \tan^2 30} = \frac{2(\frac{1}{\sqrt{3}})}{1 + (\frac{1}{\sqrt{3}})^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60$ (A)

(ii) $\frac{1 - \tan^2 45}{1 + \tan^2 45}$ (A) tan 90 (B) 1 (C) sin 45 (D) 0

$$\frac{1 - \tan^2 45}{1 + \tan^2 45} = \frac{1 - 1^2}{1 + 1^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$$
 (0)

(iii) $\sin 2A = 2 \sin A$ is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

Solution:

$$\sin 2A = 2 \sin A$$

$$\text{When } A = 0^\circ, \sin 2(0) = 2 \sin (0)$$

$$\sin 0 = 2 \sin 0$$

$$0 = 0$$

Hence the given statement is true for when A = 0°

(iv) $\frac{2 \tan 30}{1 - \tan^2 30}$

(A) cos 60 (B) sin 60 (C) tan 60 (D) sin 30

Solution:

$$\frac{2 \tan 30}{1 - \tan^2 30} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - (\frac{1}{\sqrt{3}})^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$= \sqrt{3} = \tan 60$$

3) If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$

$0 < A + B \leq 90^\circ, A > B$ find A and B.

Solution:

$$\tan(A+B) = \sqrt{3}$$

$$\tan(A+B) = \tan 60$$

$$\Rightarrow A + B = 60^\circ \text{ ————— } \textcircled{01}$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow A - B = 30^\circ \text{ ————— } \textcircled{02}$$

$$A + B = 60^\circ \text{ ————— } \textcircled{01}$$

$$A - B = 30^\circ \text{ ————— } \textcircled{02}$$

$$1 + 2 \Rightarrow 2A = 90^\circ$$

$$A = 45^\circ$$

substitute A = 45° in 1

$$45 + B = 60^\circ$$

$$B = 60 - 45 = 15^\circ$$

$$B = 15^\circ$$

4) State whether the following are true or false.

Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

Suppose A = 30°, B = 60°

$$\text{LHS} = \sin(A+B) = \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ = 1 \text{ ————— } \textcircled{01}$$

$$\text{RHS} = \sin A + \sin B = \sin 30^\circ + \sin 60^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1 + \sqrt{3}}{2} \text{ ————— } \textcircled{02}$$

From 1 and 2, LHS ≠ RHS

(ii) The value of $\sin \theta$ increases as θ increases

True. (From the table).

- (iii) The value of $\cos \theta$ increases as θ increases
False. (see the table).
- (iv) $\sin \theta = \cos \theta$ for all value of θ False. $\sin \theta = \cos \theta$ only for $\theta = 45^\circ$.
- (v) $\cot A$ is not defined for $A = 0^\circ$ True. Therefore $\cot \theta = \infty$ (not defined).

Trigonometric Identifies:-

- a) $\cos^2\theta + \sin^2\theta = 1$
- b) $1 + \tan^2\theta = \sec^2\theta$
- c) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
- d) $\cos^2\theta = 1 - \sin^2\theta$
- e) $\sec^2\theta - \tan^2\theta = 1$
- f) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Trigonometric Ratios of Complementary Angles

Note:

$$\begin{aligned} \sin(90 - \theta) &= \cos \theta & \operatorname{cosec}(90 - \theta) &= \sec \theta \\ \cos(90 - \theta) &= \sin \theta & \sec(90 - \theta) &= \operatorname{cosec} \theta \\ \tan(90 - \theta) &= \cot \theta & \cot(90 - \theta) &= \tan \theta \end{aligned}$$

Example 9: Evaluate $\frac{\tan 65}{\cot 25}$

Solution:

$$\begin{aligned} \frac{\tan 65}{\cot 25} &= \frac{\tan 65}{\cot(90-65)} \\ &= \frac{\tan 65}{\tan 65} = 1 \quad (\text{OR}) \\ \frac{\tan 65}{\cot 25} &= \frac{\tan(90-25)}{\cot 25} = \frac{\cot 25}{\cot 25} = 1 \end{aligned}$$

Example 10: If $\sin 3A = \cos(A - 26^\circ)$ where $3A$ is an acute angle, find the value of A

Solution:

Given $\sin 3A = \cos(A - 26)$ (01)

We know that $\sin 3A = \cos(90 - 3A)$

Therefore $1 \Rightarrow \cos(90 - 3A) = \cos(A - 26)$

$$\Rightarrow 90 - 3A = A - 26$$

$$A + 3A = 90 + 26$$

$$4A = 116$$

$$A = \frac{116}{4} = 29$$

Therefore $A = 29^\circ$.

Example 11: Express $\cot 85^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Solution:

$$\cot 85^\circ + \cos 75^\circ = \cot(90 - 5) + \cos(90 - 15)$$

$$= \tan 5^\circ + \sin 15^\circ$$

Therefore $\cot 85^\circ + \cos 75^\circ = \tan 5^\circ + \sin 15^\circ$

EXERCISE 8.3

1) Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin 18^\circ}{\cos(90-18)} = \frac{\sin 18^\circ}{\sin 18^\circ} = 1$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan 26^\circ}{\cot(90-26)} = \frac{\tan 26^\circ}{\tan 26^\circ} = 1$

(iii) $\cos 48^\circ - \sin 42^\circ = \cos(90 - 42) - \sin 42$
 $= \sin 42 - \sin 42 = 0$

(iv) $\operatorname{cosec} 31 - \sec 59 = \operatorname{cosec}(90 - 59) - \sec 59$
 $= \sec 59 - \sec 59 = 0$

2) Show that

(i) $\tan 48 \tan 23 \tan 42 \tan 67 = 1$

$$\begin{aligned} LHS &= \tan 48 \tan 23 \tan 42 \tan 67 \\ &= \tan(90 - 42) \cdot \tan(90 - 67) \cdot \tan 42 \tan 67 \\ &= \cot 42 \cdot \cot 67 \cdot \tan 42 \cdot \tan 67 \\ &= \frac{1}{\tan 42} \times \frac{1}{\tan 67} \times \tan 42 \cdot \tan 67 \\ &= 1 = RHS \end{aligned}$$

(ii) $\cos 38 \cos 52 - \sin 38 \sin 52 = 0$

$$\begin{aligned} LHS &= \cos 38 \cos 52 - \sin 38 \sin 52 \\ &= \cos(90 - 52) \cos(90 - 38) - \sin 38 \sin 52 \\ &= \sin 52 \sin 38 - \sin 38 \cos 52 = 0 = RHS \end{aligned}$$

3) If $\tan 2A = \cot(A - 18)$ where $2A$ is an acute angle, find the value of A .

Solution:

Given $\tan 2A = \cot(A - 18)$ (01)

We know that $\tan 2A = \cot(90 - 2A)$

Now Equation 1 becomes

$$\Rightarrow \cot(90 - 2A) = \cot(A - 18)$$

$$\Rightarrow 90 - 2A = A - 18$$

$$90 + 18 = A + 2A$$

$$3A = 108$$

$$A = \frac{108}{3} = 36$$

Therefore $A = 36^\circ$

4) If $\tan A = \cot B$, prove that $A + B = 90^\circ$.

Solution:

$$\tan A = \cot B$$

$$\cot(90 - A) = \cot B$$

$$\Rightarrow 90 - A = B \Rightarrow A + B = 90^\circ$$

Hence proved.

5) If $\sec 4A = \operatorname{cosec}(A - 20)$ where $4A$ is an acute angle, find the value of A .

Solution:

Given $\sec 4A = \operatorname{cosec}(A - 20)$ 01

We know that $\sec 4A = \operatorname{cosec}(90 - 4A)$

Therefore

$$1 \Rightarrow \operatorname{cosec}(90 - 4A) = \operatorname{cosec}(A - 20)$$

$$\Rightarrow 90 - 4A = A - 20$$

$$5A = 110$$

$$A = \frac{110}{5} = 22$$

Therefore $A = 22^\circ$

6) If A, B, C are interior angles fo triangle ABC , then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Solution:

We know that in a triangle sum of angles is 180° .

$$A + B + C = 180^\circ$$

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90^\circ$$

$$\frac{B+C}{2} = 90 - \frac{A}{2}$$

Taking \sin on both sides

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right)$$

$$\text{Therefore } \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Hence proved.

7) Express $\sin 67 + \cos 75$ in terms of trigonometric ratios of angles between 0° and 45° .

Solution:

$$\sin 67 + \cos 75$$

$$= \sin(90 - 23) + \cos(90 - 25)$$

$$= \cos 23 + \sin 25$$

$$\text{Therefore } \sin 67 + \cos 75 = \cos 23 + \sin 25$$

Example 12: Express the ratios $\cos A, \tan A$ and $\sec A$ interms of $\sin A$.

Solution:

$$\cos^2 A + \sin^2 A = 1$$

$$\text{Now } \cos^2 A = 1 - \sin^2 A$$

$$\text{Therefore } \cos A = \sqrt{1 - \sin^2 A}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$$

$$\begin{aligned} \tan A &= \frac{\sin A}{\cos A} \\ &= \frac{\sin A}{\sqrt{1 - \sin^2 A}} \end{aligned}$$

Example 13: Prove that $\sec A (1 - \sin A)(\sec A + \tan A) = 1$

Solution:

$$\text{LHS} = \sec A (1 - \sin A)(\sec A + \tan A)$$

$$\begin{aligned} &= \frac{1}{\cos A} [1 - \sin A] \left[\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right] \\ &= \frac{1}{\cos A} [1 - \sin A] \left[\frac{1 + \sin A}{\cos A} \right] \\ &= \frac{(1 - \sin A)(1 + \sin A)}{\cos^2 A} = \frac{1 - \sin^2 A}{\cos^2 A} \\ &= \frac{\cos^2 A}{\cos^2 A} = 1 \\ &= \text{RHS} \end{aligned}$$

Example 14: Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Solution:

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$= \frac{\cos A \left[\frac{1}{\sin A} - 1 \right]}{\cos A \left[\frac{1}{\sin A} + 1 \right]} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS}$$

Example 15: Prove that $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ using the identify $\sec^2 \theta = 1 + \tan^2 \theta$

Solution:

$$\begin{aligned} \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} &= \frac{\cos \theta \left[\frac{\sin \theta}{\cos \theta} - 1 + \frac{1}{\cos \theta} \right]}{\cos \theta \left[\frac{\sin \theta}{\cos \theta} + 1 - \frac{1}{\cos \theta} \right]} \\ &= \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} = \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} \\ &= \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} \times \frac{(\tan \theta - \sec \theta)}{(\tan \theta - \sec \theta)} \\ &= \frac{\tan^2 \theta - \sec^2 \theta - (\tan \theta - \sec \theta)}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \\ &= \frac{-1 - \tan \theta + \sec \theta}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \\ &= \frac{-(1 + \tan \theta - \sec \theta)}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \\ &= \frac{-1}{\tan \theta - \sec \theta} = \frac{1}{\sec \theta - \tan \theta} \\ &= \text{RHS} \end{aligned}$$

EXERCISE 8.4

1) Express the trigonometric ratios $\sin A, \sec A$ and $\tan A$ interms of $\cot A$.

Solution:

We know that, $1 + \cot^2 A = \operatorname{cosec}^2 A$

$$\text{Now, } \sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1 + \cot^2 A}}$$

We know that $1 + \tan^2 A = \sec^2 A$

Therefore, $\sec A = \sqrt{1 + \tan^2 A}$

$$\begin{aligned} &= \sqrt{1 + \frac{1}{\cot^2 A}} \\ &= \sqrt{\frac{1 + \cot^2 A}{\cot^2 A}} = \frac{\sqrt{1 + \cot^2 A}}{\cot A} \end{aligned}$$

$$\tan A = \frac{1}{\cot A}$$

Therefore $1 + \tan^2 A = \sec^2 A$

Therefore $\sec A = 1 + \tan^2 A$

2) Write all other trigonometric ratios of $\triangle A$ in terms of $\sec A$.

Solution:

We know that, $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

(i) $\sin A = \sqrt{1 - \cos^2 A}$

$$\begin{aligned} \sin A &= \sqrt{1 - \frac{1}{\sec^2 A}} \\ &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \end{aligned}$$

(ii) $\cos A = \frac{1}{\sec A}$

(iii) $\tan A = \sqrt{\tan^2 A} = \sqrt{\sec^2 A - 1}$

(iv) $\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$

(v) $\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

3) Evaluate (i) $\frac{\sin^2 63 + \sin^2 27}{\cos^2 17 + \cos^2 73}$

Solution:

$$\begin{aligned} \frac{\sin^2 63 + \sin^2 27}{\cos^2 17 + \cos^2 73} &= \frac{\sin^2 63 + \sin^2(90 - 63)}{\cos^2 17 + \cos^2(90 - 17)} \\ &= \frac{\sin^2 63 + \cos^2 63}{\cos^2 17 + \sin^2 17} = \frac{1}{1} = 1 \end{aligned}$$

identify $\sin^2 \theta + \sin^2 \theta = 1$

(ii) $\sin 25 \cos 65 + \cos 25 \sin 65$

Solution:

$$\begin{aligned} &= \sin 25 \cos 65 + \cos 25 \sin 65 \\ &= \sin 25 \cos(90 - 25) + \cos 25 \sin(90 - 25) \\ &= \sin 25 \sin 25 + \cos 25 \cos 25 \\ &= \sin^2 25 + \cos^2 25 = 1 \end{aligned}$$

4) Choose the correct option (i) $9 \sec^2 A - 9 \tan^2 A = ?$

Solution:

(i) $9 \sec^2 A - 9 \tan^2 A = ?$

- A) 1 B) 9 C) 8 D) 0

Ans:

$$\begin{aligned} &9 \sec^2 A - 9 \tan^2 A \\ &= 9(\sec^2 A - \tan^2 A) \\ &= 9(1) = 9 \end{aligned}$$

identify $\sec^2 A - \tan^2 A = 1$

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = ?$

- A) 0 B) 1 C) 2 D) -1

Ans:

$$\begin{aligned} &(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) \\ &= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\ &= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \\ &= \frac{(\cos \theta + \sin \theta)^2}{\sin \theta \cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta}{\sin \theta \cos \theta} \\ &= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2 \end{aligned}$$

(iii) $(\sec A + \tan A)(1 - \sin A) = ?$

- A) $\sec A$ B) $\sin A$ C) $\operatorname{cosec} A$ D) $\cos A$

Solution:

$$\begin{aligned} &(\sec A + \tan A)(1 - \sin A) \\ &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) \\ &= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} \\ &= \frac{\cos^2 A}{\cos A} \\ &= \cos A \end{aligned}$$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} = ?$

- A) $\sec^2 A$ B) -1 C) $\cot^2 A$ D) $\tan^2 A$

Solution:

$$\begin{aligned} \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{\sec^2 A}{\operatorname{cosec}^2 A} \\ &= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \sin^2 A \\ &= \tan^2 A \end{aligned}$$

5) Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

Solution:

$$\begin{aligned} \text{LHS} &= (\operatorname{cosec} \theta - \cot \theta)^2 \\ &= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}\right)^2 = \left(\frac{1 - \cos \theta}{\sin \theta}\right)^2 \\ &= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \end{aligned}$$

$$\begin{aligned}
 &= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} \\
 &= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} \\
 &= \frac{1-\cos\theta}{1+\cos\theta} = RHS
 \end{aligned}$$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$

Solution:

$$\begin{aligned}
 LHS &= \frac{\cos A}{1+\sin A} = \frac{1+\sin A}{\cos A} \\
 &= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A} \\
 &= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A} \\
 &= \frac{1 + 1 + 2\sin A}{(1+\sin A)\cos A} \\
 &= \frac{2 + 2\sin A}{(1+\sin A)\cos A} \\
 &= \frac{2(1+\sin A)}{(1+\sin A)\cos A} \\
 &= \frac{2}{\cos A} = 2 \sec A = RHS
 \end{aligned}$$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

Solution:

$$\begin{aligned}
 &\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} \\
 &= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}} \\
 &= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}} \\
 &= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta} \\
 &= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)} \\
 &= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta \cos\theta(\sin\theta-\cos\theta)} \\
 &= \frac{(\sin\theta-\cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta \cos\theta)}{\sin\theta \cos\theta(\sin\theta-\cos\theta)} \\
 &= \frac{1 + \sin\theta \cos\theta}{\sin\theta \cos\theta} = \frac{1}{\sin\theta \cos\theta} + \frac{\sin\theta \cos\theta}{\sin\theta \cos\theta} \\
 &= \operatorname{cosec}\theta \sec\theta + 1 \\
 &= RHS
 \end{aligned}$$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

Solution:

$$\begin{aligned}
 LHS &= \frac{1+\sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} \\
 &= \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1} \\
 &= 1 + \cos A \quad \text{--- (01)}
 \end{aligned}$$

$$\begin{aligned}
 RHS &= \frac{\sin^2 A}{1-\cos A} \\
 &= \frac{1-\cos^2 A}{1-\cos A} = \frac{(1+\cos A)(1-\cos A)}{(1-\cos A)} \\
 &= 1 + \cos A \quad \text{--- (02)}
 \end{aligned}$$

From 1 and 2 LHS = RHS

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using identity

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Solution:

$$LHS = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

on dividing Nr and Dr by $\sin A$,

$$\begin{aligned}
 &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} = \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{\cot A + \operatorname{cosec} A - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{(\cot A + \operatorname{cosec} A)[1 - (\operatorname{cosec} A - \cot A)]}{\cot A + 1 - \operatorname{cosec} A} \\
 &= \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{(\cot A + 1 - \operatorname{cosec} A)} \\
 &= (\cot A + \operatorname{cosec} A) = RHS \\
 &\text{using } a^2 - b^2 = (a+b)(a-b)
 \end{aligned}$$

Hence proved.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A.$

Solution:

$$\begin{aligned}
 \sqrt{\frac{1+\sin A}{1-\sin A}} &= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} \\
 &= \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \\
 &= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}
 \end{aligned}$$

$$= \sec A + \tan A$$

$$= RHS$$

Hence proved.

(vii) $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta.$

$$LHS = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\sin \theta [1 - 2(1 - \cos^2 \theta)]}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\sin \theta (1 - 2 + 2 \cos^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \frac{\sin \theta (2 \cos^2 \theta - 1)}{\cos \theta (2 \cos^2 \theta - 1)} = \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta = RHS$$

Hence proved

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$$= 7 + \tan^2 A + \cot^2 A$$

$$LHS = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A$$

$$+ \sec^2 A + 2 \cos A \sec A$$

$$= (\sin^2 A + \cos^2 A) + (1 + \cot^2 A)$$

$$+ 2 \sin A \cdot \frac{1}{\sin A} + (1 + \tan^2 A) + 2 \cos A \cdot \frac{1}{\cos A}$$

$$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$$

$$= 7 + \cot^2 A + \tan^2 A = RHS$$

Note: $\left[\begin{array}{l} [\operatorname{cosec}^2 A = 1 + \cot^2 A] \\ [\sec^2 A = 1 + \tan^2 A] \end{array} \right]$

Hence proved.

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

Solution:

$$LHS = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A\right) \left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right) \left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \cos A \cdot \sin A \text{ ————— } \textcircled{01}$$

$$RHS = \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \sin A \cos A \text{ ————— } \textcircled{02}$$

From 1 and 2 LHS = RHS

Hence proved.

(x) $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A.$

Solution:

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \tan^2 A \text{ ————— } \textcircled{01}$$

$$\left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \left(\frac{1 - \frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\cos A - \sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}\right)^2$$

$$= \left(\frac{\cos A - \sin A}{\cos A} \times \frac{(-\sin A)}{\cos A - \sin A}\right)^2$$

$$= \left(\frac{-\sin A}{\cos A}\right)^2 = \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A \text{ ————— } \textcircled{02}$$

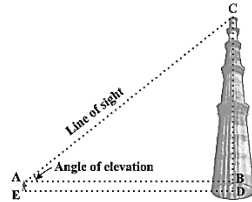
from 1 and 2

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

09 SOME APPLICATIONS OF TRIGONOMETRY

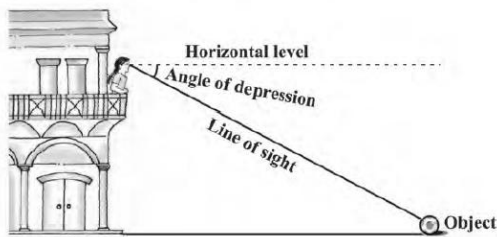
Example 1: Angle of elevation

The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal.



Angle of depression:

The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed.



Example 2: A tower stands vertically on the ground. From a point on the ground which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower, is to be 60°. Find the height of the tower.

Solution:

Let AB be the tower.
C is a point which is 15 m away from foot of the tower.

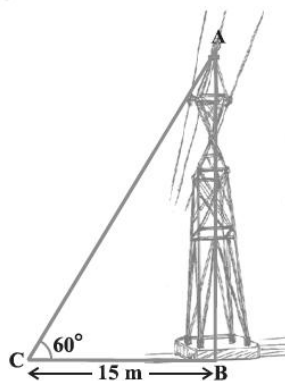
$BC = 15 \text{ m}$

In ΔABC ,

$$\tan \theta = \frac{AB}{BC}, \tan 60^\circ = \frac{AB}{15}$$

$$\sqrt{3} = \frac{AB}{15} \Rightarrow AB = 15\sqrt{3}$$

Hence height of the tower $AB = 15\sqrt{3} \text{ m}$.



Example 2: An electrician has to repair an electric fault on a pole of height 5 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work. What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal, would enable he to reach the

required position? Also, how far from the foot of the pole should he place the foot of the ladder?

Solution:

Given that $AD = 5 \text{ m}$, $AB = 1.3 \text{ m}$

Therefore $BD = AD - AB$

$$= 5 - 1.3 = 3.7 \text{ m.}$$

Here BC represents the ladder.

In ΔBCD ,

$$\sin 60^\circ = \frac{BD}{BC} = \frac{3.7}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.7}{BC} \Rightarrow BC\sqrt{3} = 3.7 \times 2$$

$$\begin{aligned} BC &= \frac{3.7 \times 2}{\sqrt{3}} \\ &= \frac{3.7 \times 2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{7.4\sqrt{3}}{3} = 2.467\sqrt{3} \\ &= 2.467 \times 1.732 = 4.27 \end{aligned}$$

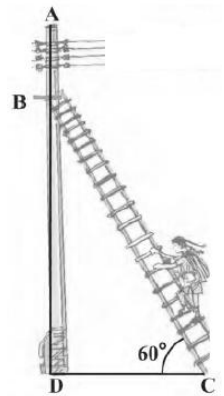
Therefore length of ladder $BC = 4.27 \text{ m}$.

Now In ΔBCD ,

$$\tan 60^\circ = \frac{BD}{DC}$$

$$\begin{aligned} \sqrt{3} &= \frac{3.7}{DC} \Rightarrow DC = \frac{3.7}{\sqrt{3}} \\ &= \frac{3.7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3.7\sqrt{3}}{3} \\ &= \frac{3.7 \times 1.732}{3} = \frac{6.4084}{3} \\ &= 2.1361 \approx 2.14 \text{ m} \end{aligned}$$

Therefore he should place the ladder at 2.14 m from the pole.



Example 3: An observer 1.5 m tall is 28.5 m away from a Chimney. The angle of elevation of the top of chimney from her eyes is 45°. What is the height of the chimney?

Solution:

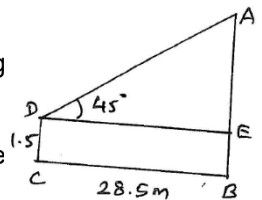
Here CD be the observer whose height is 1.5 m and AB be the tower.

Distance between tower and observer

$BC = 28.5 \text{ m}$.

In ΔADE , $DE = BC = 28.5 \text{ m}$. and angle of elevation is 45°.

Therefore $\tan 45^\circ = \frac{AE}{DE}$



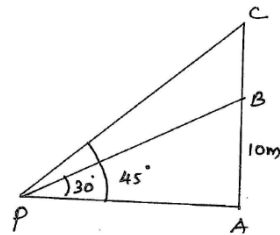
$$1 = \frac{AE}{28.5} \Rightarrow AE = 28.5 \text{ m}$$

Now height of the tower $AB = AE + EB = 28.5 + 1.5$
 $AB = 30 \text{ m}$

Example 4: From a point p on the ground the angle of elevation of the top of a 10 m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from p is 45° . Find the length of the flagstaff and the distance of the building from the point p.

Solution:

From the diagram $AB = 10 \text{ m}$
 denote the height of the building.
 BC be the height of the flagstaff.



Angle of elevation from p to top of the building is 30° and the top of flagstaff is 45° .

In ΔPAB , $\tan 30^\circ = \frac{AB}{PA}$

$$\frac{1}{\sqrt{3}} = \frac{10 \text{ m}}{PA} \Rightarrow PA = 10 \text{ m} \times \sqrt{3} = 10\sqrt{3} \text{ m.}$$

In ΔPAC , $\tan 45^\circ = \frac{AC}{PA}$

$$1 = \frac{AB + BC}{10\sqrt{3}} \Rightarrow AB + BC = 10\sqrt{3}$$

$$10 + BC = 10\sqrt{3}$$

$$\begin{aligned} BC &= 10\sqrt{3} - 10 \\ &= 10(\sqrt{3} - 1) \\ &= 10 \times (1.732 - 1) \\ &= 10 \times 0.732 \\ &= 7.32 \text{ m} \end{aligned}$$

Hence length of flagstaff $BC = 7.32 \text{ m}$.

Example 5: The shadow of a tower standing a level ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60° . Find the height of the tower.

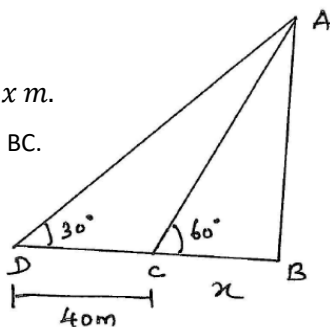
Solution:

Let AB be the tower and $BC = x \text{ m}$.
 By data, BD is 40 m longer than BC .
 Now $BD = x + 40 \text{ m}$.

In ΔABC ,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC} \Rightarrow \frac{AB}{x} = \sqrt{3}$$



$$AB = x\sqrt{3} \text{ m} \text{ ———— (01)}$$

ΔABD ,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{x+40} \Rightarrow AB = \frac{x+40}{\sqrt{3}} \text{ ———— (02)}$$

From 1 and 2 $x\sqrt{3} = \frac{x+40}{\sqrt{3}}$

$$x\sqrt{3}\sqrt{3} = x + 40$$

$$3x = x + 40 \Rightarrow 3x - x = 40$$

$$2x = 40$$

$$x = 20$$

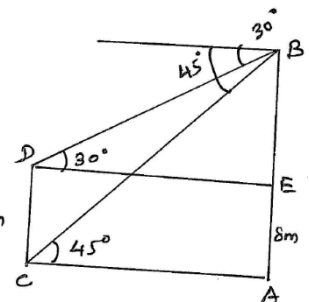
Therefore Height of the tower $AB = 20\sqrt{3} \text{ m}$.

Example 6: The angles of depression of the top and bottom of an 8 m tall building from the top of a multi-storied building are 30° and 45° respectively. Find the height of the multi story building and the distance between two buildings.

Solution:

Let AB denote the multistory building and $CD = 8 \text{ m}$ denote the tall building.

Angle of elevation from the top of multistory building to top of tall building is 30° and bottom of tall building is 45° .



In ΔABC ,

$$\tan 45^\circ = \frac{AB}{AC} \Rightarrow 1 = \frac{AB}{AC} \Rightarrow AB = AC$$

(i.e) $AE + EB = AC$

$$8 + EB = AC \text{ ———— (01)}$$

In ΔBED , $\tan 30^\circ = \frac{BE}{DE}$

$$\frac{1}{\sqrt{3}} = \frac{BE}{AC} \quad (\because DE = AC)$$

$$\frac{1}{\sqrt{3}} = \frac{BE}{8+EB} \quad (\text{from 1})$$

$$BE\sqrt{3} = 8 + BE$$

$$BE\sqrt{3} - BE = 8$$

$$BE(\sqrt{3} - 1) = 8$$

$$BE = \frac{8}{\sqrt{3} - 1} = \frac{8}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

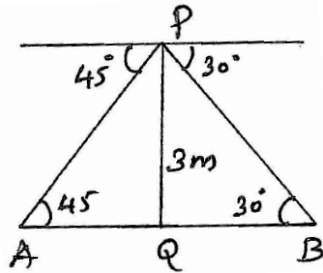
$$\begin{aligned} BE &= \frac{8(\sqrt{3} + 1)}{3 - 1} = \frac{8(1.732 + 1)}{2} = 4 \times 2.732 \\ &= 10.928 \end{aligned}$$

Therefore Height of the building $AB = AE + EB$
 $= 8 + 10.92$
 $AB = 18.92 \text{ m.}$
 Distance between two buildings $AC = 8 + EB$
 $= 8 + 10.92$
 $AC = 18.92 \text{ m.}$

Example 7: From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Solution:

Let A and B be the points on the bank of rivers.
 The width of the river is AB.
 P is the point on the bridge at a height of 3 m.
 $PQ = 3\text{m.}$



In ΔAPQ , $\tan 45^\circ = \frac{PQ}{AQ}$

$$1 = \frac{PQ}{AQ} \Rightarrow PQ = AQ$$

$$3 = AQ \text{ ————— (1)}$$

In ΔPQB , $\tan 30^\circ = \frac{PQ}{QB}$

$$\frac{1}{\sqrt{3}} = \frac{3}{QB} \Rightarrow QB = 3\sqrt{3}\text{m}$$

The width of the river $AB = AQ + QB$

$$= 3 + 3\sqrt{3} = 3(1 + \sqrt{3})$$

$$= 3(1 + 1.732) = 3(2.732)$$

$$AB = 8.196 \text{ m}$$

EXERCISE 9.1

1) A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° .

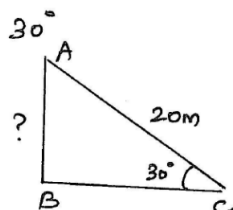
Solution: In the figure, AB be the pole and AC = 20 m is the length of the rope.

$$\text{In } \Delta ABC, \sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20} \Rightarrow 2AB = 20$$

$$AB = 10 \text{ m}$$

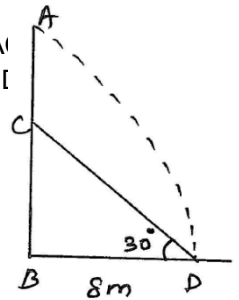
Hence the height of the pole $AB = 10 \text{ m.}$



2) A tree breaks due to storm and broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution:

Let AB be the tree whose part AC breaks and touches the ground at D.
 Given $BD = 8 \text{ m}$ and $AC = CD$.
 to find height of tree AB
 $AB = BC + AC$ or
 $BC + CD$ (Therefore $AC = CD$)



$$\text{In } \Delta BCD, \tan 30^\circ = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{BC}{8} \Rightarrow BC = \frac{8}{\sqrt{3}}$$

$$BC = \frac{8}{\sqrt{3}} = \frac{8 \times \sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

In right angled triangle BCD , $\sin 30^\circ = \frac{BC}{DC}$

$$\frac{1}{2} = \frac{8\sqrt{3}/3}{DC}$$

$$DC = 2\left(\frac{8\sqrt{3}}{3}\right)$$

$$DC = \frac{16\sqrt{3}}{3}$$

Now Height of tree $AB = BC + CD$

$$= \frac{8\sqrt{3}}{3} + \frac{16\sqrt{3}}{3} = \frac{24\sqrt{3}}{3}$$

$$AB = 8\sqrt{3} \text{ m}$$

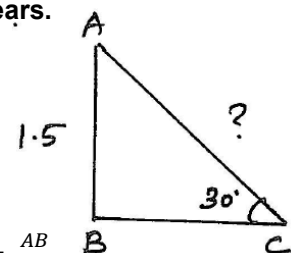
3) A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefer to have a slide whose top is at a height of 1.5 m and is inclined of an angle of 30° to the ground, where as for the elder children, she wants to have a steep slide at a height 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Solution:

Case (i) For children below 5 years.

Let AB = 1.5 m be the height of the slide.

The slide AC is inclined at an angle 30° to the ground.



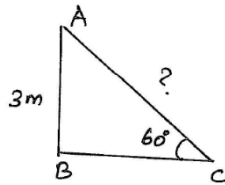
In right angled ΔABC , $\sin 30^\circ = \frac{AB}{AC}$

$$\frac{1}{2} = \frac{1.5}{AC} \Rightarrow AC = 2(1.5) = 3 \text{ m}$$

Case (ii) For elder children

Let AB = 3m be the height of the slide.

Slide AC is inclined with the ground is 60°.



In right angled ΔABC , $\sin 60^\circ = \frac{AB}{AC}$

$$\frac{\sqrt{3}}{2} = \frac{3}{AC} \Rightarrow \sqrt{3}AC = 6$$

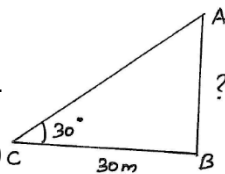
$$AC = \frac{6}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ m}$$

- 4) The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

Solution:

Let AB be the height of the tower.

From the point C, the angle of elevation of top of the tower is 30° BC = 30 m.



In right angled triangle ABC, $\tan 30^\circ = \frac{AB}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{30}$$

Therefore $AB = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$

Hence, Height of the tower AB = $10\sqrt{3} \text{ m}$.

- 5) A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution: Let AB = 60 m be the height of the kite.

AC be the length of string.

In right angled triangle ABC

$$\sin 60^\circ = \frac{AB}{AC}$$

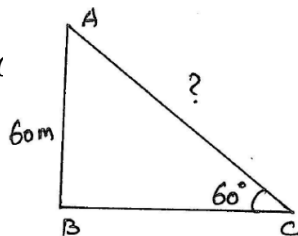
$$\frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$AC\sqrt{3} = 120$$

$$AC = \frac{120}{\sqrt{3}} = \frac{120\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{120\sqrt{3}}{3}$$

$$= 40\sqrt{3} \text{ m}$$

Hence length of the string AC = $40\sqrt{3} \text{ m}$.



- 6) A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution:

Let AB = 30 m be the height of the building.

Height of the boy CD = 1.5 m

CD = BE = 1.5

$$\text{Now, } AE = AB - BE = 30 - 1.5 = 28.5$$

Angle of elevation from D to the top of the building is 30°.

He walks $x \text{ m}$ towards the building and the angle of elevation from F to the top of the building is 60°.

In right angled triangle AED, $\tan 30^\circ = \frac{AE}{DE}$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{DF+FE} = \frac{28.5}{x+y}$$

$$x + y = 28.5\sqrt{3} \text{ ————— (01)}$$

In right angled triangle AEF, $\tan 60^\circ = \frac{AE}{EF}$

$$\sqrt{3} = \frac{28.5}{y}$$

$$y = \frac{28.5}{\sqrt{3}} \text{ m ————— (02)}$$

substitute 2 in 1

$$x + \frac{28.5}{\sqrt{3}} = 28.5\sqrt{3}$$

$$x = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} = 28.5 \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right)$$

$$= 28.5 \left(\frac{3-1}{\sqrt{3}} \right) = \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{57.0\sqrt{3}}{3}$$

$$x = 19\sqrt{3} \text{ m}$$

So, the distance he walked towards the building $x = 19\sqrt{3} \text{ m}$.

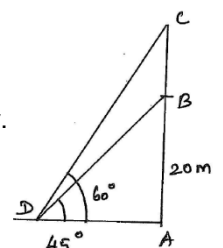
- 7) From a point on the ground, the angle of elevation of the bottom and the top of a transmission tower fixed at the top of a 20m high building are 45° and 60° respectively. Find the height of the tower. (2020)

Solution:

Let AB = 20 m be the height of the building.

Let BC be the height of the tower.

Angle of elevation from D to the bottom of tower is 45° and top of tower is 60°.



In right angled triangle ABD , $\tan 45^\circ = \frac{AB}{AD}$

$$1 = \frac{20}{AD} \Rightarrow AD = 20 \text{ m}$$

In right angled triangle ACD , $\tan 60^\circ = \frac{AC}{AD}$

$$\sqrt{3} = \frac{AB+BC}{20} \quad (\because AC = AB + BC)$$

$$20\sqrt{3} = AB + BC$$

$$20\sqrt{3} = 20 + BC$$

$$\begin{aligned} \text{Therefore } BC &= 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) \\ &= 20(1.732 - 1) = 20(0.732) \\ &= 14.640 = 14.64 \text{ m} \end{aligned}$$

Hence height of the tower $BC = 14.64 \text{ m}$.

- 8) A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of state is 60° and from the same point the angle of elevation of the top of pedestal is 45° . Find the height of pedestal.

Solution:

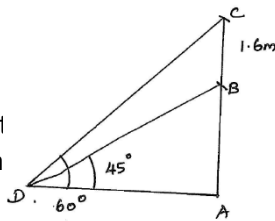
Let $BC = 1.6 \text{ m}$ be the height of the statue.

Let AB be the height of pedestal

The point C is top of statue and B is top of pedestal.

Angle of elevation from D to top of statue is 60°

and top of pedestal is 45° .



In right angled triangle $\triangle BAD$, $\tan 45^\circ = \frac{AB}{AD}$

$$\begin{aligned} 1 &= \frac{AB}{AD} \\ \Rightarrow AB &= AD \quad \text{--- (01)} \end{aligned}$$

In right angled triangle CAD , $\tan 60^\circ = \frac{AC}{AD}$

$$\sqrt{3} = \frac{AB+BC}{AD} \quad (\because AC = AB + BC)$$

$$AD\sqrt{3} = AB + 1.6 \quad (\because BC = 1.6)$$

$$AD\sqrt{3} = AD + 1.6 \quad (\because AB = AD)$$

$$AD\sqrt{3} - AD = 1.6$$

$$AD(\sqrt{3} - 1) = 1.6$$

$$AD = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$AD = \frac{1.6(1.732+1)}{3-1} = \frac{1.6(1.732+1)}{2} = 0.8 \times 2.732$$

$$AD = 2.185 \text{ m}$$

Therefore Height of the pedestal
 $AB = AD = 2.19 \text{ m}$.

- 9) The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

Solution: Let AB be the building and let $CD = 50 \text{ m}$ be the height of tower.

Angle of elevation of top of building from foot of tower is

30° and the angle of elevation of top of tower from foot of building is 60° .

To find height of building AB .

In right angled triangle ABC , $\tan 30^\circ = \frac{AB}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC} \Rightarrow AB = \frac{BC}{\sqrt{3}} \quad \text{--- (01)}$$

In right angled triangle BCD , $\tan 60^\circ = \frac{CD}{BC}$

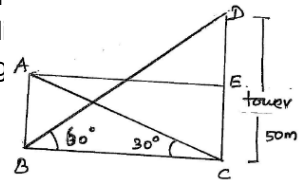
$$\sqrt{3} = \frac{50}{BC} \Rightarrow BC = \frac{50}{\sqrt{3}} \quad \text{--- (02)}$$

substitute 2 in 1

$$AB = \frac{50}{\sqrt{3}} = \frac{50}{\sqrt{3}\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore Height of the building $\frac{50}{3} =$

$16\frac{2}{3} \text{ m}$.

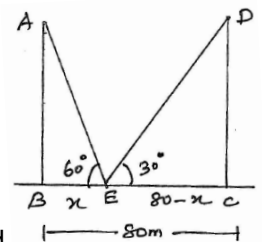


- 10) Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angle of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the poles.

Solution:

Let AB and CD be the poles of equal height. ($AB = CD$).

Distance between two poles $BC = 80 \text{ m}$.



Let E be the point on the road which is $x \text{ m}$ distance from the 1st pole and $y = (80 - x) \text{ m}$ distance from the 2nd pole.

Angle of elevation from E to top of poles are 60° and 30° respectively.

In right angled triangle ABE , $\tan 60^\circ = \frac{AB}{BE}$

$$\sqrt{3} = \frac{AB}{x} \Rightarrow AB = \sqrt{3}x \quad \text{--- (01)}$$

In right angled triangle ECD , $\tan 30^\circ = \frac{CD}{EC}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{80-x} \quad (\because AB = CD)$$

$$AB = \frac{80-x}{\sqrt{3}} \quad \text{--- (02)}$$

From 1 and 2

$$x\sqrt{3} = \frac{80-x}{\sqrt{3}}$$

$$\Rightarrow x\sqrt{3}\sqrt{3} = 80-x$$

$$3x = 80-x \Rightarrow 3x+x = 80$$

$$4x = 80$$

$$x = \frac{80}{4} = 20 \text{ m}$$

Now,

$$y = 80-x = 80-20 = 60 \text{ m}$$

Therefore Height of the poles $AB = \sqrt{3}x = 20\sqrt{3} \text{ m}$

Distances of the point E from the poles are $BE = 20 \text{ m}$ and $EC = 60 \text{ m}$.

11) A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.

Solution:

Let AB be the tower and BC be the width of the canal.

Angle of elevation from the point C to the top of the tower is 60° and from D is 30° $CD = 20 \text{ m}$ (given).

In right angled triangle ABC , $\tan 60^\circ = \frac{AB}{BC}$

$$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB = BC\sqrt{3} \quad \text{--- (01)}$$

In right angled triangle ABD , $\tan 30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC+DC}$$

$$AB\sqrt{3} = BC + DC$$

$$AB\sqrt{3} = BC + 20 \quad \text{--- (02)}$$

$$BC\sqrt{3}\sqrt{3} = BC + 20 \quad [AB = BC\sqrt{3}]$$

$$3BC - BC = 20$$

$$2BC = 20$$

Therefore $BC = 10 \text{ m}$

substitute $BC = 10$ in 1

Therefore $AB = 10\sqrt{3} \text{ units}$

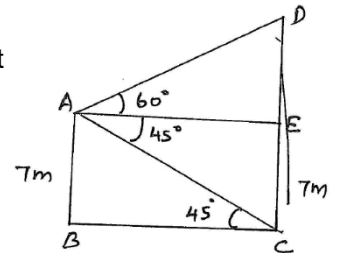
Hence Height of the tower is $AB = 10\sqrt{3} \text{ units}$ and width of the canal $BC = 10 \text{ m}$.

12) From of the top of a 7 m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower. (2020)

Solution:

Let AB = 7 m be the height of the building and CD be the height of the tower.

From the top of building A, the angle of elevation of top and bottom of the tower are 60° and 45° .



Draw $BC \parallel AE$. $AE = BC$ and $AB = BC = 7 \text{ m}$

In right angled triangle $\triangle DEA$,

$$\tan 60^\circ = \frac{DE}{AE}$$

$$\sqrt{3} = \frac{DE}{AE}$$

$$DE = AE\sqrt{3} \text{ m} \quad \text{--- (01)}$$

In right angled triangle $\triangle AEC$, or $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} \Rightarrow AB = BC.$$

$$\therefore BC = 7 \quad (\because AB = 7).$$

But, $BC = AE$, Hence $AE = 7 \text{ m}$. [$AB = BC = 7$]

Now, from 1 $DE = AE\sqrt{3} \text{ m} = 7\sqrt{3} \text{ m}$

Hence, Height of the tower $CD = CE + ED$

$$= 7 + 7\sqrt{3} = 7(1 + \sqrt{3})$$

$$= 7(1 + 1.732) = 7(2.732)$$

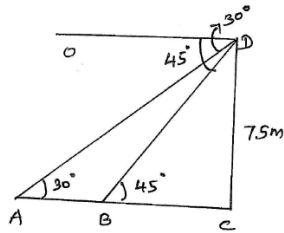
$$CD = 19.124 \text{ m}$$

- 13) As observed from the top of a 75 m high lighthouse from the sea level, the angle of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse then find the distance between two ships.

Solution:

Let $CD = 75$ m be the height of the lighthouse.

Let A and B be the position of ships.



From the point D, the top of lighthouse the angle of depression of two ships A and B are 30° and 45° respectively.

$\angle ODA = \angle DAC = 30^\circ$ (alternative angles)

$\angle ODB = \angle DBC = 45^\circ$ (alternative angles)

In right angled triangle BCD,

$$\tan 45^\circ = \frac{CD}{BC}$$

$$1 = \frac{75}{BC} \Rightarrow BC = 75 \text{ m} \quad \text{--- (1)}$$

In right angled triangle ACD,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{AB + BC}$$

$$AB + BC = 75\sqrt{3}$$

$$AB + 75 = 75\sqrt{3} \quad \text{[since } BC = 75 \text{ from 1]}$$

$$AB = 75\sqrt{3} - 75$$

$$AB = 75(\sqrt{3} - 1) = 75(1.732 - 1) \\ = 75 \times 0.732 = 54.9 \text{ m}$$

Hence distance between two ships 54.9 m.

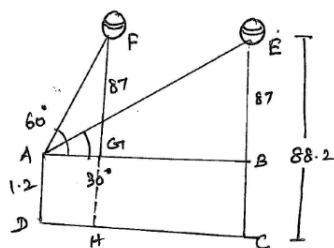
- 14) A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval. (2009).

Solution:

Let $AD = 1.2$ m be the height of girl standing on the ground DC.

Let $CE = HF = 88.2$ m be the height of balloon.

Now $AD = BC = 1.2$ m.



$$BE = CE - CB = 88.2 - 1.2 = 87 \text{ m.}$$

$$BE = GF = 87 \text{ m.}$$

In right angled triangle AGF,

$$\tan 60^\circ = \frac{FG}{AG}$$

$$\sqrt{3} = \frac{87}{AG} \Rightarrow AG = \frac{87}{\sqrt{3}}$$

$$DH = \frac{87}{\sqrt{3}} \quad (AG = DH)$$

In right angled triangle ABE,

$$\tan 30^\circ = \frac{BE}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{87}{AB} \Rightarrow AB = 87\sqrt{3} \text{ m.}$$

Distance travelled by the balloon is $GB = HC$

$$= AB - AG = 87\sqrt{3} - \frac{87}{\sqrt{3}} = 87 \left[\sqrt{3} - \frac{1}{\sqrt{3}} \right]$$

$$= 87 \left[\frac{3-1}{\sqrt{3}} \right] \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87 \times 2\sqrt{3}}{3} = 29 \times 2\sqrt{3}$$

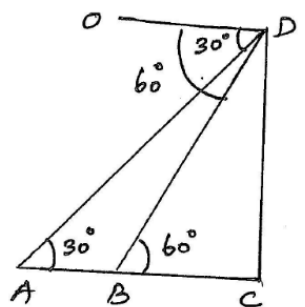
$$= 58\sqrt{3} \text{ m.}$$

- 15) A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed six seconds later the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point. (2008)

Solution:

Let CD be the height of the tower. At D , the angle of depression of the car A is 30° and at B is 60° .

Time taken by the car from A to B is 6 seconds.



$\angle ODA = \angle DAC = 30^\circ$ (alternative angles)

$\angle ODB = \angle DBC = 60^\circ$ (alternative angles)

In the right angled triangle ACD

$$\tan 30^\circ = \frac{CD}{AC} \Rightarrow CD = AC \tan 30^\circ$$

$$= \frac{AC}{\sqrt{3}} \quad \text{--- (1)}$$

In the right angled triangle BCD

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{CD}{BC} \Rightarrow CD = BC\sqrt{3}$$

$$CD = BC\sqrt{3} \text{ ————— (02)}$$

From 1 and 2 $\frac{AC}{\sqrt{3}} = BC\sqrt{3}$

$$\frac{AB+BC}{\sqrt{3}} = BC\sqrt{3} \quad [AC = AB + BC]$$

$$AB + BC = 3 BC$$

$$AB = 3 BC - BC = 2 BC$$

Let the speed of the car be k Km/sec.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{2 BC}{k}$$

$$6 k = \frac{2 BC}{k} \Rightarrow 6k = 2BC$$

Therefore $BC = \frac{6k}{2}$

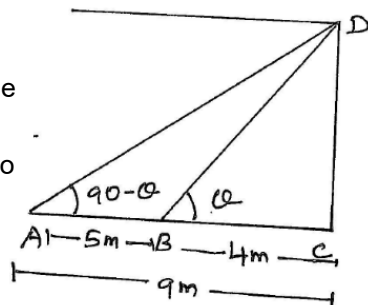
$$BC = 3 k$$

Hence the car moves from B to C in 3 sec.

- 16) The angle of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution:

Let CD be the height of the tower. AC be the horizontal line on the ground. A and B be two point on AC such that



AC = 9 cm and BC = 4 cm.

Let $\angle CBD = \theta$, $\angle CAD = (90 - \theta)$

(Since they are complementary)

In right angled triangle $\triangle BCD$, $\tan \theta = \frac{CD}{BC}$

$$\tan \theta = \frac{CD}{4} \Rightarrow CD = 4 \tan \theta \text{ ————— (01)}$$

In $\triangle ACD$, $\tan(90 - \theta) = \frac{CD}{AC}$

$$\cot \theta = \frac{CD}{9} \Rightarrow CD = 9 \cot \theta \text{ ————— (02)}$$

from 1 and 2

$$1 \times 2 \Rightarrow CD^2 = 4 \tan \theta \times 9 \cot \theta$$

$$CD^2 = 36$$

$$\Rightarrow CD = \sqrt{36} = 6$$

Hence the height of the tower is 6 m.

10 CIRCLES

Circle: A circle is a collection of all points in a plane which are a constant distance (radius) from a fixed point (centre).

CHORD: A line segment joining any two points on the circumference of the circle is called a chord.

Note: Longest chord of the circle is diameter.

Semi Circle: A diameter of a circle divides it into two equal parts or in two equal arcs. Each of these two arcs is called a semi-circle.

Circumference: The length of the complete circle is called the circumference of the circle.

Sector: The region between an arc and the two radii, joining the end of the arc to the centre is called sector.

Important Results:

- 1) The perpendicular drawn from the centre of a circle to a chord bisects it and vice versa.
- 2) Equal chords of a circle are equidistant from the centre.
- 3) The angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any point on the remaining part of the circle.
- 4) Equal chords of a circle subtend equal angles at the centre.
- 5) The angles in the semicircle is a right angle.
- 6) Angles in the same segment of a circle are equal.
- 7) The sum of any pair of opposite angles of a cyclic quadrilateral is 180° .

Tangent: A tangent to a circle is a line that intersects the circle at only one point.

Theorem: The tangent at any point of a circle is perpendicular to the radius through the point of contact [CBSE 2020, 13, 12, 11].

Given:

A circle with centre at O and a tangent AB at the point P on the circle.

To prove:

$OP \perp_r$ to AB

Construction:

Take any point Q on AB other than P.

Join OQ (Q lies outside circle).

Proof:

Let OQ intersect the circle at R

Therefore $OP = OR$ (radii)

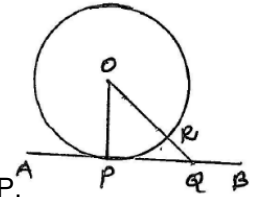
Now, $OQ = OR + RQ \Rightarrow OQ > OR$

$\Rightarrow OQ > OP$ or $OP < OQ$

OP is shorter than any other segment joining O to any point on AB.

We know that the shortest between a point and a line is perpendicular distance from the point to the line.

So, OP is \perp_r to AB and $OP \perp_r AB$



EXERCISE 10.1

- 1) How many tangents can a circle have?

Ans: Infinite.

- 2) Fill in the blanks:

- a) A tangent to a circle intersects it one point.
- b) A line intersecting a circle in two points is called secant.
- c) A circle can have two parallel tangents at most.
- d) The common point of a tangent to a circle and the circle is called point of contact.

- 3) A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. Length of PQ is

a) 12 cm b) 13 cm c) 8.5 cm d) $\sqrt{119}$ cm

Note:

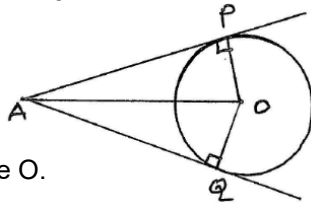
- 1) There is no tangent to a circle passing through a point lying inside the circle.
- 2) There is only one tangent to a circle passing through a point lying on the circle.
- 3) There are exactly two tangents to a circle through a point lying outside the circle.

- 4)

Theorem: The length of tangents drawn from an external point to a circle are equal.

Solution:

Let AP and AQ are two tangents from an external point A to a circle with centre O.



To prove: $AP = AQ$

Construction: Join OP, OQ.

Proof: We know that, a tangent at any point of a circle is perpendicular to the radius through the point of contact.

Here AP is tangent and OP is radius.

Therefore $OP \perp AP$

similarly $OQ \perp AQ$

Case 1: In $\triangle OAP$ and $\triangle OAQ$

$OP = OQ = \text{radius}$

OA is common and $\angle OPA = \angle OQA = 90^\circ$

Therefore $\triangle OAP \cong \triangle OAQ \Rightarrow AP = AQ$.

Case 2: In $\triangle OAP$ and $\triangle OAQ$

In $\triangle OAP, OA^2 = OP^2 + AP^2 \Rightarrow r^2 + AP^2$ — (01)

In $\triangle OAQ, OA^2 = OQ^2 + AQ^2 \Rightarrow r^2 + AQ^2$ — (02)

From 1 and 2 $r^2 + AP^2 = r^2 + AQ^2$

$\Rightarrow AP^2 = AQ^2$

$\Rightarrow AP = AQ$

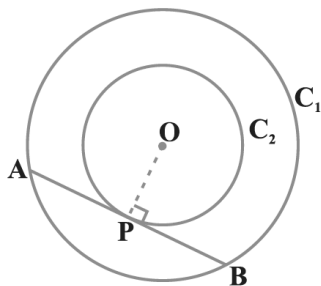
Hence proved.

Example 1: Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact. (2019, 2009)

Solution:

Let C_1 and C_2 be two given concentric circles with common centre O.

Let AB be the chord of the larger circle C_1 touching the smaller circle C_2 at P.



To prove: $PA = PB$

Construction: Join OP.

Proof: Chord AB touches the smaller circle at P.

Therefore AB is tangent at P.

OP is radius of the smaller circle C_2 .

We know that, $OP \perp AB$

Also, we know that, perpendicular from centre to the chord bisects the chord.

Here

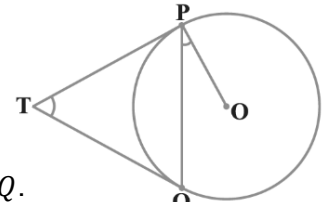
$OP \perp AB$ at P.

Therefore $PA = PB$

Hence proved.

Example 2: Two tangent TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

Given: TP and QT are two tangents to the circle from an external point T.



To prove: $\angle PTQ = 2\angle OPQ$.

Proof: $\angle PTQ + \angle POQ = 180^\circ$ — (01)

[In a Quadrilateral OPTQ, opposite angles are supplementary]

In $\triangle OPQ, \angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$2\angle OPQ + \angle POQ = 180^\circ$ — (02)

[$\angle OPQ = \angle OQP$ and $OP = OQ$

Therefore $\triangle OPA$ is isosceles]

From 1 and 2

$\angle PTQ + \angle POQ = 2\angle OPQ + \angle POQ$

$\Rightarrow \angle PTQ = 2\angle OPQ$

Hence proved.

Example 3: PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

Given: $OP = 5$ cm, $PQ = 8$ cm.

To find: Length of TP

Solution:

In $\triangle PTR$ and $\triangle QTR$

$PT = QR$ — (01)

(length of two tangents are equal from an external point)

Here OT is bisector of $\angle PTQ$

Therefore $\angle PTO = \angle QTO$ — (02)

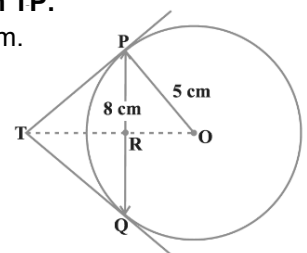
$\angle PRT = \angle QRT = 90^\circ$ — (03)

OT is $\perp r$ bisector of $PQ \Rightarrow PR = RQ = 4$ cm

In right angled $\triangle PRO$,

$OP^2 = PR^2 + OR^2$

$5^2 = 4^2 + OR^2$



Therefore $OR^2 = 25 - 16 = 9 \Rightarrow OR = \sqrt{9} = 3\text{cm}$

In right angled $\Delta^l ORP$,

$\angle RPO + \angle POR = 90^\circ$ ——— (04)

In right angled $\Delta^l OPT$,

$\angle POT + \angle OTP = 90^\circ$ ——— (05)

From 4 and 5

$\angle RPO + \angle POR = \angle POT + \angle OTP$

$\Rightarrow \angle RPO = \angle OTP$ ——— (06)

[$\therefore \angle TRP = \angle PRO = 90^\circ$ and $\angle RPO = \angle OTP$]

In ΔORP and ΔPRT

$\Delta ORP \sim \Delta PRT$ (BY AA)

$\Rightarrow \frac{TP}{OP} = \frac{PR}{OR} \Rightarrow \frac{TP}{5} = \frac{4}{3}$

$TP = \frac{5 \times 4}{3} = \frac{20}{3}$

Therefore Length of $PT = \frac{20}{3}$ unit.

EXERCISE 10.2

- 1) From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
 a) 7 cm b) 12 cm c) 15 cm

Given:

$PQ = 24\text{ cm}$

$OQ = 25\text{ cm}$

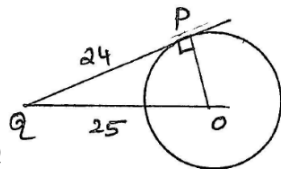
In right angled triangle OPQ

$OQ^2 = OP^2 + PQ^2$

$25^2 = OP^2 + 24^2 \Rightarrow 625 - 576 = OP^2$

$49 = OP^2 \Rightarrow OP = \sqrt{49} = 7\text{ cm}$

Hence radius $r = OP = 7\text{ cm}$.

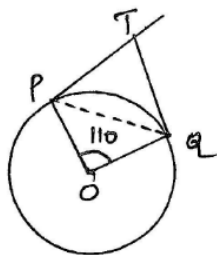


- 2) In the fig if TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^\circ$ then $\angle PTQ$ is equal to

- a) 60° b) 70° c) 80° d) 90°

Solution:

In ΔOPQ , $OP = OQ$ (radius)



Therefore, $\angle OPQ = \angle OQP = x$

In ΔOPQ $\angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$x + x + 110 = 180^\circ$

$2x = 180^\circ - 110^\circ$

$2x = 70^\circ \Rightarrow x = \frac{70}{2} = 35^\circ$

Therefore $\angle OPQ = \angle OQP = 35^\circ$

Now, $\angle OPT = 90^\circ$ (Since radius is \perp to tangent)

$\angle OPQ + \angle QPT = 90^\circ$

$35 + \angle QPT = 90^\circ$

Therefore $\angle QPT = 90 - 35 = 55^\circ$

Therefore $\angle PQT = 55^\circ$ [Since, $PT = QT$]

In ΔPTQ , $\angle PTQ + \angle QPT + \angle PQT = 180^\circ$

$\angle PTQ + 55 + 55 = 180$

$\angle PTQ = 180 - 110 = 70^\circ$

Therefore $\angle PTQ = 70^\circ$

- 3) If PA and PB from a point P to a circle with centre O are inclined to each other at of 80° , then $\angle POA$ is equal to (CBSE 2020)

- a) 50° b) 60° c) 70° d) 80°

Given: PA and PB are two tangents drawn from a point P, and $\angle APB = 80^\circ$

Join OA, OB, OP.

Clearly $OA \perp r PA$.

Similarly $OB \perp r PB$.

In Quadrilateral, OAPB.

$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$

$90 + 80 + 90 + \angle AOB = 360$

$\angle AOB = 360 - 260 = 100^\circ$

We know that tangents drawn from an external point, subtend equal angle at the centre.

Now $\angle POA = \angle POB$

$\Rightarrow \angle POA = \frac{1}{2} \angle BOA$

$= \frac{1}{2} \times 100 = 50^\circ$

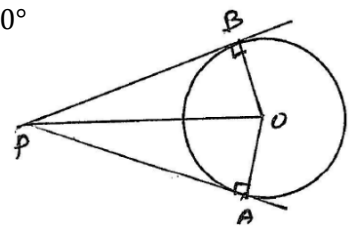
Therefore $\angle POA = 50^\circ$

- 4) Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:

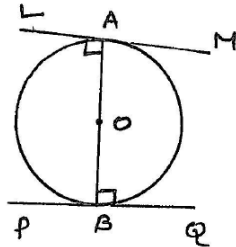
Let AB be the diameter of the circle.

LM, PQ are two tangents drawn to the circle at A and B respectively.



To prove: $LM \parallel PQ$

Proof: We know that, the tangent at any point of a circle is $\perp r$ to the radius through the point of contact.



Therefore $OA \perp r$ to LM and $OB \perp r$ to PQ

$\Rightarrow \angle LAO = 90^\circ$ and $\angle LPBO = 90^\circ$

But $\angle LAO$ and $\angle LPBO$ are alternate angles

Therefore $PQ \parallel LM$.

Hence proved.

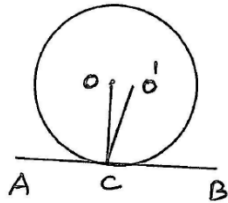
5) Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution:

Let AB be the tangent to the circle at C with centre O .

To prove: Perpendicular at C passes through the centre O .

If possible, the perpendicular passes through some other point say O' .



Construction: Join OC and OC'

Proof: We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Therefore $OC \perp AB \Rightarrow \angle LOCA = 90^\circ$

Suppose $OC' \perp AB \Rightarrow \angle LO'CA = 90^\circ$

$\Rightarrow \angle LOCA = \angle LO'CA$

Which is possible only O and O' coincide.

Hence proved.

6) The length of a tangent from a point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Solution: Let AB be the tangent to the circle at B .

$AB = 4$ cm, $OA = 5$ cm.

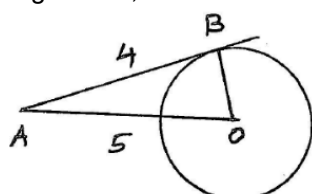
We know that $OB \perp r$ AB .

Therefore In right angled triangle OAB ,

$$OA^2 = OB^2 + AB^2$$

$$5^2 = OB^2 + 4^2$$

$$25 = OB^2 + 16$$



Therefore $OB^2 = 25 - 16 = 9$

Therefore $OB = \sqrt{9} = 3$ cm.

Therefore radius of the circle $OB = 3$ cm.

7) Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution: Let C_1 and C_2 be two circles of radii 3 cm and 5 cm respectively with centre O .

Let AB be the chord of the circle C_2 which touches the smaller circle C_1 at P .

We know that $OP \perp r$ AB .

$$\Rightarrow AP = PB.$$

In right angled triangle OPB

$$OB^2 = OP^2 + PB^2$$

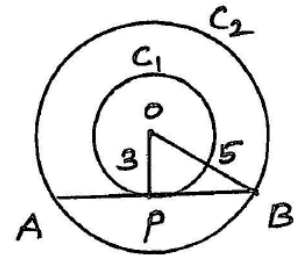
$$5^2 = 3^2 + PB^2$$

$$25 - 9 = PB^2 \Rightarrow PB^2 = 16$$

$$\Rightarrow PB = \sqrt{16} = 4$$

Therefore length of the chord $AB = 2 PB$

$$= 2(4) = 8 \text{ cm}$$



8) A Quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$

2016)

Solution: Given $ABCD$ is a quadrilateral circumscribing a circle.

To prove: $AB + CD = AD + BC$

Proof:

We know that, the length of two tangents are equal from an external point.

Therefore $AP = AS$ ——— (01)

similarly $BP = BQ$ ——— (02)

$RC = CQ$ ——— (03)

$DR = DS$ ——— (04)

Adding 1, 2, 3, 4

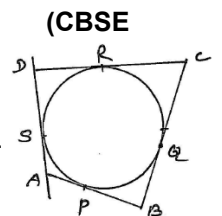
$$AP + BP + RC + DR = AS + BQ + CQ + DS$$

$$(AP + PB) + (RC + DR)$$

$$= (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC$$

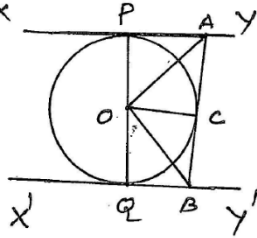
Hence proved.



9) In the figure XY and X^1Y^1 are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X^1Y^1 at B . Prove that $\angle AOB = 90^\circ$ (CBSE 2013, 12, 11)

Solution: XY and X^1Y^1 are two parallel tangents.

Another tangent AB touch the circle at C , intersect XY at A and X^1Y^1 at B .



To prove: $\angle AOB = 90^\circ$

Proof: We know that, tangents drawn from an external point to a circle are equal in length. Therefore $AP = AC$.

In $\triangle OAP$ and $\triangle OAC$

$AP = AC$, OA is common, $OP = OC$ (radius)

Therefore $\triangle OAP \cong \triangle OAC$

$$\Rightarrow \angle OAP = \angle OAC$$

$$\text{Now, } \angle PAC = 2\angle OAC \text{ ————— (01)}$$

$$\text{similarly } \angle QBC = 2\angle OBC \text{ ————— (02)}$$

$$\text{Now } \angle PAC + \angle QBC = 180^\circ \text{ ————— (03)}$$

[$XY \parallel X^1Y^1$ and AB is a transversal, sum of interior angles on the same side of transversal is 180°]

$$\text{From 3 } 2\angle OAC + 2\angle OBC = 180^\circ$$

$$\text{Therefore } 2[\angle OAC + \angle OBC] = 180$$

$$\angle OAC + \angle OBC = \frac{180}{2} = 90 \text{ ————— (04)}$$

In $\triangle AOB$,

$$\angle AOB + \angle OAC + \angle OBC = 180^\circ$$

$$\angle AOB + 90 = 180^\circ \text{ (from 4)}$$

$$\angle AOB = 180 - 90$$

$$\angle AOB = 90^\circ$$

Hence proved.

10) Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Solution: Let PQ and PR be two tangents drawn from an external point P to the circle with centre O .

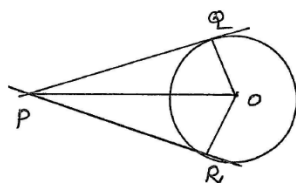
To prove: $\angle QPR + \angle QOR = 180^\circ$

Proof: In $\triangle POQ$ and $\triangle POR$

$$PQ = PR$$

OP is common

$$OQ = OR \text{ (radius)}$$



Therefore $\triangle POQ \cong \triangle POR$ (By SSS congruence)

In $\triangle POQ$,

$$\angle POQ + \angle PQQ + \angle OPQ = 180^\circ \text{ [}\angle PQQ = 90^\circ\text{]}$$

$$\therefore \angle POQ + \angle OPQ = 180^\circ - 90^\circ = 90^\circ \text{ ————— (01)}$$

In $\triangle POR$, $\angle POR + \angle PRO + \angle OPR = 180^\circ$

[But $\angle PRO = 90^\circ$]

$$\angle POR + 90 + \angle OPR = 180^\circ$$

$$\therefore \angle POR + \angle OPR = 180^\circ - 90^\circ = 90^\circ \text{ ————— (02)}$$

$$1 + 2$$

$$\Rightarrow \angle POQ + \angle OPQ + \angle POR + \angle OPR = 90 + 90$$

$$(\angle POQ + \angle POR) + (\angle OPQ + \angle OPR) = 180^\circ$$

$$\angle QOR + \angle QPR = 180^\circ$$

Hence proved.

11) Prove that the parallelogram circumscribing a circle is rhombus.

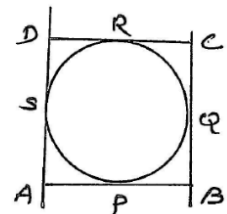
Solution:

Let $ABCD$ be a parallelogram circumscribing a circle.

To prove: $ABCD$ is a rhombus. to prove, $AB = BC = CD = AD$.

Proof:

We know that, length of tangents are equal from an external point.



$$AP = AS$$

$$BP = BQ$$

$$CR = CQ$$

$$DR = DS$$

$$\text{Adding, } AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC \text{ ————— (01)}$$

But $ABCD$ is a parallelogram

$$\text{Therefore } AB = CD \text{ and } BC = AD. \text{ ————— (02)}$$

$$\text{Therefore } 1 \Rightarrow 2AB = 2BC$$

$$\Rightarrow AB = BC \text{ ————— (03)}$$

$$\text{From 2 and 3 } AB = BC = CD = AD.$$

\Rightarrow **$ABCD$ is a rhombus.**

12) A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of length 8 cm and 6 cm respectively. Find the sides AB and AC.

Solution:

Given OD = radius = 4 cm, DC = 6 cm, DB = 8 cm.

Join OA, OB, OC,

We know that length of two tangents equal from an external point.

$$CD = CE = 6 \text{ cm}$$

$$BD = BE = 8 \text{ cm}$$

$$AE = AF = x \text{ cm}$$

Therefore sides of ΔABC are

$$AC = AE + CE = x + 6 = b$$

$$AB = AF + FB = x + 8 = c$$

$$BC = BD + DC = 8 + 6 = 14 \text{ cm} = a$$

$$S = \frac{1}{2}[AC + AB + BC]$$

$$= \frac{1}{2}[x + 6 + x + 8 + 14] = \frac{1}{2}[2x + 28]$$

$$= \frac{1}{2} \times 2[x + 14] = x + 14$$

$$S - a = x + 14 - 14 = x$$

$$S - b = x + 14 - (x + 6) = x + 14 - x - 6 = 8$$

$$S - c = x + 14 - (x + 8) = x + 14 - x - 8 = 6$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+14)x \times 8 \times 6}$$

$$= \sqrt{(x+14)x \times 48} \text{ ————— (01)}$$

But Area of $\Delta ABC = \text{Area of } \Delta OBC +$

Area of $\Delta OBA + \text{Area of } \Delta OAC$

$$= \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OF + \frac{1}{2} \times AC \times OE$$

$$= \frac{1}{2} \times 14 \times 4 + \frac{1}{2} (x+8) \times 4 + \frac{1}{2} (x+6) \times 4$$

$$= 28 + 2(x+8) + 2(x+6)$$

$$= 28 + 2x + 16 + 2x + 12$$

$$= 56 + 4x = 4(x+14) \text{ ————— (02)}$$

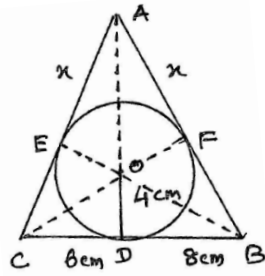
From 1 and 2

$$\sqrt{(x+14)x \times 48} = 4(x+14)$$

squaring on both sides

$$48x(x+14) = 16(x+14)^2$$

$$48x = 16(x+14)$$



$$\frac{48}{16}x = (x+14)$$

$$3x = x + 14$$

$$2x = 14$$

$$x = \frac{14}{2} = 7$$

Hence the sides of ΔABC are

$$AB = x + 8 = 7 + 8 = 15 \text{ cm}$$

$$AC = x + 6 = 7 + 6 = 13 \text{ cm}$$

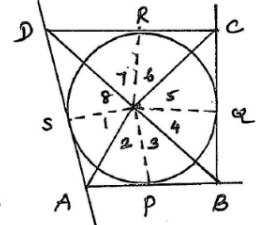
$$BC = 14 \text{ cm}$$

Hence $AB = 15 \text{ cm}$, $AC = 13 \text{ cm}$, $BC = 14 \text{ cm}$

13) Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

Let ABCD is a quadrilateral circumscribing a circle with centre O.



Let the circle touches the sides of a quadrilateral at points P, Q, R, S.

To prove: $\angle AOB + \angle COD = 180^\circ$ and

$$\angle BOC + \angle AOD = 180^\circ$$

Construction: Join OP, OQ, OR, OS

Proof:

We know that, two tangents from an external point to a circle subtend equal angles at the centre.

$$\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$$

Therefore

$$L1 + L2 + L3 + L4 + L5 + L6 + L7 + L8 = 360^\circ \text{ ————— (01)}$$

$$\Rightarrow 2(L2 + L3 + L6 + L7) = 360^\circ$$

$$(L2 + L3) + (L6 + L7) = 180^\circ$$

$$\angle AOB + \angle COD = 180^\circ \text{ ————— (02)}$$

similarly from 1

$$2(L1 + L8 + L4 + L5) = 360^\circ$$

$$(L1 + L8) + (L4 + L5) = 180^\circ$$

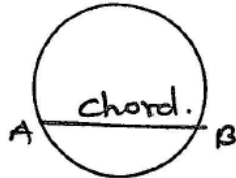
$$\angle AOD + \angle BOC = 180^\circ$$

Hence proved from 2 and 3.

12 AREAS RELATED TO CIRCLES

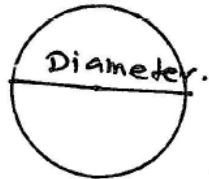
a) Chord:

A line segment joining any two points on a circle is called a chord of the circle.



Diameter:

A chord of a circle passing through the centre is called a diameter of the circle.



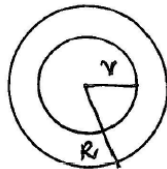
$Diameter = 2 \times radius$

Perimeter of a circle or circumference of a circle:

$Circumference\ of\ circle = 2\pi r.$

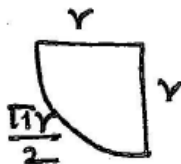
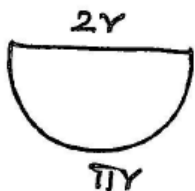
$Area\ of\ circle = \pi r^2$

$Area\ of\ Ring = \pi(R^2 - r^2)$
 $= \pi(R + r)(R - r)$



Important formula:

- 1) Circumference of semi circle = $(\pi r + 2r)$
- 2) Circumference of quadrant of a circle = $\frac{\pi r}{2} + 2r$
- 3) Distance moved or converted by a wheel in 1 rotation = Circumference of the wheel.
- 4) Number of rotation of the wheel in travelling a certain distance = $\frac{Total\ distance\ travelled\ by\ the\ wheel}{Cencumference\ of\ the\ wheel}$
- 5) Area of semi circle = $\frac{\pi r^2}{2}$
- 6) Area of quadrant of a circle = $\frac{\pi r^2}{4}$.



Example 1: The cost of fencing a circular field at the rate of Rs.24 per meter is Rs.5280. The field is to be ploughed at the rate of Rs.0.50 per m^2 . Find the cost of ploughing the field?

Solution:

Cost of fencing per meter = Rs. 24

Total cost of fencing = Rs. 5280

Therefore Length of fencing = $\frac{5280}{24} = 220\ m.$

i. e $2\pi r = 220$

$2 \times \frac{22}{7} r = 220$

$r = \frac{220 \times 7}{2 \times 22}$

$r = 5 \times 7 = 35\ m.$

Therefore radius of the field = 35 m.

Now, Area of the field = πr^2
 $= \frac{22}{7} \times 35 \times 35$

$= 110 \times 35$

Area of the field = 3850 m^2

Cost of ploughing 1 m^2 of field = Rs. 0.50

Therefore cost of ploughing = 3850×0.50

$= 3850 \times \frac{1}{2}$

Cost of ploughing = Rs. 1925

EXERCISE 12.1

1) The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles. (CBSE 2020).

Solution:

Radius of the 1st circle $r_1 = 19\ cm$

Therefore circumference of 1st circle = $2\pi r_1$
 $= 2\pi(19)$

Radius of the 2nd circle $r_2 = 9\ cm.$

Therefore circumference of 2nd circle = $2\pi r_2$
 $= 2\pi(9)$

Circumference of Required circle having radius R is
 $= 2\pi(19) + 2\pi(9)$

$2\pi R = 2\pi(19 + 9)$

$R = 28\ cm.$

Hence radius of the required circle = 28 cm.

2) The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of two circles.

Solution:

Radius of the 1st circle $r_1 = 8\ cm$

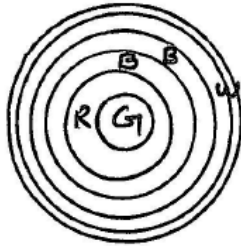
Therefore Area of the 1st circle = $\pi r_1^2 = \pi(8)^2$
 $= 64\pi\ cm^2$

Radius of the 2nd circle $r_2 = 6 \text{ cm}$
 Therefore Area of the 2nd circle $= \pi r_2^2$
 $= \pi(6)^2$
 $= 36\pi \text{ cm}^2$

Now Required Area of circle having radius
 $R = \pi R^2 = 64\pi + 36\pi(\text{Given})$
 $= 100\pi$
 $R^2 = 100 \Rightarrow R = 10 \text{ cm}$

Hence radius of the required circle $R = 10 \text{ cm}$.

3) The given figure depicts an archery target marked with its five scoring region from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other band is 10.5 cm wide. Find the area of each of five scoring regions.



Solution:

Given diameter of gold region = 21 cm.
 Therefore radius of gold region $= \frac{21}{2} = 10.5 \text{ cm}$
 Now, Area of Gold region $= \pi r^2$
 $= \frac{22}{7} \times 10.5 \times 10.5$
 $= 33.0 \times 10.5$
 $= 346.5 \text{ cm}^2$

Width of each of the other band = 10.5
 Therefore Radius for (Gold + Red) region = 10.5 + 10.5 = 21

Now, Area of Red Region $= \pi[R^2 - r^2]$
 $= \frac{22}{7} [21^2 - 10.5^2]$
 $= \frac{22}{7} [(21 + 10.5)(21 - 10.5)]$
 $= \frac{22}{7} \times 31.5 \times 10.5$
 $= 22 \times 1.5 \times 31.5$
 $= 33 \times 31.5$
 $= 1039.5 \text{ cm}^2$

Now Radius (Gold+Red+Blue) for Region
 $= 21 + 10.5 = 31.5$

Therefore Area of blue region
 $= \pi(R^2 - r^2)$
 $= \pi[(31.5)^2 - 21^2]$
 $= \frac{22}{7} [(31.5 + 21)(31.5 - 21)]$
 $= \frac{22}{7} \times 52.5 \times 10.5$
 $= 22 \times 1.5 \times 52.5$

$= 33 \times 52.5$
 $= 1732.5 \text{ cm}^2$
 Now, radius for (Gold+Red+Blue+Black) region =
 $31.5 + 10.5$
 $= 42 \text{ cm}$
 Area of black region $= \pi[R^2 - r^2]$
 $= \frac{22}{7} [42^2 - 31.5^2]$
 $= \frac{22}{7} (42 + 31.5)(42 - 31.5)$
 $= \frac{22}{7} \times 73.5 \times 10.5 = 22 \times 1.5 \times 73.5$
 $= 33 \times 73.5$

Area of black region = 2425.5 cm²
 Radius for (Gold + Red + Blue + Black + White)
 $= 42 + 10.5 = 52.5$

Now Area for white region $= \pi[R^2 - r^2]$
 $= \frac{22}{7} [52.5^2 - 42^2]$
 $= \frac{22}{7} (52.5 + 42)(52.5 - 42)$
 $= \frac{22}{7} \times 94.5 \times 10.5$
 $= 22 \times 1.5 \times 94.5$
 $= 33 \times 94.5$

Area for white region = 3118.5 cm²

4) The wheels of a car of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Solution:

Diameter of the wheel = 80 cm.
 Therefore radius of the wheel $r = \frac{80}{2}$
 $= 40 \text{ cm}$.
 Therefore Circumference of the wheel $= 2\pi r$
 $= 2 \times \frac{22}{7} \times 40 = \frac{44 \times 40}{7}$
 Circumference of the wheel $= \frac{1760}{7} \text{ cm}$.

Now speed of the car = 66 km/hr.
 $= \frac{66 \times 1000}{60} \text{ m/min}$
 $= 1100 \text{ m/min}$

Speed of the car = 11000 cm/min
 Now, wheel of the car moves in 1 min = 110000 cm.
 Therefore wheel of the car moves in 10 min
 $= 110000 \times 10 = 1100000 \text{ cm}$

Then, the number of revolution

$$= \frac{\text{Distance moved by wheel of the car in 10 min}}{\text{Circumference of the wheel}}$$

$$= \frac{1100000}{\frac{1760}{7}} = \frac{1100000}{1760} \times 7 = 625 \times 7 = 4375$$

Hence the wheel makes 4375 complete revolution in 10 min.

- 5) Tick the correct answer in the following and justify your answer: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is (A) 2 units (B) π units (C) 4 units (D) 7 units.

Solution:

$$2\pi r = \pi r^2$$

$$2r = r^2$$

$$2 = r$$

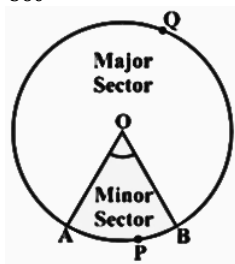
Therefore radius $r = 2$ units.

Ans: (A)

Area of Sector and segment of a circle

- Area of Sector of a circle = $\frac{\theta}{360} \pi r^2$
- Area of major sector = $\pi r^2 -$ Area of minor sector
- Area of minor sector = $\pi r^2 -$ Area of major sector
- Length of arc of a sector $l = \frac{\theta}{360} 2\pi r$
- Also, Area of Sector in

terms of Length of arc = $\frac{lr}{2}$



Example 2: Find the area of the sector of a circle with radius 4 cm and angle of 30° . Also find the area of the corresponding major sector.

Solution:

Given sector is OAPB.

Here $\theta = 30^\circ, r = 4$ cm

$$\text{Area of the sector OAPB} = \frac{\theta}{360} \pi r^2$$

$$= \frac{30}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{88}{21}$$

$$\text{Area of the sector OAPB} = 4.19 \text{ cm}^2$$

Now, Area of major sector = $\pi r^2 -$ Area of minor sector

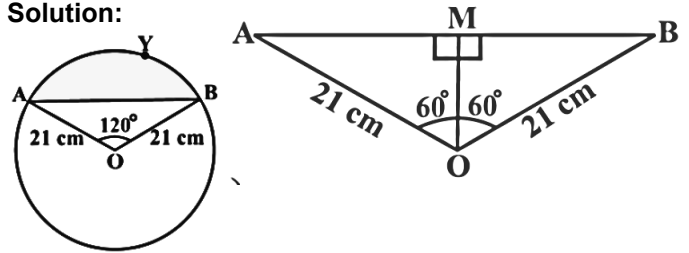
$$= 3.14 \times 4 \times 4 - 4.19$$

$$= 3.14 \times 16 - 4.19 = 50.24 - 4.19$$

$$\text{Area of major sector} = 46.05 \text{ cm}^2$$

Example 3: Find the area of the segment AYB shown in the figure, if radius of the circle is 21 cm and $\angle AOB = 120^\circ$. ($\pi = \frac{22}{7}$).

Solution:



Area of segment $AYB = \text{Area of sector } OAYB - \text{Area of } \Delta OAB$.

$$\text{Now Area of sector } OAYB = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= 22 \times 21 = 462 \text{ cm}^2$$

In $\Delta OAB, OA = OB = 21$ cm. $OM \perp r$ to AB .

Therefore $\Delta OAM \simeq \Delta OMB$.

$\Rightarrow M$ is mid point of AB .

$$\text{In } \Delta OAB, \sin 60^\circ = \frac{AM}{OA}$$

$$\frac{\sqrt{3}}{2} = \frac{AM}{21}$$

$$\text{Therefore } AM = \frac{21\sqrt{3}}{2} \text{ cm}$$

$$\cos 60^\circ = \frac{OM}{OA}$$

$$\frac{1}{2} = \frac{OM}{21}$$

$$OM = \frac{21}{2} \text{ cm}$$

Therefore Area of $\Delta OAB = 2 \times \text{Area of } \Delta OAM$

$$= 2 \times \frac{1}{2} \times AM \times OM$$

$$= \frac{21\sqrt{3}}{2} \times \frac{21}{2} = \frac{441\sqrt{3}}{4}$$

$$\text{Therefore Area of segment } AYB = 462 - \frac{441\sqrt{3}}{4}$$

[From 1]

$$= \frac{1848 - 441\sqrt{3}}{4} \text{ cm}^2$$

$$= \frac{31}{4} (88 - 21\sqrt{3}) \text{ cm}^2$$

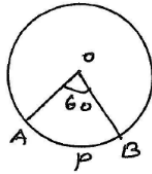
EXERCISE 12.2

- 1) Find the area of a sector of a circle with radius 6 cm if angle of sector is 60° .

Solution:

Radius of the sector = 6 cm.

Central angle $\theta = 60^\circ$



Therefore Area of the sector = $\frac{\theta}{360} \pi r^2$

$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6 = \frac{22 \times 6}{7} = \frac{132}{7}$$

Area of the sector = 18.85 cm^2

- 2) Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution:

Circumference of a circle = 22 cm

$$2\pi r = 22$$

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{ cm}$$

$$\text{Area of the quadrant} = \frac{\pi r^2}{4} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{1}{4}$$

$$= \frac{11 \times 49}{14 \times 4} = \frac{77}{8} \text{ cm}^2$$

- 3) The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:

Radius = length of minute hand

$$= 14 \text{ cm}$$

The angle described by minute hand in 60 min =

$$360^\circ$$

Therefore the angle described by minute hand in 5 min

$$= \frac{360}{60} \times 5 = 30^\circ$$

The area swept by the minute hand in 5 min

= Area of the sector of angle 30° with radius 14 cm.

$$= \frac{\theta}{360} \pi r^2 = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14$$

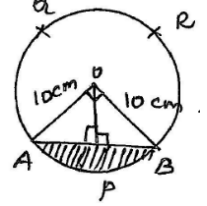
$$= \frac{11 \times 14}{3} = \frac{154}{3} \text{ cm}^2$$

- 4) A chord of a circle of radius 10 cm subtends a right angled at the centre. Find the area of the corresponding. (i) minor segment (ii) major sector.

Solution:

Given, radius of the circle

$r = 10 \text{ cm}$ and $\angle AOB = 90^\circ$



Therefore Area of $\Delta AOB = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 10 \times 10$$

$$= \frac{100}{2} = 50 \text{ cm}^2$$

Area of the sector $OAPBO = \frac{\theta}{360} \times \pi r^2$

$$= \frac{90}{360} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{11 \times 5 \times 10}{7}$$

$$= \frac{550}{7} = 78.57 \text{ cm}^2$$

- (i) Area of minor segment APB =

Area of sector $OAPBO - \text{Area of } \Delta OAB$

$$= 78.57 - 50 = 28.57 \text{ cm}^2$$

- (ii) Area of major sector AQRB = Area of circle -

Area of sector $OAPBO$

$$= \pi r^2 - 78.57 \text{ cm}^2$$

$$= \frac{22}{7} \times 10 \times 10 - 78.57$$

$$= \frac{2200}{7} - 78.57$$

$$= 314.285 - 78.57$$

$$= 314.29 - 78.57$$

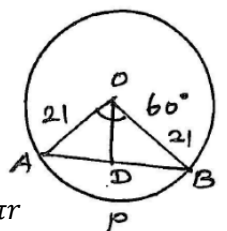
Area of major sector AQRB = 235.72 cm^2

- 5) In a circle of radius 21 cm, an arc subtends on angle of 60° at the centre. Find (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord.

Solution:

radius $r = 21 \text{ cm}$

central angle $\theta = 60^\circ$



- (i) Length of the arc APB = $\frac{\theta}{360} 2\pi r$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$

Length of the arc APB = 22 cm

(ii) Area of the sector, formed by the arc

$$\begin{aligned} (\text{OAPBO}) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 = 11 \times 21 = 231 \text{ cm}^2 \end{aligned}$$

(iii) Given $\angle AOB = 60^\circ$, $OA = OB = \text{radii} = 21 \text{ cm}$

Let $\angle LOAB = \angle LOBA = x$

[Since OAB is an isosceles triangle]

In $\triangle OAB$,

$$\angle ABO + \angle LOAB + \angle LOB = 180^\circ$$

$$x + x + 60 = 180^\circ$$

$$2x = 120 \Rightarrow x = 60^\circ$$

Therefore $\triangle OAB$ is an equilateral triangle.

$$\begin{aligned} \text{Area of the } \triangle OAB &= \frac{\sqrt{3}}{4} a^2 \\ &= \frac{\sqrt{3}}{4} \times 21 \times 21 \\ &= \frac{441\sqrt{3}}{4} \text{ cm}^2 \end{aligned}$$

Therefore Area of the segment formed by the chord = Area of the sector OAPBO – Area of $\triangle OAB$.

$$= \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

6) A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. ($\pi = 3.14, \sqrt{3} = 1.73$).

Solution:

Given AB is a chord, subtends an angle 60° at the centre. $\angle AOB = 60^\circ$

Radius $OA = OB = 15 \text{ cm}$

In $\triangle OAB$, $OA = OB$

Therefore $\angle LOAB = \angle LOBA = x$

In $\triangle OAB$,

$$\angle LOAB + \angle LOBA + \angle LOB = 180^\circ$$

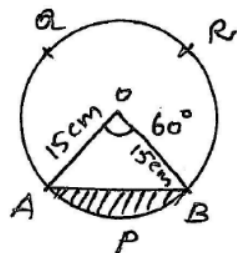
$$x + x + 60 = 180$$

$$2x + 60 = 180 \Rightarrow 2x = 120$$

$$x = 60^\circ$$

Therefore $\triangle AOB$ is an equilateral triangle

$$\text{Therefore Area of } \triangle AOB = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 15^2$$



$$\begin{aligned} &= \frac{\sqrt{3}}{4} \times 225 \\ &= \frac{1.73 \times 225}{4} = 0.4325 \times 225 \\ &= 97.3125 \text{ cm}^2 \end{aligned}$$

$$\text{Area of the sector AOBP} = \frac{\theta}{360} \times \pi r^2$$

$$\begin{aligned} &= \frac{60}{360} \times 3.14 \times 15 \times 15 = \frac{3.14 \times 225}{6} \\ &= \frac{706.5}{6} = 117.75 \text{ cm}^2 \end{aligned}$$

Therefore Area of minor segment APB = Area of the sector AOBP – Area of triangle OAB

$$= 117.75 - 97.3125$$

$$= 20.4375 \text{ cm}^2$$

Area of major segment = Area of circle – Area of minor segment.

$$= \pi r^2 - 20.4375$$

$$= 3.14 \times 15^2 - 20.4375$$

$$= 3.14 \times 225 - 20.4375$$

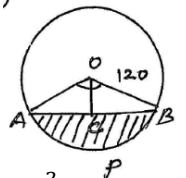
$$= 706.5 - 20.4375$$

$$= 686.0625 \text{ cm}^2$$

7) A chord of a circle of radius 12 cm subtends an angle 120° at the centre. Find the area of the corresponding segment of the circle, ($\pi = 3.14, \sqrt{3} = 1.73$).

Solution:

Given, $r = 12 \text{ cm}$ and $\angle AOB = 120^\circ$



Therefore Area of sector OAPBO = $\frac{\theta}{360} \pi r^2$

$$= \frac{120}{360} \times 3.14 \times 12 \times 12$$

$$= 3.14 \times 4 \times 12$$

$$= 12.56 \times 12$$

$$= 150.72 \text{ cm}^2$$

In $\triangle OAB$, Draw $OC \perp AB$.

In $\triangle OAC$ and $\triangle OBC$

$$OA = OB \text{ (radii)}$$

$$OC = OC \text{ common}$$

$$\angle LOCA = \angle LOCB = 90^\circ$$

Therefore By SAS criteria $\triangle OAC \cong \triangle OBC$

Therefore $\angle LOAC = \angle LOBC = \frac{1}{2} \angle LOAB = 60^\circ$

In $\triangle OAC$, $\sin 60^\circ = \frac{AC}{OA}$

$$\frac{\sqrt{3}}{2} = \frac{AC}{12} \Rightarrow AC = \frac{12\sqrt{3}}{2} = 6\sqrt{3} \text{ cm}$$

$$\cos 60^\circ = \frac{OC}{OA}$$

$$\frac{1}{2} = \frac{OC}{12} \Rightarrow OC = \frac{12}{2} = 6 \text{ cm}$$

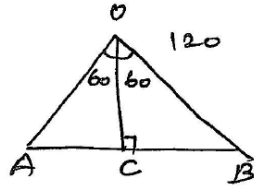
Now $AB = 2AC = 2(6\sqrt{3})$

$$\begin{aligned} \text{Area of the } \Delta OAB &= \frac{1}{2} \times AB \times OC \\ &= \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3} \text{ cm}^2 \end{aligned}$$

Now Area of the segment APB = Area of sector OAPBO – Area of ΔOAB

$$\begin{aligned} &= 150.72 - 36\sqrt{3} = 150.72 - 36 \times 1.73 \\ &= 150.72 \times 62.28 \end{aligned}$$

Area of the segment APB = 88.44 cm²

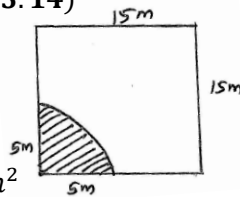


8) A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze. (ii) the increasing in the grazing area if the rope were 10 m long instead of 5 m. ($\pi = 3.14$)

Solution:

Side of the square = 15 m.

$$\begin{aligned} \text{Area of the square} &= \text{side} \times \text{side} \\ &= 15 \times 15 = 225 \text{ m}^2 \end{aligned}$$



Also given length of rope = 5 m

radius of arc = 5 m

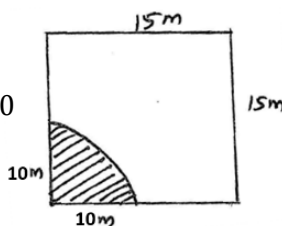
(i) Therefore, Area of the field graze by the horse

$$\begin{aligned} A_1 &= \frac{\theta}{360} \pi r^2 = \frac{90}{360} \times 3.14 \times 5 \times 5 \\ &= 0.785 \times 25 \\ &= 19.625 \text{ m}^2 \end{aligned}$$

(ii) If length of the rope r = 10 m

Therefore, Area of the field graze by the horse

$$\begin{aligned} A_2 &= \frac{\theta}{360} \pi r^2 \\ &= \frac{90}{360} \times 3.14 \times 10 \times 10 \\ &= 3.14 \times 25 \\ &= 78.50 \text{ m}^2 \end{aligned}$$



$$\begin{aligned} \text{Now increase in grazing area} &= A_2 - A_1 \\ &= 78.50 - 19.625 \\ &= 58.875 \text{ m}^2 \end{aligned}$$

9) A brooch is made up of silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure. Find (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

Solution:

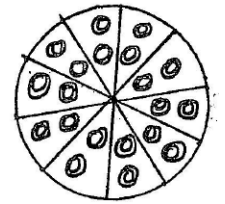
Diameter of circular wire = 35 mm

Circumference of wire = $2\pi r$

$$= \pi d$$

$$= \frac{22}{7} \times 35 = 22 \times 5$$

$$= 110 \text{ mm}$$



Now length of 5 diameter = 5×35

$$= 175 \text{ mm}$$

(i) Total length of silver wire = $110 + 175$

$$= 285 \text{ mm}$$

(ii) The circle divides into 10 equal sectors.

Therefore angle of each sector = $\frac{360}{10}$

$$= 36^\circ$$

Area of each sector = $\frac{\theta}{360} \pi r^2$

$$= \frac{36}{360} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= \frac{11 \times 35}{4} = 96.25 \text{ mm}^2$$

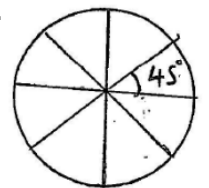
10) An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Solution:

Given, umbrella to be a flat circle.

Central angle of umbrella is 360° .

The umbrella has 8 ribs.



Therefore Angle between two ribs = $\frac{360}{8} = 45^\circ$

Now Area between two ribs = $\frac{\theta}{360} \pi r^2$

$$= \frac{45}{360} \times \frac{22}{7} \times 45 \times 45$$

$$= \frac{45 \times 45 \times 22}{56}$$

$$= \frac{2025 \times 11}{28}$$

Area between two ribs = $\frac{22275}{28} \text{ cm}^2$

11) A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades.
Solution:

Given length of wiper blade $r = 25$ cm
 Angle made by the blade $\theta = 115^\circ$
 Therefore Area cleaned by one blade

$$\begin{aligned} &= \frac{\theta}{360} \times \pi r^2. \\ &= \frac{115}{360} \times \frac{22}{7} \times 25^2 \\ &= \frac{115}{360} \times \frac{22}{7} \times 25 \times 25 \end{aligned}$$

Now Area cleaned by two blade

$$= 2 \times \frac{115}{360} \times \frac{22}{7} \times 25 \times 25$$

25

$$\begin{aligned} &= \frac{23 \times 11 \times 625}{7 \times 18} \\ &= \frac{14375 \times 11}{126} \end{aligned}$$

$$\text{Area cleaned by two blade} = \frac{158125}{126} \text{ cm}^2$$

12) To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle of 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned ($\pi = 3.14$).
Solution:

Given, distance = $r = 16.5$
 central angle $\theta = 80^\circ$

$$\begin{aligned} \text{Therefore Area of sector} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{80}{360} \times 3.14 \times (16.5)^2 \\ &= \frac{6.28}{9} \times 16.5 \times 16.5 \\ &= \frac{6.28 \times 272.25}{9} \end{aligned}$$

Which is required area of the sea which the ship are warned

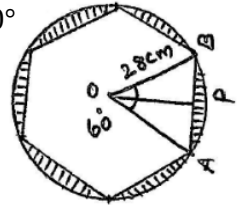
$$\begin{aligned} &= \frac{1709.73}{9} \\ &= 189.97 \text{ km}^2 \end{aligned}$$

13) A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹0.35 per cm^2 . ($\sqrt{3} = 1.732$)
Solution:

Central angle of the sector (circle) = 360°

$$\text{Angle of each sector} = \frac{360}{6} = 60^\circ$$

In $\triangle OAB$, $\angle AOB = 60^\circ$ and
 $OA = OB = 28$ (radii)



Let $\angle OAB = \angle OBA = x$ (say)

Therefore angles opposite to equal sides of a triangle are equal.

$$\text{In } \triangle OAB, \angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$60 + x + x = 180^\circ$$

$$2x = 180 - 60$$

$$2x = 120$$

$$x = 60^\circ$$

$$\Rightarrow \angle OAB = \angle OBA = 60^\circ$$

Therefore $\triangle OAB$ is an equilateral triangle.

$$\text{Therefore Area of } \triangle OAB = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \times 28^2$$

$$= \frac{1.732}{4} \times 28 \times 28$$

$$= 1.732 \times 196$$

$$= 339.472 \text{ cm}^2$$

$$\text{Area of the sector AOBPA} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28$$

$$= \frac{11 \times 4 \times 28}{3} = \frac{44 \times 28}{3}$$

$$= \frac{1232}{3} = 410.66$$

$$\text{Area of the sector AOBPA} = 410.67 \text{ cm}^2$$

Now Area of segment APBA = Area of sector

AOBPA – Area of $\triangle AOB$

$$= 410.67 - 339.472$$

$$\text{Area of segment APBA} = 71.198 \text{ cm}^2$$

Therefore Area of 6 segment = 6×71.198

$$\text{Area of 6 segment} = 427.188 \text{ cm}^2$$

Cost of making the design is ₹ 0.35 per cm^2 .

Therefore Total cost = 427.188×0.35

$$\text{Total cost} = ₹149.52$$

14) Tick the correct answer in the following question. Area of a sector of angle P (in degree) of a circle with radius R is

(i) $\frac{P}{180} 2\pi R$

(ii) $\frac{P}{180} \times \pi R^2$

(iii) $\frac{P}{360} 2\pi R$

(iv) $\frac{P}{720} \times 2\pi R^2$

Solution:

angle of sector $\theta = P$, radius $r = R$

Therefore Area of the sector

$$= \frac{\theta}{360} \times \pi r^2$$

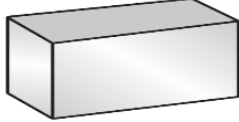
$$= \frac{P}{360} \times \pi R^2$$

$$\text{Area of the sector} = \frac{P}{720} \times 2\pi R^2 \text{ (iv)}$$

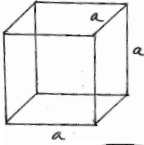
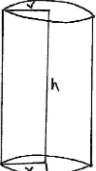
13 SURFACE AREAS AND VOLUMES

Important formulae:

1) Cuboid:

- a) Total surface area of cuboid
TSA = $2(lb+bh+lh)$ sq unit. 
- b) Lateral surface area of cuboid = $2(l+b)h$ sq unit.
- c) Diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.
- d) Volume of cuboid = $l \times b \times h$ cu unit.

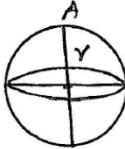
2) Cube:

- a) TSA of cube = $6a^2$ sq units. 
- b) LSA of cube = $4a^2$ sq units.
- c) Diagonal of cube = $\sqrt{3}a$ units.
- d) Volume of cube = a^3 cu units. 

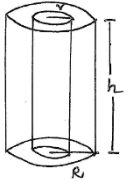
3) Right circular cylinder:

- a) Curved surface area of cylinder = $2\pi rh$ sq units.
- b) Total surface area of cylinder = $2\pi r(h + r)$ sq units.
- c) Volume of cylinder = $\pi r^2 h$ cu units.

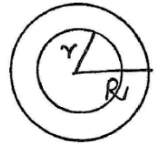
4) Sphere:

- Surface area of a sphere = $4\pi r^2$ sq units. 
- Volume of the sphere = $\frac{4}{3}\pi r^3$ cu units.

5) Right Circular Hollow Cylinder:

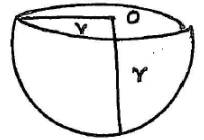
- a) CSA of hollow cylinder = $2\pi rh + 2\pi Rh$
= $2\pi(r + R)h$ 
- b) TSA of hollow cylinder
= $2\pi(R + r)[h + R - r]$ sq unit.
- c) Volume of hollow cylinder = $\pi(R^2 - r^2)h$ cu units.

6) Spherical Shell:



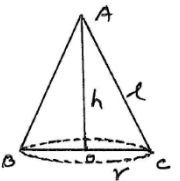
- a) Volume of hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$
- b) Outer surface area = $4\pi r^2$ sq unit.

7) Hemisphere:



- a) Curved surface area of hemisphere = $2\pi r^2$ sq unit.
- b) TSA of hemisphere = $3\pi r^2$ sq unit.
- c) Volume of hemisphere = $\frac{2}{3}\pi r^3$ cu unit.

8) Cone:

- a) Slant height $l = \sqrt{h^2 + r^2}$ 
- b) CSA of cone = πrl sq unit.
- c) TSA of cone = $\pi r(l + r)$ sq units.
- d) Volume of cone = $\frac{1}{3}\pi r^2 h$ cu units.

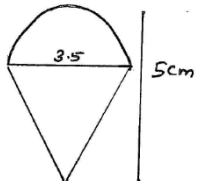
Example 1: Rasheed got a playing top as his birthday present, which is surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour.

Solution:

TSA of top = CSA of hemisphere + CSA of cone. — 1

Hemisphere: radius $r = \frac{3.5}{2}$ cm

Cone: radius $r = \frac{3.5}{2}$ cm

height $h = 5 - \frac{3.5}{2} = \frac{10-3.5}{2} = \frac{6.5}{2}$ 

Slant height $l = \sqrt{h^2 + r^2} = \sqrt{\left(\frac{6.5}{2}\right)^2 + \left(\frac{3.5}{2}\right)^2}$
 $= \sqrt{(3.25)^2 + \left(\frac{3.5}{2}\right)^2} = \sqrt{10.5625 + 3.0625}$
 $= \sqrt{13.6250} = 3.69 \approx 3.7$ cm

CSA of hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$ cm²

CSA of cone = $\pi rl = \frac{22}{7} \times \frac{3.5}{2} \times 3.7$ cm²

Therefore TSA of top = CSA of hemisphere + CSA of cone

$$\begin{aligned}
 &= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + \frac{22}{7} \times \frac{3.5}{2} \times 3.7 \\
 &= \frac{22}{7} \times \frac{3.5}{2} [2 \times \frac{3.5}{2} + 3.7] \\
 &= \frac{5.5 \times 7.2}{110} \\
 &\quad \underline{385} \\
 &\quad \underline{39.60}
 \end{aligned}$$

TSA of top = 39.6 cm²

Note: TSA of top is not the sum of TSA of cone and hemisphere.

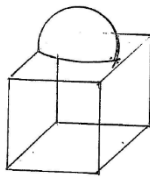
Example 2: The decorative block shown in the figure is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block.

Solution:

Note that the part of cube where the hemisphere attached is not included in the surface area.

Therefore the surface area of the block = TSA of the cube – base area of the hemisphere + CSA of hemisphere. ——— A

$$\begin{aligned}
 1. \text{ Total surface area of cube} &= 6a^2 \\
 &= 6 \times 5 \times 5 \\
 &= 150 \text{ cm}^2
 \end{aligned}$$



$$\begin{aligned}
 2. \text{ Base area of the hemisphere} &= \pi r^2 \\
 &= \frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2} \\
 &= 1.26 \times 11 \\
 &= 13.86 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 3. \text{ CSA of hemisphere} &= 2\pi r^2 \\
 &= 2(13.86) \text{ cm}^2 \\
 &= 27.72 \text{ cm}^2
 \end{aligned}$$

From A, 1, 2, 3

$$\begin{aligned}
 \text{Therefore surface area of the block} \\
 &= 150 - 13.86 + 27.72 \\
 &= 150 + 13.86
 \end{aligned}$$

Surface area of the block = 163.86 cm²

Example 3: A wooden toy rocket is in the shape of a cone mounted on a cylinder as in the figure. The height of the entire rocket is 26 cm. While the height of conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base of diameter of cylindrical portion is 3 cm. If the conical portion is to be painted orange and the

cylindrical portion yellow, find the area of the rocket painted with each of these colors.

($\pi = 3.14$)

Solution:

Cone:

$$\text{radius of the cone } r = \frac{5}{2} \text{ cm.}$$

$$\text{height of the cone } h = 6 \text{ cm.}$$

$$\text{slant height of cone } l = \sqrt{h^2 + r^2}$$

$$= \sqrt{6^2 + \left(\frac{5}{2}\right)^2} = \sqrt{36 + \frac{25}{4}}$$

$$= \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5 \text{ cm}$$

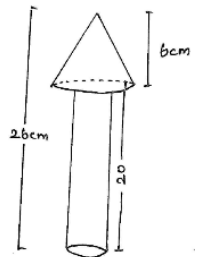
Cylinder:

$$\text{radius of the cylinder } R = \frac{3}{2} \text{ cm}$$

$$\text{Height of the cylinder } H = 26 - 6 = 20 \text{ cm}$$

The area to be painted orange (conical part) = CSA of cone + base area of cone – base area of cylinder

$$\begin{aligned}
 &= \pi r l + \pi r^2 - \pi R^2 \\
 &= \frac{22}{7} \times \frac{5}{2} \times 6.5 + \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} - \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \\
 &= \frac{22}{7} \left[\frac{5}{2} \times \frac{13}{2} + \frac{5}{2} \times \frac{5}{2} - \frac{3}{2} \times \frac{3}{2} \right] \\
 &= \frac{22}{7} \left[\frac{65}{4} + \frac{25}{4} - \frac{9}{4} \right] \\
 &= \frac{22}{7} \times \frac{81}{4} = 3.14 \times 20.25
 \end{aligned}$$



Area to be painted orange (conical part) =

63.58 cm²

Now, the area to be painted yellow = CSA of the cylinder + area of base of the cylinder.

$$\begin{aligned}
 &= 2\pi RH + \pi R^2 \\
 &= \pi R [2H + R] \\
 &= 3.14 \times \frac{3}{2} \left[2 \times 20 + \frac{3}{2} \right] \\
 &= 1.57 \times 3 [40 + 1.5] \\
 &= 4.71 \times 41.5
 \end{aligned}$$

Area to be painted yellow = 195.465 cm²

Example 4: Mayank made a bird-bath for his garden in the shape of cylinder with a hemispherical depression at one end. The height of cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird bath. ($\pi = \frac{22}{7}$)

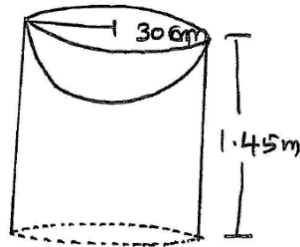
Solution:

Total surface area of bird-bath = CSA of cylinder +

CSA of hemisphere ——— (01)

Cylinder:

Height $h = 1.45\text{ m}$
 $= 145\text{ cm}$
 radius $r = 30\text{ cm}$



Hemisphere:

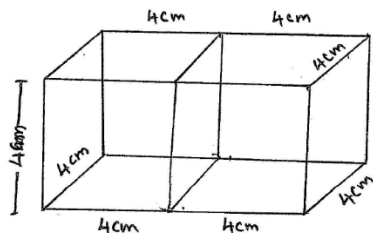
radius $r = 30\text{ cm}$

$$\begin{aligned} \text{From 1 TSA of bird-bath} &= 2\pi rh + 2\pi r^2 \\ &= 2\pi r(h + r) \\ &= 2 \times \frac{22}{7} \times 30(145 + 30) \\ &= 2 \times \frac{22}{7} \times 30 \times 175 \\ &= 2 \times 30 \times 25 \times 22 \\ &= 1500 \times 22 \\ &= 33000\text{ cm}^2 \end{aligned}$$

TSA of bird-bath = 3.3 m^2

EXERCISE 13.1

- 1) 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.



Solution:

Given volume of each cube is 64 cm^3

$$\begin{aligned} a^3 &= 64 \\ \Rightarrow a &= 4\text{ cm} \end{aligned}$$

When two cubes are joined together, then we get a cuboid whose length $l = (4 + 4) = 8\text{ cm}$ breadth $b = 4\text{ cm}$, height $h = 4\text{ cm}$.

$$\begin{aligned} \text{Now, surface area of the cuboid} &= 2(lb + bh + lh) \\ &= 2[8 \times 4 + 4 \times 4 + 8 \times 4] \\ &= 2[32 + 16 + 32] \end{aligned}$$

Surface area of the cuboid = $2 \times 80 = 160\text{ cm}^2$

- 2) A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm . Find the inner surface area of the vessel.

Solution:

Total height of the vessel = 13 cm .

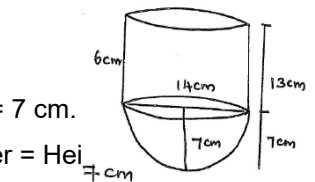
radius of the cylinder =

radius of the hemisphere =

Height of the hemisphere = 7 cm .

Therefore Height of cylinder = Height of the vessel -

Height of the hemisphere



$$= 13 - 7 = 6\text{ cm}.$$

Cylinder: $h = 6\text{ cm}$, radius $r = 7\text{ cm}$.

Hemisphere: radius $r = 7\text{ cm}$.

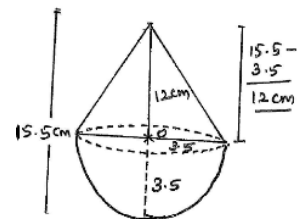
Inner surface area of vessel = CSA of cylinder + CSA of hemisphere

$$\begin{aligned} &= 2\pi rh + 2\pi r^2 = 2\pi r(h + r) \\ &= 2 \times \frac{22}{7} \times 7[6 + 7] = 2 \times 22 \times 13 \\ &= 44 \times 13 \end{aligned}$$

Inner surface area of vessel = 572 cm^2

- 3) A toy is in the form of a cone of radius 3.5 cm surrounded on a hemisphere of same radius. The total height of the toy is 15.5 cm . Find the total surface area of the toy (2020).

Solution:



Given Total height of the toy is 15.5 cm .

Radius of the cone = 3.5 cm

Radius of the hemisphere = 3.5 cm

Height of hemisphere = 3.5 cm .

Therefore Height of cone = Total height of toy - radius of hemisphere

$$= 15.5 - 3.5 = 12\text{ cm}$$

Conical part: radius $r = 3.5\text{ cm}$

height $h = 12\text{ cm}$

Hemisphere: radius $r = 3.5\text{ cm}$

Total surface area of the toy = CSA of cone + CSA of hemisphere.

$$= \pi rl + 2\pi r^2 \text{ ————— (1)}$$

In cone $l = \sqrt{h^2 + r^2}$

$$= \sqrt{12^2 + \left(\frac{7}{2}\right)^2} = \sqrt{144 + \frac{49}{4}} = \sqrt{\frac{576+49}{4}}$$

$$= \sqrt{\frac{625}{4}} = \frac{25}{2} = 12.5$$

Therefore from 1

Total surface area of the toy = $\pi rl + 2\pi r^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{22}{7} \times \frac{7}{2} \left[\frac{25}{2} + 2 \times \frac{7}{2} \right] = 11 \left[\frac{25+14}{2} \right]$$

$$= \frac{11 \times 39}{2} = \frac{429}{2}$$

Total surface area of the toy = 241.5 cm^2

- 4) A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the surface area of the solid?

Solution:

For cubical portion:

Edge = 7 cm

For hemisphere portion:

Diameter = 7 cm.

Therefore radius $r = \frac{7}{2}$ cm

Surface area of solid = TSA of cuboid + CSA of hemisphere – Area of base of hemisphere

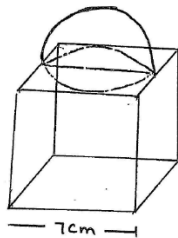
$$= 6a^2 + 2\pi r^2 - \pi r^2$$

$$= 6 \times 7^2 + \pi r^2$$

$$= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + \frac{77}{2} \Rightarrow 294 + 38.5$$

$$= 332.5 \text{ cm}^2$$



- 5) A hemispherical depression is cut-out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:

Given:

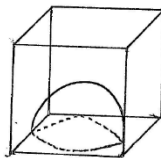
Edge of cube = Diameter of hemisphere.

$$= l$$

Therefore radius of hemisphere = $\frac{l}{2}$

Hemispherical depression is cut-out from one face of cubical block.

Therefore surface area of remaining solid = TSA of cube + CSA of hemisphere – base area of hemisphere.



$$= 6(\text{Edge})^2 + 2\pi r^2 - \pi r^2$$

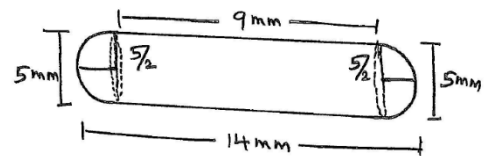
$$= 6l^2 + \pi r^2$$

$$= 6l^2 + \pi \left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \frac{\pi l^2}{4} = \frac{24l^2 + \pi l^2}{4} = \frac{l^2}{4}(24 + \pi) \text{ sq unit.}$$

Surface area of remaining solid = $\frac{l^2}{4}(24 + \pi)$ sq unit

- 6) A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Solution:

Given:

Medicine capsule is combination of two hemisphere and a cylinder.

Length of capsule = 14 mm

$$\begin{aligned} \text{Therefore Length of cylindrical part} &= 14 - 2.5 - 2.5 \\ &= 14 - 5 \\ &= 9 \text{ mm} \end{aligned}$$

radius of cylindrical part = $\frac{5}{2}$

radius of hemisphere $r = \frac{5}{2}$

Now surface area of capsule = CSA of cylinder + 2 × CSA of hemisphere.

$$= 2\pi rh + 2 \times 2\pi r^2$$

$$= 2\pi rh + 4\pi r^2$$

$$= 2\pi r[h + 2r]$$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \left[9 + 2 \times \frac{5}{2} \right]$$

$$= \frac{110}{7} [14] = \frac{1540}{7}$$

$$= 220 \text{ mm}^2$$

- 7) A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m, respectively and the slant height of the top is 2.8 m, then find the area of the canvas used for making the tent. Also, find the cost of the

canvas of the tent at the rate of ₹500 per m^2 .
(2016).

Solution:

Given that, the tent is combination of cone and the cylinder.

Cylinder:

$$\text{radius } r = \frac{4}{2} = 2 \text{ m}$$

$$\text{height } h = 2.1 \text{ m}$$

Cone:

$$\text{radius } r = 2 \text{ m}$$

$$\text{slant height } l = 2.8 \text{ m}$$

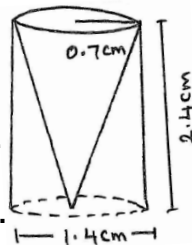
Required surface area of the tent = CSA of cone + CSA of cylinder.

$$\begin{aligned} &= \pi r l + 2\pi r h \\ &= \pi r [l + 2h] \\ &= \frac{22}{7} \times 2 [2.8 + 2(2.1)] \\ &= \frac{22}{7} \times 2 [2.8 + 4.2] \\ &= \frac{44}{7} \times 7 = 44m^2 \end{aligned}$$

Cost of canvas per meter sq. = Rs.500

Therefore cost = $44 \times 500 = \text{Rs. } 22000$

8) From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 .



Solution:

Radius of cylinder = radius of conical cavity

$$= \frac{1.4}{2} = 0.7 \text{ cm}$$

Height of cylinder = Height of conical cavity

$$= 2.4 \text{ cm}$$

$$\begin{aligned} \text{Slant height of conical cavity } l &= \sqrt{h^2 + r^2} \\ &= \sqrt{(2.4)^2 + (0.7)^2} \\ &= \sqrt{5.76 + 0.49} \\ &= \sqrt{6.25} = 2.5 \text{ cm} \end{aligned}$$

Now TSA of remaining solid = CSA of cavity + CSA of cylinder + Area of the base of the cylinder

$$\begin{aligned} &= \pi r l + 2\pi r h + \pi r^2 \\ &= \pi r (l + 2h + r) \end{aligned}$$

$$\begin{aligned} &= \frac{22}{7} \times 0.7(2.5 + 2(2.4) + 0.7) \\ &= 22 \times 0.1(2.5 + 4.8 + 0.7) \\ &= 2.2 \times 8.0 \\ &= 17.6 \approx 18cm^2 \end{aligned}$$

TSA of remaining solid = $17.6 \approx 18cm^2$

9) A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in adjacent figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, then find the total surface area of the article.

Solution:

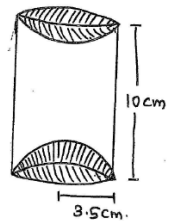
Given wooden article is combination of cylinder + two hemisphere.

Height of the cylinder $h = 10 \text{ cm}$.

radius of cylinder = radius of hemisphere

$$r = 3.5 \text{ cm.}$$

Now TSA of the wooden article = CSA of cylinder + $2 \times$ CSA of hemisphere.



$$\begin{aligned} &= 2\pi r h + 2 \times 2\pi r^2 \\ &= 2\pi r h + 4\pi r^2 \\ &= 2\pi r (h + 2r) \\ &= 2 \times \frac{22}{7} \times 3.5 [10 + 2(3.5)] \\ &= 2 \times \frac{22}{7} \times \frac{7}{2} \times 17 = \\ &17 \times 22 \\ &= 374 \text{ cm}^2 \end{aligned}$$

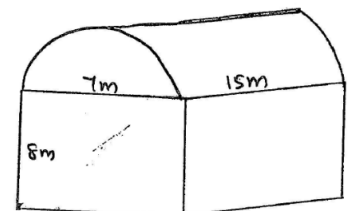
Example 5: Shanta runs an industry in a shed which is in the shape of cuboid surmounted by a half cylinder. If the base of the shed is of dimension $7 \text{ m} \times 15 \text{ m}$ and the height of the cuboid portion is 8 m . Find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300 m^3 , and there are 20 workers, each of whom occupy about 0.08 m^3 space on an average. Then how much air is in the shed? ($\pi = \frac{22}{7}$)

Solution: The volume of air inside the shed is the volume of air inside cuboid and volume of air in the half cylinder together.

Cuboid:

length = 15 m.

breadth = 7 m.



Height = 8 m.

Cylinder:

radius of cylinder $r = \frac{7}{2} m$

Height $h = 15 m$

Therefore the required volume = volume of cuboid + $\frac{1}{2}$ volume of cylinder.

$$\begin{aligned} &= lbh + \frac{1}{2}\pi r^2 h \\ &= 15 \times 7 \times 8 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15 \\ &= 840 + \frac{1155}{4} = 840 + 288.75 \\ &= 1128.75 m^3 \end{aligned}$$

Total space occupied by the machinery = $300 m^3$

Space occupied by the 20 workers = $20 \times 0.08 = 1.6 m^3$

Therefore space occupied by machinery and 20 workers

$$= 300 + 1.6 = 301.6 m^3$$

Therefore volume of air inside the shed = total volume – space occupied by the machinery and 20 workers.

$$= 1128.75 - 301.6$$

Volume of air inside the shed = $827.15 m^3$.

Example 6: A juice seller was serving his customers using glasses as shown in the figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. ($\pi = 3.14$)

Solution:

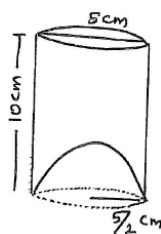
Actual capacity of the glass = volume of cylindrical glass – volume of hemisphere.

Cylinder:

radius $r = \frac{5}{2} cm$

Height $h = 10 cm$

$$\begin{aligned} \text{Volume of cylindrical glass} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 10 \\ &= 3.14 \times 2.5 \times 2.5 \times 10 \\ &= 31.4 \times 6.25 \\ &= 196.25 \end{aligned}$$



$$\begin{aligned} \text{Volume of hemisphere} &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times 3.14 \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \\ &= \frac{1.57 \times 125}{6} = \frac{196.25}{6} \\ &= 32.71 \end{aligned}$$

Therefore $1 \Rightarrow 196.25 - 32.71$

$$= 163.54 cm^3$$

Example 7: A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the toy and cylinder. ($\pi = 3.14$)

Solution:

Let ABC be the cone standing on the base of hemisphere BPC.

Cone:

radius of the cone $r = 2 cm$.

Height of the cone $h = 2 cm$.

Hemisphere:

radius of hemisphere $r = 2 cm$.

Therefore volume of the toy = Volume of hemisphere + Volume of cone

$$\begin{aligned} &= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h \\ &= \frac{2}{3} \times 3.14 \times 2^3 + \frac{1}{3} \times 3.14 \times 2^2 \times 2 \\ &= 2 \times \frac{3.14}{3} [8 + 4] \\ &= \frac{2}{3} \times 3.14 \times 12 \\ &= 3.14 \times 8 \\ &= 25.12 cm^3 \end{aligned}$$

Cylinder:

radius of the cylinder $r = 2 cm$

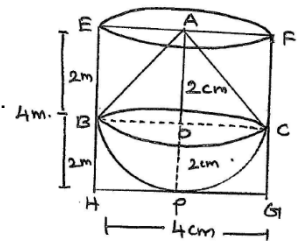
Height of cylinder $h = OA + OP$

$$= 2 + 2 = 4 cm.$$

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h = 3.14 \times 2^2 \times 4 \\ &= 3.14 \times 4 \times 4 = 3.14 \times 16 \\ &= 50.24 cm^3 \end{aligned}$$

Now Difference between the cylinder and the toy is

$$50.24 - 25.12 = 25.12 cm^3$$



EXERCISE 13.2

- 1) A solid in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π .

Solution: Solid is a combination of cone and a hemisphere.

Cone:

radius $r = 1$ cm

Height $h = 1$ cm

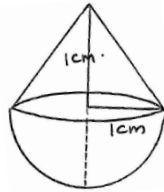
Hemisphere:

radius $r = 1$ cm.

Now volume of solid = Volume of cone + Volume of hemisphere.

$$\begin{aligned} &= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 \\ &= \frac{1}{3}\pi \times 1^2 \times 1 + \frac{2}{3}\pi \times 1^3 \\ &= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi \text{ cm}^3 \end{aligned}$$

Volume of solid = $\pi \text{ cm}^3$



- 2) Rachel, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, then find the volume of air contained in the model that Rachel made.

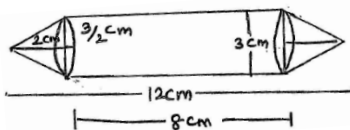
Solution:

Model is combination cylinder and two cones.

Cone:

Height $h_1 = 2$ cm

radius $r_1 = \frac{3}{2}$ cm.



Cylinder:

Height $h_2 = 12 - 2 - 2 = 12 - 4 = 8$ cm

Radius $r_2 = \frac{3}{2}$ cm.

Now volume of air inside the model = Volume of air in (Cylinder + 2 cones)

$$\begin{aligned} &= \pi r_2^2 h_2 + 2 \times \frac{1}{3}\pi r_1^2 h_1 \\ &= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 8 + \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 2 \\ &= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \left[8 + \frac{4}{3} \right] = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{28}{3} = 66 \text{ cm}^3 \end{aligned}$$

Volume of air inside the model = 66 cm^3

- 3) A Gulab jamun, contains sugar syrup upto 30% of its volume. Find approximately how much syrup would be found in 45 Gulab jamun, each shaped like a cylinder with two hemispherical ends with lengths 5 cm and diameter 2.8 cm.

Solution:

Gulab jamun is a combination of cylinder with two hemisphere.

Length of gulabjamun = 5 cm.

diameter = 2.8 cm.

Therefore radius $r = \frac{2.8}{2} = 1.4$ cm

Radius of hemispherical part = radius of cylinder = 1.4 cm.

Therefore Height of cylinder = $5 - (1.4 + 1.4) = 5 - 2.8 = 2.2$ cm.

Now volume of one gulabjamun = Volume of cylinder + 2 × Volume of hemisphere.

$$\begin{aligned} &= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right) \\ &= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + 2\left(\frac{2}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4\right) \\ &= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \times \frac{22}{10} + 2\left(\frac{2}{3} \times \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \times \frac{14}{10}\right) \\ &= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \left[\frac{22}{10} + 2 \times \frac{2}{3} \times \frac{14}{10} \right] \\ &= \frac{11 \times 14}{25} \left[\frac{22}{10} + \frac{56}{30} \right] = \frac{11 \times 14}{25} \times \frac{66 + 56}{30} = \frac{11 \times 14}{25} \times \frac{122}{30} \end{aligned}$$

Therefore volume of 45 Gulab jamun

$$\begin{aligned} &= 45 \times \frac{11 \times 14}{25} \times \frac{122}{30} = \frac{3 \times 11 \times 14 \times 61}{25} = \frac{33 \times 14 \times 61}{25} \\ &= \frac{28182}{25} = 1127.28 \text{ cm}^3 \end{aligned}$$

Volume of 45 Gulab jamun = 1127.28 cm^3

Now, Volume of cyrub = 30% of volume of 45 Gulab jamun

$$= \frac{30}{100} \times 1127.28 = \frac{3381.84}{10} = 338.184$$

Volume of cyrub = 338 cm^3

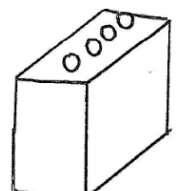
- 4) A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm × 10 cm × 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

Solution:

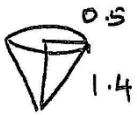
Given length of cuboid $l = 15$ cm

Breadth $b = 10$ cm

Height $h = 3.5$ cm



Volume of wooden cuboid = $l \times b \times h$



$$\begin{aligned} &= 15 \times 10 \times 3.5 \\ &= 15 \times 35 \\ &= 525 \text{ cm}^3 \end{aligned}$$

Radius of conical depression $r = 0.5 \text{ cm}$

Height of conical depression $h = 1.4 \text{ cm}$

$$\begin{aligned} \text{Volume of conical depression} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{14}{10} \\ &= \frac{11}{30} \text{ cm}^3 \end{aligned}$$

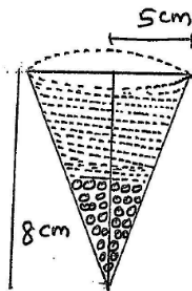
$$\begin{aligned} \text{Now, Volume of 4 conical depression} &= 4 \times \frac{11}{30} \\ &= \frac{22}{15} \text{ cm}^3 \end{aligned}$$

Volume of wood in the entire stand = Volume of cuboid – Volume of 4 conical depression

$$\begin{aligned} &= 525 - \frac{22}{15} \\ &= 525 - 1.47 \\ &= 523.53 \text{ cm}^3 \end{aligned}$$

- 5) A vessel in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open is 5 cm. It is filled with water upto the brim. When the lead shots each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of water flows out. Find the number of lead shots dropped in the vessel.

Given: A cone and spherical lead shots are given.



Cone:

Radius $r = 5 \text{ cm}$

Height $h = 8 \text{ cm}$

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 \\ &= \frac{110 \times 40}{21} = \frac{4400}{21} \text{ cm}^3 \end{aligned}$$

Volume of water filled in the vessel = Volume of cone

$$= \frac{4400}{21} \text{ cm}^3$$

Spherical lead shot:

radius $r = 0.5 = \frac{1}{2} \text{ cm}$

$$\begin{aligned} \text{Therefore volume of one lead shot} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \end{aligned}$$

$$= \frac{44}{84} = \frac{22}{42} = \frac{11}{21}$$

Note that volume of water flows' out is equal to volume of lead shot.

Let n be the number of lead shot dropped into the vessel.

According to the question,

$$n \times \frac{11}{21} = \frac{1}{4} \text{ of volume of water}$$

$$\frac{11n}{21} = \frac{1}{4} \times \frac{4400}{21}$$

$$n = \frac{1}{4} \times \frac{4400}{21} \times \frac{21}{11}$$

$$= 100$$

Therefore Number of lead shots dropped into the water = 100.

- 6) A solid iron pole consists of cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximat: mass. ($\pi = 3.14$)

Solution:

Given solid is a combination two cylinder.

Cylinder I

Height of 1st cylinder $h_1 = 220 \text{ cm}$

radius of 1st cylinder $r_1 = \frac{24}{2} = 12 \text{ cm}$

$$\begin{aligned} \text{Volume of 1}^{\text{st}} \text{ cylinder} &= \pi r_1^2 h_1 \\ &= 3.14 \times 12 \times 12 \times 220 \\ &= 3.14 \times 144 \times 220 \\ &= 3.14 \times 31680 \\ &= 99475.20 \text{ cm}^3 \end{aligned}$$

Cylinder II

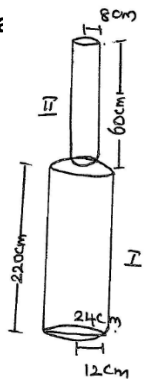
radius $r_2 = 8 \text{ cm}$

Height $h_2 = 60 \text{ cm}$

$$\begin{aligned} \text{Volume of 2}^{\text{nd}} \text{ cylinder} &= \pi r_2^2 h_2 \\ &= 3.14 \times 8 \times 8 \times 60 \\ &= 3.14 \times 64 \times 60 \\ &= 3.14 \times 3840 \\ &= 12057.60 \text{ cm}^3 \end{aligned}$$

Now, Volume of iron pole = Volume of 1st cylinder + Volume of 2nd cylinder.

$$\begin{aligned} &= 99475.2 + 12057.6 \\ &= 111532.8 \text{ cm}^3 \end{aligned}$$



Given, mass of 1 cm^3 of iron pole = 8 gm .

$$\begin{aligned} \text{Therefore mass of } 111532.8 \text{ cm}^3 \text{ of iron} \\ &= 111532.8 \times 8 \text{ gm} \\ &= 111532.8 \times \frac{8}{100} \text{ kg} \\ &= \frac{892262.4}{1000} \text{ kg} \\ &= 892.2624 \text{ kg} \\ &= \mathbf{892.26 \text{ kg}} \end{aligned}$$

7) A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm .

Solution:

Given solid is a combination of cone and a hemisphere and is placed into a right circular cylinder.

Cylinder:

Height of cylinder $h_1 = 180 \text{ cm}$

Radius $r = 60 \text{ cm}$

Cone:

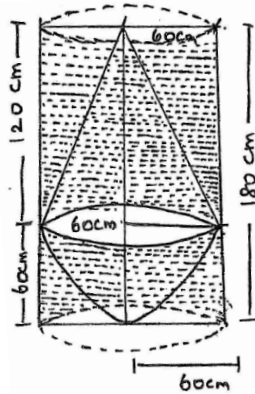
Height of cone $h_2 = 120 \text{ cm}$

radius $r = 60 \text{ cm}$

Hemisphere:

radius $r = 60 \text{ cm}$

Note that radius of cone, hemisphere, cylinder are same.



$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h_1 \\ &= \frac{22}{7} \times 60 \times 60 \times 180 \\ &= \frac{22 \times 6 \times 6 \times 18}{7} \times 1000 \\ &= \frac{22 \times 36 \times 18}{7} \times 1000 \\ &= \frac{792 \times 18}{7} \times 1000 \\ &= \frac{14256}{7} \times 1000 \\ &= \frac{14256000}{7} \text{ cm}^3 \\ &= \frac{14.256}{7} \text{ m}^3 \end{aligned}$$

For conical portion:

radius $r = 60 \text{ cm}$

$$\begin{aligned} \text{volume of conical portion} &= \frac{1}{3} \pi r^2 h_2 \\ &= \frac{1}{3} \times \frac{22}{7} \times 60 \times 60 \times 120 \\ &= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 12 \times 1000 \\ &= \frac{22 \times 144 \times 1000}{7} \\ &= \frac{3168000}{7} \text{ cm}^3 \\ &= \frac{3.168}{7} \text{ m}^3 \end{aligned}$$

For hemispherical portion:

Radius $r = 60 \text{ cm}$

$$\begin{aligned} \text{Volume of hemispherical portion} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 60 \\ &= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 6 \times 1000 \\ &= \frac{88 \times 36 \times 1000}{7} \\ &= \frac{3168000}{7} \text{ cm}^3 \\ &= \frac{3.168}{7} \text{ m}^3 \end{aligned}$$

Volume of water left in the cylinder = Volume of cylinder – (Volume of cone + Volume of hemisphere)

$$\begin{aligned} &= \frac{14.256}{7} - \left[\frac{3.168}{7} + \frac{3.168}{7} \right] \\ &= \frac{14.256}{7} - \left[\frac{6.336}{7} \right] \\ &= \frac{14.256 - 6.336}{7} = \frac{7.920}{7} \text{ m}^3 \\ &= \frac{7.92}{7} = 1.13143 \end{aligned}$$

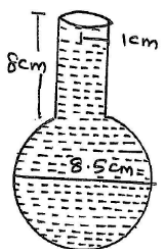
Volume of water left in the cylinder = 1.131 m^3

8) A spherical glass vessel has a cylinder neck 8 cm long, 2 cm in diameter, the diameter of spherical part is 8.5 cm . By measuring the amount of water holds, a child finds its volume to be 345 cm^3 . Check whether she is correct, taking the above as the inside measurements and $\pi = 3.14$.

Solution:

Given: The spherical glass vessel is a combination of cylinder + sphere.

Now, volume of water filled in the vessel = volume of water in cylinder + volume of water in sphere.



For cylindrical portion:

Height of cylinder $h = 8\text{ cm}$

radius of cylinder $r = 1\text{ cm}$

$$\begin{aligned} \text{Therefore volume of water in the cylinder} &= \pi r^2 h \\ &= 3.14 \times 1 \times 1 \times 8 \\ &= 25.12\text{ cm}^3 \end{aligned}$$

For Sphere:

radius $r = \frac{8.5}{2}\text{ cm}$

$$\begin{aligned} \text{Volume of water in the sphere} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times 3.14 \times \frac{8.5}{2} \times \frac{8.5}{2} \times \frac{8.5}{2} \\ &= \frac{1.57 \times 8.5 \times 8.5 \times 8.5}{3} \\ &= \frac{964.17625}{3} \\ &= 321.39\text{ cm}^3 \end{aligned}$$

From 1 Volume of water in the vessel = Volume of water in the cylinder + Volume of water in the sphere.

$$\begin{aligned} &= 25.12 + 321.39 \\ &= 346.51\text{ cm}^3 \end{aligned}$$

So, the correct answer is 346.51 cm^3 .

Conversion of Solid from one shape to Another

Example 8: A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

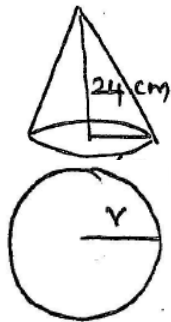
Given:

Cone:

Height $h = 24\text{ cm}$

radius $r = 6\text{ cm}$

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi \times 6 \times 6 \times 24 \\ &= 12 \times 24 \times \pi \\ &= 288\pi\text{ cm}^3 \end{aligned}$$



Now the cone reshaped into the sphere

⇒ volume of sphere = volume of cone

$$\begin{aligned} \frac{4}{3}\pi r^3 &= 288\pi \\ r^3 &= \frac{288 \times 3}{4} = 72 \times 3 \\ r^3 &= 2^3 \times 3^2 \times 3 \\ &= 2^3 \times 3^3 \end{aligned}$$

Therefore $r = 2 \times 3 = 6\text{ cm}$.

Therefore radius of the sphere $r = 6\text{ cm}$.

Example 9: Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump. Which is in the shape of a cuboid. The sump has dimension $1.57\text{ m} \times 1.44\text{ m} \times 95\text{ cm}$. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump. ($\pi = 3.14$)

Solution:

The volume of water in the overhead tank is equal to volume water removed from the sump.

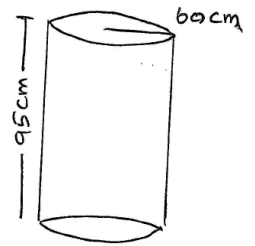
Cylindrical tank:

Height $h = 95\text{ cm}$

$r = 60\text{ cm}$

Volume of water in the overhead

$$\begin{aligned} \text{cylindrical tank} &= \pi r^2 h \\ &= 3.14 \times 60 \times 60 \times 95 \end{aligned}$$



Cuboid shape of sump

length = $1.57\text{ m} = 157\text{ cm}$

Breadth = $1.44\text{ m} = 144\text{ cm}$

Height = 95 cm

Volume of water in the sump

when full = $l \times b \times h$

$$= 157 \times 144 \times 95\text{ cm}^3$$

Now, volume of water left

$$\begin{aligned} &= 157 \times 144 \times 95 - 3.14 \times 60 \times 60 \times 95 \\ &= 157 \times 12 \times 12 \times 95 - 3.14 \times 12 \times 5 \times 12 \times 5 \times 95 \\ &= 12 \times 12 \times 95 [157 - 3.14 \times 5 \times 5] \\ &= 12 \times 12 \times 95 [157 - 3.14 \times 25] \\ &= 12 \times 12 \times 95 [157 - 78.50] \\ &= 12 \times 12 \times 95 \times 78.5 \end{aligned}$$

$$l \times b \times h = 12 \times 12 \times 95 \times 78.5$$

$$1.57\text{ m} \times 1.44\text{ m} \times h = 12 \times 12 \times 95 \times 78.5$$

$$h = \frac{12 \times 12 \times 95 \times 78.5}{157 \times 144} = \frac{95}{2}$$

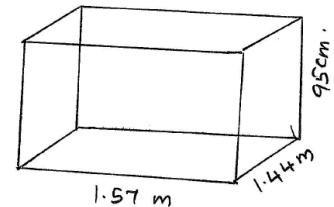
$h = 47.5\text{ cm}$

Therefore height of the water left in the tank = 47.5 cm

Now, the capacity of the tank with that of sump,

$$\begin{aligned} \frac{\text{capacity of the tank}}{\text{capacity of the sump}} &= \frac{3.14 \times 60 \times 60 \times 95}{157 \times 144 \times 95} = \frac{3.14 \times 60 \times 60}{157 \times 144} \\ &= \frac{3.14 \times 25}{157} = \frac{78.50}{157} = \frac{1}{2} \end{aligned}$$

Therefore Capacity of the tank is half the capacity of the sump.



Example 10: A copper rod of diameter 1 cm and length 8 cm is drawn into wire of length 18 m of uniform thickness. Find the thickness of wire.

Cylindrical rod:

Length $l = 8 \text{ cm}$.

radius $r = \frac{1}{2} \text{ cm}$.

$$\begin{aligned} \text{Volume of rod} &= \pi r^2 h = \pi \times \frac{1}{2} \times \frac{1}{2} \times 8 = \frac{8}{4} \pi \text{ cm}^3 \\ &= 2\pi \text{ cm}^3 \end{aligned}$$

Wire:

Length of wire $h = 18 \text{ m} = 1800 \text{ cm}$.

Let r be the radius of wire.

$$\begin{aligned} \text{Therefore Volume of wire} &= \pi r^2 h \\ &= \pi \times r^2 \times 1800 \text{ m} \\ &= \pi \times r^2 \times 1800 \text{ cm}^3 \end{aligned}$$

Given, rod is reshaped into wire.

Now, Volume of wire = Volume of rod.

$$\begin{aligned} \pi r^2 \times 1800 &= 2\pi \\ r^2 &= \frac{2}{1800} = \frac{1}{900} \end{aligned}$$

Therefore $r = \frac{1}{30}$

$$\begin{aligned} \text{Thickness of the wire} &= 2r \\ &= 2 \times \frac{1}{30} \\ &= \frac{1}{15} \text{ cm} \\ &\approx 0.666 \approx 0.67 \text{ mm (app)} \end{aligned}$$

Example 11: A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres persecond. How much time will it take to empty half the tank, if it is 3 m in diameter.

Given:

radius of the tank = $\frac{3}{2} \text{ m}$.

$$\begin{aligned} \text{Volume of hemispherical tank} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \\ &= \frac{99}{14} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of water to be emptied} &= \frac{1}{2} \text{ of the tank} \\ &= \frac{1}{2} \times \frac{99}{14} \text{ m}^3 \\ &= \frac{99}{28} \times 1000 \text{ litres} \end{aligned}$$

Given volume of water emptied in 1 sec

$$\begin{aligned} &= 3\frac{4}{7} \text{ litres} \\ &= \frac{25}{7} \text{ litres} \end{aligned}$$

So, $\frac{25}{7}$ litres of water is emptied in 1 sec.

Therefore 1 litre of water is emptied in $\frac{1}{\frac{25}{7}}$

$$= \frac{7}{25} \text{ sec}$$

Now, $\frac{99}{28} \times 1000$ litres of water is emptied

$$\begin{aligned} &\frac{7}{25} \times \frac{99}{28} \times 1000 \text{ sec} \\ &= 990 \text{ sec} \\ &= \frac{990}{60} = \frac{99}{6} \text{ min} \\ &= 16.5 \text{ min} \end{aligned}$$

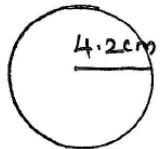
EXERCISE 13.3

1) A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution:

radius of the sphere $r = 4.2 \text{ cm}$

$$\begin{aligned} \text{Therefore volume of sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \pi \times 4.2 \times 4.2 \times 4.2 \text{ cm}^3 \end{aligned}$$



Cylinder:

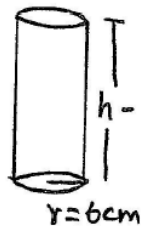
radius $r = 6 \text{ cm}$

Let Height of the cylinder be $h \text{ cm}$.

Given that the sphere is melted and recast into cylinder.

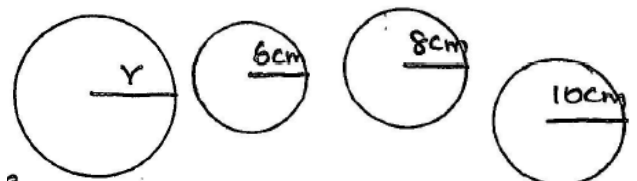
Volume of cylinder = Volume of sphere

$$\begin{aligned} \pi r^2 h &= \frac{4}{3} \times \pi \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \\ 6 \times 6 \times h &= \frac{4}{3} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \\ h &= \frac{1}{6 \times 6} \times \frac{4}{3} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \text{ cm} \\ h &= \frac{4 \times 7 \times 7 \times 14}{1000} = \frac{28 \times 98}{1000} \text{ cm} \\ &= \frac{2744}{1000} \\ h &= 2.744 \text{ cm.} \end{aligned}$$



2) Metallic sphere of radii 6 cm, 8 cm, 10 cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution: Let $r_1 = 6 \text{ cm}$, $r_2 = 8 \text{ cm}$, $r_3 = 10 \text{ cm}$ be radius of three metallic spheres. Let r be the radius of a single solid sphere.



Volume of single solid sphere = Volume of three metallic sphere.

$$\begin{aligned} \frac{4}{3}\pi r^3 &= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 \\ &= \frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3] \\ r^3 &= 6^3 + 8^3 + 10^3 = 216 + 512 + 1000 \\ &= 728 + 1000 = 1728 = 12^3 \end{aligned}$$

$$\Rightarrow r = 12 \text{ cm}$$

Therefore radius of resulting sphere = 12 cm.

- 3) A 20 m deep well with diameter 7 m is dig and the earth from digging is evenly spread out to form a plat form 22 m × 14 m. Find the height of the platform.

Solution:

Well is in the form of cylinder.

Height of the cylinder $h = 20 \text{ m}$.

radius of cylinder $r = \frac{7}{2} \text{ m}$.

$$\begin{aligned} \text{Volume of cylinder (Well)} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 \\ &= 77 \times 10 = 770 \text{ m}^3 \end{aligned}$$

Length of platform $l = 24 \text{ m}$.

Breadth of platform $b = 14 \text{ m}$.

Let h be the height of plat form

$$\begin{aligned} \text{Therefore volume of platform} &= lbh \\ &= 22 \times 14 \times h \end{aligned}$$

Given that the earth is removed and spread out to form a plat form.

Therefore volume of plat form = Volume of cylinder.

$$\begin{aligned} 22 \times 14 \times h &= 770 \\ h &= \frac{770}{22 \times 14} = \frac{5}{2} \\ &= 2.5 \text{ m} \end{aligned}$$

Therefore Height of platform = 2.5 m.

- 4) A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form embankment. Find the height of the embankment.

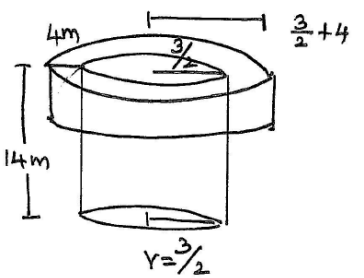
Solution:

The well is in the form of cylinder.

Height of cylindrical well $h = 14 \text{ m}$.

Radius of the well $r = \frac{3}{2} \text{ m}$.

Therefore volume of earth taken out = $\pi r^2 h$

$$\begin{aligned} &= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 14 \text{ m}^3 \\ &= 99 \text{ m}^3 \end{aligned}$$


We know that the embankment form a hollow cylinder.

Therefore volume of embankment = Volume of hollow cylinder

Hollow Cylinder:

Inner radius $r = \frac{3}{2} \text{ m}$.

Outer radius $R = 4 + \frac{3}{2} = \frac{11}{2} \text{ m}$.

Volume of hollow $\pi(R^2 - r^2)h_2$

Here h_2 be the height of Hollow Cylinder.

$$\begin{aligned} \text{Therefore } \frac{22}{7} \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2 \right] h_2 &= \frac{22}{7} \left[\frac{121}{4} - \frac{9}{4} \right] h_2 = \frac{22}{7} \left[\frac{121-9}{4} \right] h_2 \\ &= \frac{22}{7} \left[\frac{112}{4} \right] h_2 \end{aligned}$$

Now volume of embankment = Volume of earth taken out.

$$\frac{22}{7} \times \frac{112}{4} \times h_2 = 99$$

$$\text{Therefore } h_2 = \frac{99 \times 4 \times 7}{22 \times 112} = \frac{9}{8} = 1.125 \text{ m}$$

Therefore Height of embankment $h_2 = 1.125 \text{ m}$.

- 5) A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Solution:

Container (Cylinder):

Height of the container $h = 15 \text{ cm}$.

radius of the container $r = \frac{12}{2} = 6 \text{ cm}$.

Therefore Volume of ice cream in the container =

$$\pi r^2 h = \pi \times 6 \times 6 \times 15 \text{ cm}^3$$

Now, volume of cone full of ice-cream = Volume of cone + Volume of hemisphere

Cone:

Height $h = 12 \text{ cm}$.

radius $r = 3 \text{ cm}$.

$$\begin{aligned} \text{Therefore volume of cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi \times 3 \times 3 \times 12 \\ &= 36\pi \text{ cm}^3 \end{aligned}$$

Hemisphere:

radius $r = 3 \text{ cm}$.

$$\begin{aligned} \text{Therefore volume of hemisphere} &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3}\pi \times 3 \times 3 \times 3 \\ &= 18\pi \end{aligned}$$

$$\begin{aligned} \text{Therefore volume of cone full of ice cream} \\ &= 36\pi + 18\pi = 54\pi \end{aligned}$$

Let n be the number of full of ice cream cone, then the volume of ice cream container = $n \times$ Volume of cone full of ice-cream.

$$\pi \times 6 \times 6 \times 15 = n \times 54 \times \pi$$

$$\text{Therefore } n = \frac{6 \times 6 \times 15}{54} = 10$$

Hence the number of cone is 10

- 6) How many silver coins, 1.75 cm in diameter and of thickness 2 mm must be melted to form a cuboid of dimension 5.5 cm × 10 cm × 3.5 cm?

Solution:

coin is in the cylinder.

$$\text{radius of coin} = \frac{1.75}{2} \text{ cm} = \frac{17.5}{2} \text{ mm.}$$

Height of coin $h = 2 \text{ mm}$.

$$\begin{aligned} \text{Therefore volume of the coin} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{17.5}{2} \times \frac{17.5}{2} \times 2 \text{ mm}^3 \\ &= 11 \times 2.5 \times 17.5 \end{aligned}$$

$$\begin{aligned} \text{Volume of cuboid} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 550 \times 100 \times 35 \text{ mm}^3 \end{aligned}$$

Let n be the number of coins required to make the cuboid.

Therefore $n \times$ volume of coin = volume of cuboid.

$$n \times 11 \times 2.5 \times 17.5 = 550 \times 100 \times 35$$

$$n = \frac{550 \times 100 \times 35}{11 \times 2.5 \times 17.5} = \frac{550 \times 100 \times 35}{11 \times 2.5 \times 17.5} = 4000 \text{ mm}$$

$$= 400$$

Therefore 400 silver coins must be melted to form a cuboid.

- 7) A cylindrical bucket 32 cm high and with radius of base 18 cm, is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. If the height of conical heap is

24 cm. Then find the radius and slant height of the heap.

Solution:

The sand filled in cylindrical bucket form a conical heap on the ground.

Therefore volume of cylindrical bucket = volume of cone.

$$\pi r^2 h = \frac{1}{3}\pi r^2 h$$

$$18 \times 18 \times 32 = \frac{1}{3} \times r^2 \times 24$$

$$\text{Therefore } r^2 = \frac{18 \times 18 \times 32 \times 3}{24}$$

$$r^2 = 9 \times 2 \times 9 \times 2 \times 2 \times 2$$

$$r = \sqrt{9^2 \times 2^4} = 9 \times 2^2 = 9 \times 4$$

$$= 36 \text{ cm.}$$

Slant height of heap $l = \sqrt{h^2 + r^2}$

$$= \sqrt{24^2 + 36^2} = \sqrt{576 + 1296}$$

$$= \sqrt{1872} = \sqrt{2^4 \times 3^2 \times 13} = 4 \times 3\sqrt{13}$$

$$= 12\sqrt{13} \text{ cm.}$$

Hence radius and slant height of heap are 36 cm and $12\sqrt{13}$ cm respectively.

- 8) Water in a canal, 6 m wide and 1.5 m deep is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 min, if 8 cm of standing water is needed.

Solution:

Given speed of water = 10 km/hr.

$$= 10 \times 1000 \text{ m/hr.}$$

Length of water flow in 1 hour = 10 × 1000 m

Length of water flow in $\frac{1}{2}$ hour = $\frac{10 \times 1000}{2}$ m.

$$= 5000 \text{ m.}$$

Now, volume of water flowing in 30 min = volume of cuboid of length 5000 m, width 6 m and height 1.5 m.

$$= 5000 \times 6 \times 1.5 = 5000 \times 9 = 45000 \text{ m}^3$$

Now, The area covered for irrigation with 8 cm

$$\text{standing water} = \frac{45000}{8 \text{ cm}} = \frac{45000}{\frac{8}{100}} \text{ m} = \frac{45000 \times 100}{8}$$

$$= \frac{4500000}{8} = 562500 \text{ m}^2$$

$$= \frac{562500}{10000} \text{ hec}$$

Area covered for irrigation with 8 cm standing water = 56.25 hec

Note: 1 hec = 1000 m²

9) A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m in deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled?

Solution:

$$\begin{aligned} \text{Given speed of flow of water} &= 3 \text{ km/hr.} \\ &= 3000 \text{ m/hr.} \end{aligned}$$

Diameter of the pipe = 20 cm.

$$\begin{aligned} \text{Therefore radius of the pipe} &= \frac{20}{2} = 10 \text{ cm} \\ &= \frac{10}{100} \text{ m} \\ &= \frac{1}{10} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Now, volume of water flow in 1 hour} &= \pi r^2 h \\ &= \pi \times \left(\frac{1}{10}\right)^2 \times 3000 \\ &= \pi \times \frac{1}{100} \times 3000 \\ &= 30\pi \text{ m}^3 \end{aligned}$$

For Cylindrical tank:

Diameter = 10m, Therefore radius = 5 m.

Height = 2 m.

$$\begin{aligned} \text{Therefore volume of cylindrical tank} &= \pi r^2 h \\ &= \pi(5)^2 \times 2 \\ &= \pi \times 25 \times 2 = 50\pi \text{ m}^3 \end{aligned}$$

Therefore Required time =

$$\begin{aligned} &\frac{\text{volume of cylindrical tank}}{\text{volume of water flows in 1 hour}} \\ &= \frac{50\pi}{30\pi} h = \frac{50}{30} \times 60 \text{ min} = 100 \text{ min} \end{aligned}$$

Hence time taken to fill the tank = 100 min.

Frustum of a cone: Let h be the height, l the slant height and r_1 and r_2 the radii of ends of the frustum of a cone. Then we can directly find the volume, the curved surface area and the total surface area of frustum by using the formula.

- 1) Volume of the frustum of the cone

$$= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$$
- 2) CSA of frustum of the cone = $\pi l (r_1 + r_2)$
 Where $l = \sqrt{h^2 + (r_1 - r_2)^2}$.
- 3) Total surface area of frustum of a cone =

$$\pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$
 Where $l = \sqrt{h^2 + (r_1 - r_2)^2}$.

Example 12: The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm. Find its volume, the curved surface area and the total surface area. ($\pi = \frac{22}{7}$).

Solution:

$$\text{Given } r_1 = 28 \text{ cm, } r_2 = 7 \text{ cm, } h = 45 \text{ cm.}$$

$$\begin{aligned} \text{Therefore } l &= \sqrt{h^2 + (r_1 - r_2)^2} \\ &= \sqrt{45^2 + (28 - 7)^2} \\ &= \sqrt{2025 + 21^2} \\ &= \sqrt{2025 + 441} = \sqrt{2466} = 49.65 \end{aligned}$$

$$\begin{aligned} \text{(i) Volume of frustum} &= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2) \\ &= \frac{1}{3} \times \frac{22}{7} \times 45 [28^2 + 7^2 + 7 \times 28] \\ &= \frac{1}{3} \times \frac{22}{7} \times 45 [784 + 49 + 196] \\ &= \frac{22}{7} \times 15 [1029] \\ &= 22 \times 15 \times 147 \end{aligned}$$

$$\text{Volume of frustum} = 48510 \text{ cm}^3$$

$$\begin{aligned} \text{(ii) CSA of frustum} &= \pi (r_1 + r_2) l \\ &= \frac{22}{7} [28 + 7] 49.65 \\ &= \frac{22}{7} \times 35 \times 49.65 \\ &= 110 \times 49.65 \end{aligned}$$

$$\text{CSA of frustum} = 5461.5 \text{ cm}^2.$$

Total surface area of the frustum

$$\begin{aligned} &= \pi (r_1 + r_2) l + \pi r_1^2 + \pi r_2^2 \\ &= 5461.5 + \frac{22}{7} \times 28^2 + \frac{22}{7} \times 7^2 \\ &= 5461.5 + \frac{22}{7} \times 28 \times 28 + \frac{22}{7} \times 7 \times 7 \\ &= 5461.5 + 22 \times 112 + 22 \times 7 \\ &= 5461.5 + 2464 + 154 \end{aligned}$$

$$\text{Total surface area of the frustum} = 8079.5 \text{ cm}^2$$

Example 13: Hanumappa and his wife Gangamma are busy making Jaggery out of sugarcane juice. They have processed the sugarcane juice to make the molasses, which is poured into moulds in the shape of a frustum of a cone having the diameters of its two circular faces as 30 cm and 35 cm and the vertical height to the mould is 14 cm. If each cm³ of molasses has mass about 1.2 g, find the mass of the molasses that can be poured into each mould.

Solution:

Since the mould is in the shape of a frustum of a cone.

The quantity that molasses that can be poured into it is nothing but the volume of the frustum.

$$= \frac{\pi}{3} h(r_1^2 + r_2^2 + r_1 r_2)$$

Here $r_1 = \frac{35}{2} \text{ cm}$, $r_2 = \frac{30}{2} \text{ cm}$, $h = 14 \text{ cm}$.

Therefore volume

$$= \frac{1}{3} \times \frac{22}{7} \times 14 \left[\left(\frac{35}{2}\right)^2 + \left(\frac{30}{2}\right)^2 + \left(\frac{35}{2}\right) \times \left(\frac{30}{2}\right) \right]$$

$$= \frac{1}{3} \times 44 \left[\frac{1225}{4} + \frac{900}{4} + \frac{1050}{4} \right]$$

$$= \frac{1}{3} \times 44 \left[\frac{3175}{4} \right]$$

$$= \frac{11 \times 3175}{3} = \frac{34925}{3} = 11641.666$$

$$= \mathbf{11641.67 \text{ cm}^3}$$

Given that mass of 1 cm^3 molasses = 1.2 g

Therefore mass of 11641.67 cm^3 molasses =

$$11641.67 \times 1.2$$

$$= 139700004 = 13970.004 \text{ gm.}$$

Mass of 11641.67 cm^3 molasses = 14 kg (app).

Example 14 : An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 45 cm and 25 cm the total vertical height of the bucket is 40 cm and that the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle bucket. Also find the volume of water the bucket can hold?

Solution:

The total height of the bucket is 40 cm which includes the height of the base 6 cm.

Therefore Height of the frustum = 40 – 6 = 34 cm.

Here $r_1 = \frac{45}{2}$, $r_2 = \frac{25}{2}$.

Now, $l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{34^2 + (22.5 - 12.5)^2}$

$$= \sqrt{1156 + 10^2} = \sqrt{1156 + 100}$$

$$= \sqrt{1256}$$

$$= \mathbf{35.44 \text{ cm}}$$

The area of metallic sheet used = curved surface area of frustum + base area of frustum + CSA of cylinder.

$$= \pi(r_1 + r_2)l + \pi r^2 + 2\pi r h$$

Frustum:	Cylinder:	Base area:
$r_1 = 22.5$	$r = 12.5 \text{ cm}$	radius $r = 12.5 \text{ cm}$
$r_2 = 12.5$	$h = 6 \text{ cm}$	
$h = 34$		

Area of metallic sheet used = $\pi[(r_1 + r_2)l + r^2 + 2rh]$

$$= \pi [(22.5 + 12.5)35.44 + (12.5)^2 + 2 \times 12.5 \times 6]$$

$$= \frac{22}{7} [35 \times 35.44 + 156.25 + 150]$$

$$= \frac{22}{7} [1240.4 + 156.25 + 150]$$

$$= \frac{22}{7} [1546.65]$$

$$= 22 \times 220.95$$

The area of metallic sheet used = 4860.9 cm^2

Now, volume of bucket = $\frac{\pi}{3} \times h[r_1^2 + r_2^2 + r_1 r_2]$

$$= \frac{22}{7} \times \frac{34}{3} [(22.5)^2 + (12.5)^2 + 22.5 \times 12.5]$$

$$= \frac{22}{7} \times \frac{34}{3} [506.25 + 156.25 + 281.25]$$

$$= \frac{22}{7} \times \frac{34}{3} \times 943.75 = 33615.48 \text{ cm}^3$$

Volume of bucket = 33.62 ltrs.

EXERCISE 13.4

- 1) A drinking slats is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution:

Given $h = 14 \text{ cm}$.

$r_1 = 2 \text{ cm}$.

$r_2 = 1 \text{ cm}$.

Now, volume of drinking glass

$$= \frac{1}{3} \pi r h [r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 [4 + 1 + 2(1)]$$

$$= \frac{1}{3} \times 44 \times 7 = \frac{308}{3}$$

$$\mathbf{\text{Volume of drinking glass} = 102 \frac{2}{3} \text{ cm}^3}$$

- 2) The slant height of a frustum of a cone is 4 cm and the perimeter of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution:

Given slant height $l = 4 \text{ cm}$.

Perimeter of one circular end $2\pi r_1$

$$2\pi r_1 = 18 \text{ cm.}$$

$$\text{Therefore } r_1 = \frac{18}{2\pi} = \frac{9}{\pi} \text{ cm.}$$

Perimeter of another circular end $2\pi r_2 = 6 \text{ cm}$.

$$r_2 = \frac{6}{2\pi} = \frac{3}{\pi} \text{ cm.}$$

Hence CSA of the frustum = $\pi(r_1 + r_2)l$

$$= \pi \left[\frac{9}{\pi} + \frac{3}{\pi} \right] \times 4 = \pi \times \frac{12}{\pi} \times 4 = 4 \times 12$$

$$\text{CSA of the frustum} = 48 \text{ cm}^2$$

- 3) A fez, the cap used by the turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, then find the area of material used for making it.

Solution:

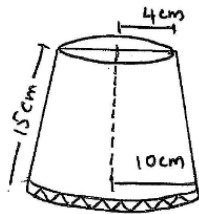
Given $l = 15 \text{ cm}, r_1 = 10 \text{ cm},$

$r_2 = 4 \text{ cm}.$

Hence area of material used for making fez = CSA of frustum + Area of upper closed ends.

$$\begin{aligned} &= \pi(r_1 + r_2)l + \pi r_2^2 \\ &= \pi[10 + 4]15 + \pi(4)^2 \\ &= \pi[210 + 16] = \frac{22}{7} \times 226 \\ &= \frac{4972}{7} = 710 \frac{2}{7} \text{ cm}^2. \end{aligned}$$

Area of material used for making fez = $710 \frac{2}{7} \text{ cm}^2$



- 4) A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹20 per litre. Also find the cost of metal sheet used to make the container, if it cost ₹8 per 100 cm².

($\pi = 3.14$)

Solution:

Container is in the shape of frustum.

Given $h = 16 \text{ cm}, r_1 = 20, r_2 = 8 \text{ cm}.$

Now, volume of container = Volume of frustum

$$\begin{aligned} &= \frac{1}{3} \pi h [r_1^2 + r_2^2 + r_1 r_2] \\ &= \frac{1}{3} \times 3.14 \times 16 [20^2 + 8^2 + 20 \times 8] \\ &= \frac{1}{3} \times 3.14 \times 16 [400 + 64 + 160] \\ &= \frac{1}{3} \times 3.14 \times 16 \times 624 = 3.14 \times 16 \times 208 \\ &= 10449.92 \text{ cm}^3 = \frac{10449.92}{1000} \text{ litres} \\ &= 10.449 \approx 10.45 \text{ litres.} \end{aligned}$$

Cost of 1 liter milk = ₹20

Therefore cost of 10.45 liters of milk = $10.45 \times 20 = 209.00$

Cost of 10.45 liters of milk = ₹209

$$\begin{aligned} \text{Now, slant height } l &= \sqrt{h^2 + (r_2 - r_1)^2} \\ &= \sqrt{16^2 + (20 - 8)^2} \\ &= \sqrt{256 + 144} = \sqrt{400} \end{aligned}$$

Slant height $l = 20 \text{ cm}.$

Area of metal used for making the container = CSA of frustum + Area of its Lower end.

$$\begin{aligned} &= \pi(r_1 + r_2)l + \pi r_1^2 \\ &= 3.14(8 + 20) \times 20 + 3.14 \times 8^2 \\ &= 3.14 \times 28 \times 20 + 3.14 \times 64 \\ &= 3.14[560 + 64] \\ &= 3.14 \times 624 = 1959.36 \text{ cm}^2 \end{aligned}$$

Cost of metal sheet per 100 cm² = ₹8

Therefore, cost of metal sheet per cm² = ₹ $\frac{8}{100}$

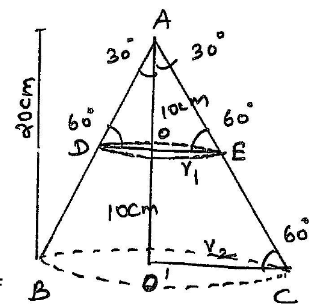
Therefore cost of metal sheet used to make the container = $1959.36 \times \frac{8}{100} = \frac{15674.88}{100} = 156.7488$

Cost to make metal sheet for the container = ₹156.75

- 5) A metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16} \text{ cm}$, find the length of the wire.

Solution:

A cone of height 20 cm is cut into two parts, one, the upper part is cone and the other part is like frustum.



Therefore Height of 1st part of the cone is 10 cm and

Height of the frustum $h = 10 \text{ cm}.$

For finding radii r_1 , consider the triangle.

$$\begin{aligned} OAE, \tan 60^\circ &= \frac{OA}{OE} \\ \sqrt{3} &= \frac{10}{r_1} \Rightarrow r_1 = \frac{10}{\sqrt{3}} \text{ cm.} \end{aligned}$$

For finding radii r_2 , consider the triangle,

$$\begin{aligned} AO'C, \tan 60^\circ &= \frac{O'A}{O'C} \\ \sqrt{3} &= \frac{20}{r_2} \Rightarrow r_2 = \frac{20}{\sqrt{3}} \text{ cm.} \end{aligned}$$

$$\begin{aligned} \text{Volume of frustum} &= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1r_2] \\ &= \frac{1}{3}\pi \times 10 \left[\left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2 + \frac{10}{\sqrt{3}} \times \frac{20}{\sqrt{3}} \right] \\ &= \frac{1}{3}\pi \times 10 \left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3} \right] \\ &= \frac{10\pi}{3} \times \frac{700}{3} = \frac{7000\pi}{9} \text{ cm}^3 \end{aligned}$$

Given, the frustum can be drawn into cylindrical wire of diameter $\frac{1}{16}$ cm.

Therefore Radius of wire = $\frac{1}{32}$ cm.

Let 'n' be the height of cylindrical wire

Therefore volume of wire = $\pi r^2 h$

$$= \pi \times \left(\frac{1}{32}\right)^2 \times h$$

According to the question,

Volume of wire = Volume of frustum

$$\pi \times \frac{1}{32} \times \frac{1}{32} \times h = \frac{7000\pi}{9}$$

$$\begin{aligned} \text{Therefore } l &= \frac{7000\pi}{9} \times \frac{32 \times 32}{\pi} \text{ cm} \\ &= \frac{7000 \times 32 \times 32}{9 \times 100} \text{ m} \\ &= \frac{71680}{9} = 7964.44 \text{ m} \end{aligned}$$

Hence length of wire $l = 7964.44$ m.

14 STATISTICS

MEAN:

(i) Let $x_1, x_2, x_3 \dots x_n$ be n observations, then their arithmetic Mean is given by $\bar{x} = \frac{x_1+x_2+\dots+x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$

(ii) Let $x_1, x_2, x_3 \dots x_n$ be n observations and let $f_1, f_2, f_3 \dots f_n$ be their corresponding frequencies, then their arithmetic mean is given by $\bar{x} = \frac{f_1x_1+f_2x_2+\dots+f_nx_n}{f_1+f_2+\dots+f_n} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$

(iii) Short cut Method: $\bar{x} = A + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$, where A is Assumed Mean $d_i = (x_i - A)$.

Example 1: The marks obtained by 30 students of class X of certain school in a mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the

Students Marks obtained x_i	10	20	36	40	50	56	60	70	72	80	88	92	95
Number of students f_i	1	1	3	4	3	2	4	4	1	1	2	3	1

Solution:

x_i	f_i	$f_i x_i$
10	1	1
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
$\sum f_i = 30$		$\sum f_i x_i = 1779$

$$\begin{aligned} \text{Mean } \bar{x} &= \frac{\sum f_i x_i}{\sum f_i} \\ &= \frac{1779}{30} \\ &= 59.3 \end{aligned}$$

Therefore, Mean $\bar{x} = 59.3$

Example 2: The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and when territories of India. Find the mean percentage of female teachers by all the three method discussed in this section.

Percentage of female Teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Numbers of states/U.T	6	11	7	4	4	2	1

Solution:

Percentage of female Teachers	Numbers of states/U.T f_i	x_i	$d_i = x - A$	$u_i = \frac{x - A}{10}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
15-25	6	20	-30	-3	120	-180	-18
25-35	11	30	-20	-2	330	-220	-22
35-45	7	40	-10	-1	280	-70	-7
45-55	4	50 A	0	0	200	0	0
55-65	4	60	10	1	240	40	4
65-75	2	70	20	2	140	40	4
75-85	1	80	30	3	80	30	3
	$\sum f_i = 35$				$\sum f_i x_i = 1390$	$\sum f_i d_i = -360$	$\sum f_i u_i = -36$

From the table

$$\sum f_i = 35$$

$$\sum f_i x_i = 1390$$

$$\sum f_i d_i = -360$$

$$\sum f_i u_i = -36$$

(i) using direct method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1390}{35}$
 $= 39.71$

(ii) using assumed mean method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$
 $= 50 + \frac{(-360)}{35} = 50 - 10.28$
 $= 39.71$

(iii) using step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$
 $= 50 + \frac{-36}{35} \times 10 = 50 - 10.28$
 $= 39.71$

Example 3: The distribution below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?

Number of Wickets	20 - 60	60 -100	100 -150	150 -250	250 -350	350 -450
Number of Bowlers	7	5	16	12	2	3

Solution:

Number of Wicket taken	Numbers of bowlers	x_i	$x_i - A$ $d = x_i - 200$	$u_i = \frac{d_i}{20}$	$u_i f_i$
20-60	7	40	-160	-8	-56
60-100	5	80	-120	-6	-30
100-150	16	125	-75	-3.75	-60
150-250	12	200 - A	0	0	0
250-350	2	300	100	5	10
350-450	3	400	200	10	30
	$\sum f_i = 45$				$\sum u_i f_i = -106$

Here Assumed Mean $a = 200$ $h = 20$

$$\text{Mean } \bar{x} = a + \frac{\sum u_i f_i}{\sum f} = 200 + \frac{-106}{45} \times 20 = 200 - \frac{106 \times 4}{9} = 200 - 47.11$$

Mean $\bar{x} = 152.89$

EXERCISE 14.1

1) A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of Houses	1	2	1	5	6	2	3

Solution:

Class	Frequency f_i	x_i	$d = \frac{x_i - A}{x_i - 7}$	$f_i x_i$
0-2	1	1	-6	-6
2-4	2	3	-4	-8
4-6	1	5	-2	-2
6-8	5	7 A	0	0
8-10	6	9	2	12
10-12	2	11	4	8
12-14	3	13	6	18
	$\sum f_i = 20$			$\sum f_i x_i = 22$

$$\sum f_i x_i = 38 - 16 = 22$$

Here a = 7

$$\sum f_i = 20$$

$$\sum f_i x_i = 22$$

$$\begin{aligned} \text{Mean } \bar{x} &= a + \frac{\sum f_i x_i}{\sum f} = 7 + \frac{22}{20} \\ &= 7 + 1.1 = 8.1 \end{aligned}$$

2) Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages in Rs	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Solution:

Daily Wages	f_i	x_i	$d = \frac{x - A}{x - 150}$	$u_i = \frac{d}{20}$	$u_i f_i$
100-120	12	110	-40	-2	-24
120-140	14	130	-20	-1	-14
140-160	8	150-A	0	0	0
160-180	6	170	20	1	6
180-200	10	190	40	2	20
Total	$\sum f_i = 50$				$\sum u_i f_i = -12$

Here a = 150

$$\sum f_i = 50$$

$$\sum u_i f_i = -12$$

$$h = 20$$

$$\begin{aligned} \text{Mean } \bar{x} &= a + \frac{\sum f_i x_i}{d_i} \times h = 150 + \frac{-12}{50} \times 20 \\ &= 150 - \frac{24}{5} = 150 - 4.8 = 145.2 \end{aligned}$$

$$\text{Mean } \bar{x} = 145.2$$

3) The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance in Rs18. Find the missing frequency.

Daily pocket allowance (in ₹)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	F	5	4

Solution:

CI	f	x	fx
11 - 13	7	12	84
13 - 15	6	14	84
15 - 17	9	16	144
17 - 19	13	18	234
19 - 21	f	20	$20f$
21 - 23	5	22	110
23 - 25	4	24	96
	$\sum f = 44 + f$		$\sum fx = 752 + 20f$

Here, $\sum f_x = 752 + 20f$

Given that mean = 18

$$\frac{\sum fx}{\sum f} = 18 \Rightarrow \frac{752+20f}{44+f} = 18$$

$$\Rightarrow 752 + 20f = 18(44 + f)$$

$$752 + 20f = 792 + 18f$$

$$20f - 18f = 792 - 752$$

$$2f = 40$$

$$f = 20$$

Therefore the missing frequently is $f = 20$.

- 4) Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Find the mean heart beats per minute for those women, choosing a suitable method.

Number of heart beat per minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women	2	4	3	8	7	4	2

Solution:

CI	f	x_i	fx_i
65-68	2	66.5	133
68-71	4	69.5	278
71-74	3	72.5	217.5
74-77	8	75.5	604.0
77-80	7	78.5	549.5
80-83	4	81.5	326.0
83-86	2	84.5	169.0
$\sum f = 30$		$\sum f_i x_i = 2227$	

$$\text{Mean } \bar{x} = \frac{\sum fx_i}{\sum f} = \frac{2227}{30}$$

$$\text{Mean } \bar{x} = 75.9$$

- 5) In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was distribution of mangoes according to the number of boxes. Find the mean number of mangoes kept in a packing box.

Number of Mangoes	50-52	53-55	56-58	59-61	62-64
Number of boxes	15	110	135	115	25

Solution:

CI	f_i	x_i	$d = x_i - A$ $x_i - 57$	$f_i d_i$
50-52	15	51	-6	-90
53-55	110	54	-3	-330
56-58	135	57 - A	0	0
59-61	115	60	3	345
62-64	25	63	6	150
$\sum f_i = 400$				$\sum f_i d_i = 75$

$$\text{Mean } \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 57 + \frac{75}{400} = 57 + \frac{3}{16} = 57 + 0.18$$

$$\text{Mean } \bar{x} = 57.18$$

- 6) The table below shows the daily expenditure on food of 25 households in locality. Find the mean daily expenditure on food.

Daily expenditure	100-150	150-200	200-250	250-300	300-350
Number of households	4	5	12	2	2

Solution:

CI	f_i	x_i	$d = x_i - A$ $x_i - 225$	$f_i d_i$
100 - 150	4	125	-100	-400
150 - 200	5	175	-50	-250
200 - 250	12	225 A	0	0
250 - 300	2	275	50	100
300 - 350	2	325	100	200
	$\sum f = 25$			$\sum f_i d_i = -350$

Here $\sum f_i = 25$
 $\sum f_i d_i = -350, A = 225$

Mean $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 225 + \frac{-350}{25} = 225 - 14$
Mean $\bar{x} = 211$

7) To find out the concentration of SO_2 in the air the data was collected for 30 localities in a certain city is presented below. Find the mean concentration of SO_2 in the air.

Concentration of SO_2	0.00 - 0.04	0.04 - 0.08	0.08 - 0.12	0.12 - 0.16	0.16 - 0.20	0.20 - 0.24
Frequency	4	9	9	2	4	2

Solution:

CI	f_i	x_i	$f_i x_i$
0.00 - 0.04	4	0.02	0.08
0.04 - 0.08	9	0.06	0.54
0.08 - 0.12	9	0.10	0.90
0.12 - 0.16	2	0.14	0.28
0.16 - 0.20	4	0.18	0.72
0.20 - 0.24	2	0.22	0.44
	$\sum f_i = 30$		$\sum f_i x_i = 2.96$

Here $\sum f_i = 30$
 $\sum f_i x_i = 2.96$
 Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f} = \frac{2.96}{30}$

$= 0.0986$
 $= 0.099$
Mean $\bar{x} = 0.099$

8) A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 - 6	6-10	10-14	14-20	20-28	28-38	38-40
Number of students	11	10	7	4	4	3	1

Solution:

CI	f_i	x_i	$f_i x_i$
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	17	68
20-28	4	24	96
28-38	3	33	99
38-40	1	39	39
	$\sum f_i = 40$		$\sum f_i x_i = 499$

Here $\sum f_i = 40$
 $\sum f_i x_i = 499$

Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f} = \frac{499}{40} = 12.475 = 12.48$
Mean $\bar{x} = 12.48$

9) The following table gives the literacy rate of 35 cities. Find the mean literacy rate.

Literacy rate in %	45-55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

Solution:

CI	f	x	$d = \frac{x - A}{x - 70}$	fd
45-55	3	50	-20	-60
55-65	10	60	-10	-100
65-75	11	70 A	0	0
75-85	8	80	10	80
85-95	3	90	20	60
	$\Sigma f = 35$			$\Sigma fd = -20$

Here $\Sigma f = 35$

$\Sigma fd = -20$

$$\begin{aligned} \text{Mean } \bar{x} &= a + \frac{\Sigma fd}{\Sigma f} \\ &= 70 + \frac{(-20)}{35} \\ &= 70 - \frac{4}{7} \\ &= 70 - 0.57 = 69.43\% \end{aligned}$$

Mean $\bar{x} = 69.43\%$

Mode: The value of observation having maximum frequency is called Mode.

Mode of grouped data:

In grouped data, $\text{Mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h$

Where l - lower limit of the modal class

h - size of the class interval

f_1 - frequency of the modal class

f_0 - frequency of the class preceding the modal class

f_2 - frequency of the class succeeding the modal class.

Example 5: A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a house hold. Find the mode of this data.

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Solution:

Here the maximum frequency $f = 8$

Corresponding Modal class – 3-5

lower limit of Modal class $l = 3$

Class size $h = 2$

Frequency of Modal class $f_1 = 8$

f_0 - frequency of preceding Modal class = 7

f_2 - frequency of succeeding Modal class = 2

$$\begin{aligned} \text{Mode} &= l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h \\ &= 3 + \left\{ \frac{8 - 7}{16 - 7 - 2} \right\} \times 2 \\ &= 3 + \frac{1}{7} \times 2 \\ &= 3 + \frac{2}{7} \\ &= \frac{23}{7} = 3.286 \end{aligned}$$

Mode = 3.286

Example 6: The marks distribution of 30 students in a mathematics examination are given below. Find the mode of this data. Also compare and interpret the mode and the mean.

CI	10 - 25	25 - 40	40 - 55	55 - 70	70 - 85	85 - 100
Number of students	2	3	7	6	6	6

Solution:

Here $f_1 = 7$ (Highest frequency)

$l = 40$ (lower limit of corresponding frequency)

$f_0 = 3$ (frequency of preceding Modal class)

$f_2 = 6$ (frequency of succeeding Modal class)

$h = 15$ (class size)

$$\begin{aligned} \text{Mode} &= l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h = 40 + \left\{ \frac{7 - 3}{14 - 3 - 6} \right\} \times 15 \\ &\Rightarrow 40 + \left\{ \frac{4}{5} \right\} \times 15 \Rightarrow 40 + 12 = 52 \end{aligned}$$

Mode = 52

For Mean:

CI	f	x	f_x
10-25	2	17.5	35
25-40	3	32.5	97.5
40-55	7	47.5	332.5
55-70	6	62.5	375.0
70-85	6	77.5	465.0
85-100	6	92.5	555.0
	$\Sigma f = 30$		$\Sigma f_x = 1860$

Here $\Sigma f = 30$

$$\Sigma f_x = 1860$$

$$\text{Mean } \bar{x} = \frac{\Sigma f_x}{\Sigma f} = \frac{1860}{30} = 62$$

Mean = 62

Mode = 52

EXERCISE 14.2

1) The following table shows the ages of the patients admitted in a hospital during a year.

Age in years	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Solution:

CI	f_i	x_i	$u_i = \frac{x_i - A}{10}$	$f_i u_i$
5 - 15	6	10	-2	-12
15 - 25	11	20	-1	-11
25 - 35	21	30 A	0	0
35 - 45	23	40	1	23
45 - 55	14	50	2	28
55 - 65	5	60	3	15
Total	$\Sigma f_i = 80$			$\Sigma f_i u_i = 43$

Here $\Sigma f_i = 80$

$$a = 30, \Sigma f_i u_i = 43, h = 10$$

By step deviation method $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$$= 30 + \frac{43}{80} \times 10$$

$$= 30 + 5.37 = 35.37$$

For Mode:

Maximum frequency $f_1 = 23$

So, Modal class is 35-45

$$l = 35, f_0 = 21, f_2 = 14, h = 10$$

$$\text{Mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h = 35 + \left\{ \frac{23 - 21}{46 - 21 - 14} \right\} \times 10$$

$$= 35 + \left\{ \frac{2}{11} \right\} \times 10 = 35 + \frac{20}{11} = 35 + 1.818$$

$$= 35 + 1.82 = 36.82$$

Hence Maximum number of patients admitted in the hospital are of age 36.82 years.

Average age of a patient admitted in the hospital is 35.37

2) The following data gives the information on the observed lifetime of 225 electrical components. Determine the modal lifetimes of components.

Lift time in hours	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Solution:

Maximum frequency $f_1 = 61$

Modal class 60-80

So, $l = 60, f_1 = 61, f_0 = 52, f_2 = 38, h = 20$

$$\text{Mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h = 60 + \left\{ \frac{61 - 52}{122 - 52 - 38} \right\} \times 20$$

$$= 60 + \frac{9 \times 20}{32} = 60 + \frac{9 \times 5}{8} = 60 + \frac{45}{8} = 60 + 5.625$$

Mode = 65.63

3) The following data gives The distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure In Rs	1000 - 1500	1500 - 2000	2000 - 2500	2500 - 3000	3000 - 3500	3500 - 4000	4000 - 4500	4500 - 5000
	24	40	33	28	30	22	16	7

Solution:

Here maximum frequency $f_1 = 40$

So, Modal class = 1500-2000

$l = 1500, f_0 = 24, f_1 = 40, f_2 = 33, h = 500$

$$\text{Modal} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h$$

$$= 1500 + \left\{ \frac{40 - 24}{80 - 24 - 33} \right\} \times 500$$

$$= 1500 + \frac{16 \times 500}{23}$$

$$= 1500 + 347.826 = 1847.83$$

Modal = 1847.83

4) The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and Mean of this data. Interpret the two.

Number of students per teacher	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50	50 - 55
Number of states/UT	3	8	9	10	3	0	0	2

Solution:

Mode:

From the data, Maximum frequency $f_1 = 10$

So, the modal class is 30-35

Here $l = 30, f_1 = 10, f_0 = 9, f_2 = 3, h = 5$

$$\text{Therefore Mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h$$

$$= 30 + \left\{ \frac{10 - 9}{20 - 9 - 3} \right\} \times 5$$

$$= 30 + \frac{1 \times 5}{8} = 30 + 0.625 = 30.63$$

For Mean

CI	f_i	x_i	$u_i = \frac{x-d}{h} = \frac{x-32.5}{5}$	$f_i u_i$
15 - 20	3	17.5	-3	-9
20 - 25	8	22.5	-2	-16
25 - 30	9	27.5	-1	-9
30 - 35	10	32.5 a	0	0
35 - 40	3	37.5	1	3
40 - 45	0	42.5	2	0
45 - 50	0	47.5	3	0
50 - 55	2	52.5	4	8
Total	$N = \sum f_i = 35$			$\sum f_i u_i = -23$

$$\begin{aligned} \text{Mean} &= a + \frac{\sum f_i u_i}{N} \times h = 32.5 + \frac{-23}{35} \times 5 \\ &= 32.5 - \frac{23}{7} = 32.5 - 3.28 \end{aligned}$$

Mean = 29.22

Most of the states/UT have a student ratio is 30.6 and the average ratio is 29.22

5) The given distribution shows the number of runs scored by some top batsman of the world in one day international cricket matches. Find the mode of the data.

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000	10000-11000
Number of batsmen	4	18	9	7	6	3	1	1

Solution:

The maximum frequency is 18.

So, corresponding Modal class is 4000-5000.

we have $l = 4000, f_0 = 4, f_1 = 18, f_2 = 9, h = 1000$

$$\text{Mode} = l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h = 4000 + \left\{ \frac{18 - 4}{36 - 4 - 9} \right\} \times 1000$$

$$= 4000 + \frac{14}{23} \times 1000 = 4000 + \frac{14000}{23} \Rightarrow 4000 + 608.69$$

Mode = 4608.7

6) A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

Solution:

The maximum frequency is 20

Corresponding modal class 40-50

So, $l = 40, f_0 = 12, f_1 = 20, f_2 = 11, h = 10$

$$\begin{aligned} \text{Mode} &= l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h = 40 + \left\{ \frac{20 - 12}{40 - 12 - 11} \right\} \times 10 \\ &= 40 + \frac{8 \times 10}{17} = 40 + \frac{80}{17} = 40 + 4.7 \end{aligned}$$

Mode = 44.7 cars

Median of Grouped Data: For finding the median of ungrouped data, we first arrange the data values of the observations in ascending order.

Note: If n is odd, the median is the $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation. And, if n is even, then the median will be the average of the $\frac{n}{2}^{\text{th}}$ and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ observations.

Example 7: A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained find the median height.

Height in cm	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Solution:

CI	f	cf
Below 140	4	4
140 - 145	7	11
145 - 150	18	29
150 - 155	11	40
155 - 160	6	46
160 - 165	5	51

Here $n = 51$ (odd no)

Therefore $\frac{n}{2} = \frac{51}{2} = 25.5$

So, 25.5 lies in the class 145-150.

Here $l = 145$ (lower limit), $f = 18$,

$$\begin{aligned} cf &= (\text{Cumulative frequency of the class preceding } 145-150) \\ &= 11, \end{aligned}$$

$H = 5$ class size.

$$\begin{aligned} \text{Therefore Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 145 + \left(\frac{25.5 - 11}{18} \right) \times 5 \\ &= 145 + \frac{14.5 \times 5}{18} = 145 + \frac{72.5}{18} \\ &= 145 + 4.03 \\ &= 149.03 \end{aligned}$$

Median Height of the girl is 149.03

Example 8: The median of the following data is 525. Find the value of x and y , if the total frequency is 100.

CI	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Solution:

CI	f	cf
0 - 100	2	2
100 - 200	5	7
200 - 300	x	$7 + x$
300 - 400	12	$19 + x$
400 - 500	17	$36 + x$
500 - 600	20	$56 + x$
600 - 700	y	$56 + x + y$
700 - 800	9	$65 + x + y$
800 - 900	7	$72 + x + y$
900 - 1000	4	$76 + x + y$

Given that $n = 100$

$$76 + x + y = 100$$

$$x + y = 100 - 76 = 24$$

$$x + y = 24 \text{ ——— } 1$$

The median is 525,

Which lies in the class 500-600.

$$\text{So, } l = 500, f = 20, cf = 36 + x, h = 100.$$

$$\text{Therefore Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$525 = 500 + \frac{50 - 36 - x}{20} \times 100$$

$$525 = 500 + (14 - x)5$$

$$525 = 500 - 70 - 5x$$

$$5x = 500 + 70 - 525$$

$$5x = 45$$

$$x = \frac{45}{5} = 9$$

(Substitute) $x = 9$ in 1

$$9 + y = 24$$

$$y = 24 - 9 = 15$$

Therefore $x = 9, y = 15$

EXERCISE 14.3

1) The following frequency distribution gives the monthly consumption of electricity of 68 consumers of locality. Find the median mean and mode of the data and compare them.

Monthly consumption	65 - 85	85 - 105	105 - 125	125 - 145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

Solution:

(i) Median:

CI	f	cf
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	8	64
185 - 205	4	68

Here $n=68$

$$\text{Therefore } \frac{n}{2} = 34$$

corresponding class 125-145

$$\text{Therefore } l = 125, f = 20, cf = 22, h = 20$$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 125 + \frac{34 - 22}{20} \times 20 \\ &= 125 + 12 = \mathbf{137 \text{ units.}} \end{aligned}$$

(ii) Mean:

CI	f	x	$u = \frac{x - 135}{20}$	fu
65-85	4	75	-3	-12
85-105	5	95	-2	-10
105-125	13	115	-1	-13
125-145	20	135 a	0	0
145-165	14	155	1	14
165-185	8	175	2	16
185-205	4	195	3	12
	68			7

Here $n=68$

$$\sum f_i u_i = 7$$

$$\begin{aligned} \text{Mean} &= a + \frac{\sum f_i u_i}{n} \times h = 135 + \frac{7 \times 20}{68} = 135 + \frac{35}{17} \\ &= 135 + 2.05 \\ &= \mathbf{137.05} \end{aligned}$$

(iii) Mode:

Here maximum frequency $f_1 = 20$.

corresponding class interval 125-145.

$$l = 125, f_0 = 13, f_2 = 14, h = 20$$

$$\begin{aligned} \text{Therefore Mode} &= l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h \\ &= 125 + \frac{20 - 13}{40 - 13 - 14} \times 20 \\ &= 125 + \frac{7}{13} \times 20 \Rightarrow 125 + \frac{140}{13} \\ &= 125 + 10.77 = \mathbf{135.77} \end{aligned}$$

Hence **Median = 137**

Mean = 137.05

Mode = 135.77

2) If the median of the distribution given below is 28.5, find the value of x and y .

CI	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	y	5	60

Solution:

CI	f	cf
0-10	5	5
10-20	x	$5 + x$
20-30	20	$25 + x$
30-40	15	$40 + x$
40-50	y	$40 + x + y$
50-60	5	$45 + x + y$

Given Median = 28.5

Which lies in the class 20-30.

Also given $n=60$

(i.e) $45 + x + y = 60$

$$x + y = 60 - 45$$

$$x + y = 15 \quad \text{---} \quad 1$$

Here $l = 20, f = 20, cf = 5 + x, h = 10$

$$\text{Therefore Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 20 + \frac{30 - 5 - x}{20} \times 10 = 28.5$$

$$= 20 + \frac{25 - x}{2} = 28.5$$

$$\frac{25 - x}{2} = 8.5 \Rightarrow 25 - x = 17$$

$$x = 25 - 17 = 8$$

Substitute $x = 8$ in 1

$$8 + y = 15$$

$$y = 7$$

$$x = 8$$

3) A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age in years	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
No of Policy Holders	2	6	24	45	78	89	92	98	100

Solution:

CI	f	$c.f$
Below 20	2	2
20 - 25	4	6
25 - 30	18	24
30 - 35	21	45
35 - 40	33	78
40 - 45	11	89
45 - 50	3	92
50 - 55	6	98
55 - 60	2	100

$n = 100,$

$$\text{So, } \frac{n}{2} = 50$$

This observation lies in the class 35-40.

Here $l = 35, f = 33, cf = 45, h = 5$

$$\text{Therefore Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 35 + \frac{50 - 45}{33} \times 5$$

$$= 35 + \frac{5 \times 5}{33} = 35 + \frac{25}{33}$$

$$= 35 + 0.757$$

$$= 35.76$$

4) The length of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table. Find the median length of the leaves.

Length in mm	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of leaves	3	5	9	12	5	4	2

Solution:

Length in mm	Number of leaves f	$c.f$
117.5 - 126.5	3	3
126.5 - 135.5	5	8
135.5 - 144.5	9	17
144.5 - 153.5	12	29
153.5 - 162.5	5	34
162.5 - 171.5	4	38
171.5 - 180.5	2	40

		N=40
--	--	------

here $n = 40,$

$$\text{So, } \frac{n}{2} = 20$$

Maximum frequency $f = 12, l = 144.5, cf = 17, h = 9$

$$\text{Therefore Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 144.5 + \frac{20 - 17}{12} \times 9$$

$$= 144.5 + \frac{3}{12} \times 9 = 144.5 + 2.25$$

$$= 146.75 \text{ mm}$$

5) The following table gives the distribution of the life time of 400 neon lamps. Find the median of life time.

Life time (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of lamps	14	56	60	86	74	62	48

Solution:

CI	f	c.f
1500 - 2000	14	14
2000 - 2500	56	70
2500 - 3000	60	130
3000 - 3500	86	216
3500 - 4000	74	290
4000 - 4500	62	352
4500 - 5000	48	400

Here $n = 400, \frac{n}{2} = 200$.

maximum frequency $f = 86$.

corresponding class 3000-3500.

$$l = 3000, cf = 130, h = 500.$$

$$\begin{aligned} \text{Therefore Median} &= l + \left\{ \frac{\frac{n}{2} - cf}{f} \right\} \times h \\ &= 3000 + \left\{ \frac{200 - 130}{86} \right\} \times 500 \\ &= 3000 + \frac{70}{86} \times 500 \\ &= 3000 + \frac{35000}{86} \\ &= 3000 + 406.97 \\ &= 3406.97 \end{aligned}$$

6) 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows.

Number of Letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
Number of surnames	6	30	40	16	4	4

Determine (i) median number of letters in the surnames. (ii) Mean number of letters in the surnames. (iii) Modal size of the surnames.

Solution:

(i) Median

CI	f	c.f
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100
		N=100

Here $n = 100$,

$$\text{So, } \frac{n}{2} = 50.$$

Maximum frequency $f = 40$.

corresponding class is 7-10.

So, $l = 7, f = 40, cf = 36, h = 3$

$$\begin{aligned} \text{Median} &= l + \left\{ \frac{\frac{n}{2} - cf}{f} \right\} \times h \\ &= 7 + \left\{ \frac{50 - 36}{40} \right\} \times 3 \\ &= 7 + \frac{14}{40} \times 3 = 7 + \frac{42}{40} \\ &= 7 + 1.05 \end{aligned}$$

$$\text{Median} = 8.05$$

(ii) Mean:

CI	f_i	x_i	$f_i x_i$
1-4	6	2.5	15.0
4-7	30	5.5	165.0
7-10	40	8.5	340.0
10-13	16	11.5	184.0
13-16	4	14.5	58.0
16-19	4	17.5	70.0
	100		832

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{\sum f} = \frac{832}{100} = 8.32$$

Therefore **Mean = 8.32**

(iii) For Mode:

Maximum frequency $f_1 = 40$.

corresponding Modal class = 7-10.

$$f_0 = 30, f_2 = 16, l = 7, h = 3.$$

$$\begin{aligned} \text{Mode} &= l + \left\{ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right\} \times h = 7 + \left\{ \frac{40 - 30}{80 - 30 - 16} \right\} \times 3 \\ &= 7 + \left\{ \frac{10}{34} \right\} \times 3 = 7 + \frac{30}{34} = 7 + 0.88 \end{aligned}$$

$$= 7.88$$

Therefore **Mode = 7.88**

7) The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight in Kg	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of Students	2	3	8	6	6	3	2

Solution:

CI	f	$c.f$
40 - 45	2	2
45 - 50	3	5
50 - 55	8	13
55 - 60	6	19
60 - 65	6	25
65 - 70	3	28
70 - 75	2	40

Here maximum frequency $f = 8$.

corresponding class 50-55.

$$\text{So, } l = 50, f = 8, cf = 5, h = 5, n = 40, \frac{n}{2} = 20$$

$$\begin{aligned} \text{Median} &= l + \left\{ \frac{\frac{n}{2} - cf}{f} \right\} \times h = 50 + \left\{ \frac{20 - 5}{8} \right\} \times 5 \\ &= 50 + \frac{15 \times 5}{8} = 50 + \frac{75}{8} = 50 + 9.375 \\ &= 50 + 9.38 = \mathbf{59.38} \end{aligned}$$

$$\mathbf{\text{Median} = 59.38}$$

15 PROBABILITY

Experiment: An operation which produces some well-defined outcomes, is called an experiment. Eg: Tossing a coin.

Sample space:

Outcomes of an experiment is called sample space. The theoretical probability of the event E is given by

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of outcomes}}$$

Example 1: Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Solution:

When a coin is tossed once, the possible outcomes are

$$S = \{H, T\}, n(S) = 2$$

let A denote the event of getting a tail.

$$A = \{T\}, n(A) = 1$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

Example 2: A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Krithika takes out a ball from the bag without looking into it. When the probability that she takes out the a) yellow ball b) red ball c) blue ball.

Solution:

$$S = \{1 \text{ Red}, 1 \text{ blue}, 1 \text{ yellow}\}$$

$$n(S) = 3$$

a) Let A denote the event of getting yellow ball.

$$A = \{1 \text{ yellow}\}$$

$$n(A) = 1$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{1}{3}$$

b) Let B denote the event of getting red ball.

$$B = \{1 \text{ Red}\}$$

$$n(B) = 1$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{1}{3}$$

c) Let C denote the event of getting blue ball.

$$C = \{1 \text{ blue}\}$$

$$n(C) = 1$$

$$\text{Therefore } P(C) = \frac{n(C)}{n(S)} = \frac{1}{3}$$

Example 3: Suppose we throw a die once. (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

Solution:

When a die is thrown, the possible outcomes are

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

(i) Let A denote the event of getting a number greater than 4.

$$A = \{5, 6\}, n(A) = 2$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Let B denote the event of getting a number less than or equal to 4.

$$B = \{1, 2, 3, 4\}, n(B) = 4$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Example 4: One card is drawn from a well shuffled deck of 52 cards. Calculate the probability that the card will a) be an ace b) not be an ace.

Solution:

$$n(S) = 52$$

(i) let A denote the event of getting an ace card.

$$n(A) = 4$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) let \bar{A} denote the event of getting a card not be an ace.

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{1}{13} = \frac{12}{13}$$

Example 5: Two players, Sangeetha and Reshma, play tennis match. It is known that the probability of Sangeetha winning the match is 0.62. What is the probability of Reshma winning the match?

Solution:

Let S and R denote the event that Sangeetha wins the match and Reshma wins the match respectively.

$$\text{Therefore } P(S) = 0.62$$

$$\text{Now } P(B) = 1 - P(S) = 1 - 0.62 = 0.38$$

Example 6: Savita and Hamida are friends what is the probability that both will have (i) different birthdays (ii) the same birthday?

Solution:

Savita's birthday can be any of the day of the year 365 days.

Similarly Hamida's birthday can be any of the day of the year 365 days.

(i) If Hamida's birthday is different from Savita's birthday, number of outcomes for Hamida's birthday is $365 - 1 = 364$.

$$\left. \begin{array}{l} \text{Therefore } P(\text{Hamida's birthday different} \\ \text{from Savita's birthday}) \end{array} \right\} = \frac{364}{365}$$

(ii) $P(\text{Savita and Hamida have the same birthday}) = 1 - P(\text{both have different birthday})$

$$= 1 - \frac{364}{365} = \frac{1}{365}$$

Example 7: There are 40 students in class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?

Solution:

There are 40 students.

Therefore $n(s) = 40$.

Let A denote the event of getting a card with Girl name.

$n(A) = 25$.

$$\text{Therefore } P(A) = \frac{n(A)}{n(s)} = \frac{25}{40} = \frac{5}{8}$$

Let B denote the event of getting a card with Boy name.

$n(B) = 15$.

$$\text{Therefore } P(B) = \frac{n(B)}{n(s)} = \frac{15}{40} = \frac{3}{8}$$

Example 8: A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be a) white b) blue c) red.

Solution:

Number of possible outcomes = $3 + 2 + 4 = 9$

$n(s) = 9$

a) Let W denote the event of getting a white marble.

$n(A) = 2$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{2}{9}$$

b) Let B denote the event of getting a blue marble.

$n(B) = 3$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{3}{9}$$

c) Let R denote the event of getting a white marble.

$n(R) = 4$

$$\text{Therefore } P(R) = \frac{n(R)}{n(S)} = \frac{4}{9}$$

Example 9: Harpreet tosses two different coins simultaneously. What is the probability that she gets atleast one head?

Solution:

When two coins are tossed together the sample space is

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$

$n(S) = 4$

Let A denote the event of getting atleast one Head.

$$A = \{(H, H), (H, T), (T, H)\}$$

$n(A) = 3$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

Example 12: A carton consist of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects, Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton what is the probability that

(i) It is acceptable to Jimmy?

(ii) It is acceptable to Sujatha?

Solution:

$n(S) = 100$.

let A denote the number of outcomes acceptable to Jimmy.

$n(A) = 88$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{88}{100} = 0.88$$

let B denote the number of outcomes acceptable to Sujatha.

$n(B) = 100 - 4 = 96$ or

$$88 + 8 = 96$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{96}{100} = 0.96$$

Example 13: Two dice, one blue and one gray are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8? (ii) 13? (iii) less than or equal to 12?
Solution:

When two dice are thrown, the possible outcomes are

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

$$n(S) = 36$$

(i) let A denote the event of getting sum is 8.

$$A = \{(2,6)(3,5)(4,4)(5,3)(6,2)\}$$

$$n(A) = 5$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

(ii) let B denote the event of getting, sum of the faces is 13.

$$B = \{ \}$$

$$\text{Therefore } n(B) = 0$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{36} = 0$$

(iii) let C denote the event of getting sum is less than or equal to 12.

$$C = \{(1,1)(1,2) \dots \dots (6,6)\}$$

$$n(C) = 36$$

$$\text{Therefore } P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

EXERCISE 15.1

1) Complete the following statements:-

- (i) Probability of an event + probability of the event "not E" = 1
- (ii) The probability of an event that cannot happen is 0 such an event is called impossible event.

- (iii) The probability of an event that is certain to happen is 1 such on event is called sure event.
- (iv) The sum of the probability of all the elementary events of an experiment is 1.
- (v) The probabilities of an event is greater than or equal to 0 and less than or equal to 1.

2) Which of the following experiments have equally likely outcomes. Explain

(i) A driver attempts to start a car. The car starts or does not start.

Ans: not equally likely. Since the car starts normally. If any defect, it does not starts.

(ii) A player attempts to shoot a basketball.

She/he shoots or misses the shot.

Ans: Not equally likely. Since the outcomes depends on training of the player, quality of baseket ball etc.,

(iii) A trial is made to answer a true-false question. The answer is right or wrong.

Ans: The outcomes are True or false question. Therefore It is equally likely.

(iv) A baby is born. It is a boy or girl.

Ans: The outcomes are equally likely. Since new baby is either boy or girl.

3) Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game.

Ans: Tossing a coin is equally likely. We get either head or tail. So, the result is unbiased.

4) Which of the following cannot be the probability of an event?

- (i) $\frac{2}{3}$ (ii) -1.5 (iii) 15% (iv) 0.7

Ans: The probabilities lies between 0 and 1. The event cannot be negative. Therefore Ans is -1.5.

5) If $P(E) = 0.05$, What is the probability of 'not E'.

Solution:

$$P(E) = 0.05$$

$$\text{probability of not E} = P(\bar{E}) = 1 - P(E) = 1 - 0.05$$

$$P(\bar{E}) = 0.95$$

6) A bag contains lemon flavored candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out (i) an orange flavored candy? (ii) a lemon-flavored candy?

Solution:

A bag contains only lemon-flavored candies only is $n(s) = \text{lemon}$. Let A denote the event of getting orange flavored candy

$$A = \{\}$$

$$n(A) = 0$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{0}{n(S)} = 0.$$

let B denote the event of getting lemon flavoured candy.

$$B = \text{lemon flavoured candy}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{n(B)}{n(B)} = 1$$

7) It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Solution:

The probability of 2 students not having the same birthday = 0.992.

Therefore the probability of 2 students have the same birthday = $1 - 0.992 = 0.008$.

8) A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red (ii) not red.

Solution:

$$\text{Sample space } S = \{3R, 5B\}$$

$$n(S) = 8$$

let A denote the event of getting a red ball

$$n(A) = 3$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

let \bar{A} denote the event of getting not red.

$$\text{Therefore } P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}$$

9) A bag contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red (ii) white

(iii) not green.

Solution:

$$\text{Sample space } S = \{5R, 8W, 4G\}$$

$$\text{Therefore } n(S) = 5 + 8 + 4 = 17$$

(i) Let R denote the event of getting red marble.

$$R = 5 \text{ red}$$

$$n(R) = 5$$

$$\text{Therefore } P(R) = \frac{n(R)}{n(S)} = \frac{5}{17}$$

(ii) Let W denote the event of getting white marble.

$$n(W) = 8$$

$$\text{Therefore } P(W) = \frac{n(W)}{n(S)} = \frac{8}{17}$$

(iii) Let G denote the event of getting red marble.

$$n(G) = 4$$

$$\text{Therefore } P(G) = \frac{n(G)}{n(S)} = \frac{4}{17}$$

Therefore \bar{G} denote the event of getting not green

$$\text{Therefore } P(\bar{G}) = 1 - P(G)$$

$$= 1 - \frac{4}{17} = \frac{17-4}{17} = \frac{13}{17}$$

10) A piggy bank contains hundred 50 p coins fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. If it is equally likely that one of the wins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be 50 p coin? (ii) will not be a ₹5 coin?

Solution:

Given:

$$\text{Number of 50 p coin} = 100.$$

$$\text{Number of ₹1 coin} = 50.$$

$$\text{Number of ₹2 coin} = 20.$$

$$\text{Number of ₹5 coin} = 10.$$

$$\text{Therefore total number of coins } n(S) = 180.$$

(i) Let A denote the event of getting 50 p coin.

$$\text{Therefore } n(A) = 100.$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{100}{180} = \frac{10}{18} = \frac{5}{9}$$

(ii) Number of coins which are not ₹5 = Total number of coins – Number of ₹5 coin = $180 - 10 = 170$.

$$\text{Therefore the probability that the coin will not be a ₹5} = \frac{170}{180} = \frac{17}{18}$$

- 11) Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?

Solution:

Number fish in the tank = 5 male + 8 female
= 13 fish.

$$n(S) = 13.$$

let A denote the event of getting a male fish =

$$n(A) = 5$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{5}{13}.$$

- 12) A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. What is the probability that it will point at a) 8 b) an odd numbers c) a number greater than 2? d) a number less than 9?

Solution:

Sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$n(S) = 8$$

- a) Let A denote the event of getting the number 8.

$$n(A) = 1$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

- b) Let B denote the event of getting arrow at an odd number.

$$B = \{1, 3, 5, 7\}$$

$$n(B) = 4$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

- c) Let C denote the event of getting arrow at a number greater than 2.

$$C = \{3, 4, 5, 6, 7, 8\}$$

$$n(C) = 6$$

$$\text{Therefore } P(C) = \frac{n(C)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

- d) Let D denote the event of getting arrow at less than 9.

$$D = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$n(D) = 8$$

$$\text{Therefore } P(D) = \frac{n(D)}{n(S)} = \frac{8}{8} = 1$$

- 13) A die is thrown once. Find the probability of getting.

- a) a prime number b) a number lying, between 2 and 6 c) an odd number. (2020, 19)

Solution:

When a die is thrown, the Sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6.$$

- a) let A denote the event of getting a prime number.

$$A = \{2, 3, 5\}$$

$$\text{Therefore } n(A) = 3$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

- b) let B denote the event of getting a number lying between 2 and 6.

$$\text{Therefore } B = \{3, 4, 5\}$$

$$\text{Therefore } n(B) = 3$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

- c) let C denote the event of getting an odd number.

$$C = \{1, 3, 5\}$$

$$\text{Therefore } n(C) = 3$$

$$\text{Therefore } P(C) = \frac{n(C)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

- 14) One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting

- a. a king of red colour.
b. a face card.
c. a red face card.
d. the Jack of hearts.
e. a spade.
f. the queen of diamonds. (2015)

Solution:

Number of cards = 52.

$$n(S) = 52.$$

- a) Let A denote event of getting a king of red colour.

$$\text{Number of red king} = n(A) = 2.$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

- b) Let B denote event of getting of face card.

$$[\text{face card} = 4 \text{ king} + 4 \text{ Jacks} + 4 \text{ queen}]$$

$$= n(B) = 12.$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

- c) Let C denote event of red face card. [Red face card = 2 king + 2 Jacks + 2 queen]

$$= n(C) = 6.$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

- d) Let D denote event of getting a Jack of heart.
[There are 4 Jack cards namely 1 heart + 1 club + 1 space + 1 diamond]

$$= n(D) = 1.$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{52}$$

- e) Let E denote event of getting a spade. [In 52 cards 13 spades, 13 clubs, 13 hearts, 13 diamonds]

$$= n(E) = 13.$$

$$\text{Therefore } P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

- f) Let F denote event of getting the queen of diamonds.

$$= n(F) = 1.$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{1}{52}.$$

- 15) Five cards – the ten, Jack, queen, King, and ace of diamonds, are well-shuffled with their facer downwards. One card is then picked up at random.

- a) What is the probability that the card is the queen?
b) If the queen is drawn and put aside what is the probability that the second card picked up is (i) an ace (ii) a queen? (2014)

Solution:

Total number of cards = 5

$$n(S) = 5$$

- a) Let A denote the event of getting a queen card.

$$n(A) = 1$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{1}{5}$$

- b) Queen card is drawn and put a side. Now total number of cards = 4.

$$n(S) = 4.$$

- (i) let B denote the event of getting an ace card.

$$n(B) = 1$$

$$\text{Therefore } P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}.$$

- (ii) let C denote the event of getting a queen card.

$$n(C) = 0$$

$$\text{Therefore } P(C) = \frac{n(C)}{n(S)} = \frac{0}{4} = 0.$$

(Queen card is already drawn and put a side).

- 16) 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at a random from this lot. Determine the probability that the pen taken out is a good one.

Solution:

Number of good pens = 132.

Number of defective pens = 12.

Therefore total number of pens = $n(S) = 144$.

Let G denote the event of getting a good pen.

$$n(G) = 132.$$

$$\text{Therefore } P(G) = \frac{n(G)}{n(S)} = \frac{132}{144} = \frac{11}{12}.$$

- 17) (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that the bulb is defective.

Solution:

Total number of bulbs $n(S) = 20$.

- (i) Let D denote the event of getting a defective bulb.

$$n(D) = 4$$

$$\text{Therefore } P(D) = \frac{n(D)}{n(S)} = \frac{4}{20} = \frac{1}{5}.$$

- (ii) Suppose the bulb drawn in (a) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective.

Given drawn bulb is not defective and is not replaced.

Therefore the number of bulbs without replacement is 19.

$$n(S) = 19.$$

Number of defective bulbs = 4

$$n(D) = 4.$$

Now number of good bulbs = $19 - 4 = 15$.

$$n(G) = 15.$$

Let G denote the event of getting good bulb.

$$\text{Therefore } P(G) = \frac{n(G)}{n(S)} = \frac{15}{19}.$$

- 18) A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bear (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

Solution:

Total number of discs in the box = $n(S) = 90$

[numbered from 1 to 90].

let A denote the event of getting a two digit number.

Therefore $A = \{10, 11, 12 \dots 90\}$

$$n(A) = 81 \text{ [90 – single digit number is } 90 - 9 = 81]$$

Therefore $P(A) = \frac{n(A)}{n(S)} = \frac{81}{90} = \frac{9}{10}$

let B denote the event of getting a perfect square number.

Therefore $B = \{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

$$n(B) = 9$$

Therefore $P(B) = \frac{n(B)}{n(S)} = \frac{9}{90} = \frac{1}{10}$

let C denote the event of getting a number divisible by 5.

$C = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90\}$

Therefore $n(C) = 18$

$$P(C) = \frac{n(C)}{n(S)} = \frac{18}{90} = \frac{2}{10} = \frac{1}{5}$$

19) A child has a die whose six faces show the letters as given below.

A	B	C	D	E	A
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The die is thrown once. What is the probability of getting (i) A? (ii) D?

Solution:

When a die is thrown once. The possible outcomes are $S = \{A, B, C, D, E, A\}$

$$n(S) = 6.$$

Let A denote the event of getting the letter A.

$$A = \{A, A\}, n(A) = 2$$

Therefore $P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Let B denote the event of getting the letter D.

$$B = \{D\} \quad n(B) = 1$$

Therefore $P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$

20) Suppose you a drop a die at random on the rectangular region shown in the figure. What is the probability that it will land inside the circle with diameter 1 m?

Solution:

Area of rectangle = $3 \times 2 = 6 \text{ m}^2$

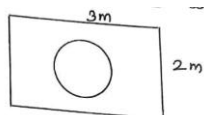
Given diameter of circle = 1 m

Therefore radius of the circle = $\frac{1}{2} \text{ m}$.

Area of circle = πr^2

$$= \pi \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ m}^2$$

Now the probability that the die land inside the



$$\begin{aligned} \text{circle} &= \frac{\text{Area of circle}}{\text{Area of rectangle}} \\ &= \frac{\pi}{6} = \frac{\pi}{24} \end{aligned}$$

21) A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) she will buy it? (ii) she will not buy it?

Solution:

Total number of balls = 144

$$n(S) = 144$$

Number of defective pens = 20

Therefore Number of good pens = $144 - 20 = 124$.

(i) let A denote the event of getting good pen.

$$n(A) = 124$$

Therefore $P(A) = \frac{n(A)}{n(S)} = \frac{124}{144} = \frac{31}{36}$

(ii) Let B denote the event of getting defective pens.

$$n(B) = 20$$

Therefore $P(B) = \frac{n(B)}{n(s)} = \frac{20}{144} = \frac{5}{36}$

22) A game consist of tossing a one rupee coin 3 times and nothing its outcome each time. Hanif wins if all the tosses give the same result. Three heads of three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution:

A coin is tossed three times. The possible outcomes are

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$n(S) = 8$$

let A denote the event that the Hanif will lose the game.

All the tosses do not have the same result.

Therefore $A = \{HHT, HTH, THH, HTT, THT, TTH\}$

$$n(A) = 6$$

Therefore $P(A) = \frac{n(A)}{n(S)} = \frac{6}{8} = \frac{3}{4}$

- 23) A die is thrown twice. What is the probability that
 (i) 5 will not come up either time. (ii) 5 will come up atleast once.

Solution:

Given a die is thrown twice.

Therefore the possible outcomes are

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

$$n(S) = 36.$$

- (i) Let A denote the event of getting 5 will come atleast one.

$$A = \left\{ \begin{array}{l} (1,5), (2,5), (3,5), (4,5), (5,5), (6,5) \\ (5,1), (5,2), (5,3), (5,4), (5,6) \end{array} \right\}$$

$$n(A) = 11$$

$$\text{Therefore } P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}.$$

- (ii) Let \bar{A} denote the event of getting 5 will not come up either time.

$$\text{Therefore } P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{11}{36} \Rightarrow \frac{36-11}{36} = \frac{25}{36}$$
